

The following example is from [Trappe & Washington \(2006\)](#), with some details added.

We want to factor $n = 455839$. Let's choose the elliptic curve $y^2 = x^3 + 5x - 5$, with the point $P = (1, 1)$ on it, and let's try to compute $(10!)P$.

The slope of the tangent line at some point $A=(x, y)$ is $s = (3x^2 + 5)/(2y) \pmod{n}$. Using s we can compute $2A$. If the value of s is of the form a/b where $b > 1$ and $\gcd(a,b) = 1$, we have to find the [modular inverse](#) of b . If it does not exist, $\gcd(n,b)$ is a non-trivial factor of n .

First we [compute \$2P\$](#) . We have $s(P) = s(1,1) = 4$, so the coordinates of $2P = (x', y')$ are $x' = s^2 - 2x = 14$ and $y' = s(x - x') - y = 4(1 - 14) - 1 = -53$, all numbers understood $\pmod{n}$. Just to check that this $2P$ is indeed on the curve: $(-53)^2 = 2809 = 14^3 + 5 \cdot 14 - 5$.

Then we compute $3(2P)$. We have $s(2P) = s(14, -53) = -593/106 \pmod{n}$. Using the [Euclidean algorithm](#):

$455839 = 4300 \cdot 106 + 39$, then $106 = 2 \cdot 39 + 28$, then $39 = 28 + 11$, then $28 = 2 \cdot 11 + 6$, then $11 = 6 + 5$, then $6 = 5 + 1$. Hence $\gcd(455839, 106) = 1$, and working backwards (a version of the [extended Euclidean algorithm](#)): $1 = 6 - 5 = 2 \cdot 6 - 11 = 2 \cdot 28 - 5 \cdot 11 = 7 \cdot 28 - 5 \cdot 39 = 7 \cdot 106 - 19 \cdot 39 = 81707 \cdot 106 - 19 \cdot 455839$. Hence $106^{-1} = 81707 \pmod{455839}$, and

$-593/106 = -133317 \pmod{455839}$. Given this s , we can compute the coordinates of $2(2P)$, just as we did above:

$4P = (259851, 116255)$. Just to check that this is indeed a point on the curve: $y^2 = 54514 = x^3 + 5x - 5 \pmod{455839}$. After this, we can compute $3(2P) = 4P \boxplus 2P$.

We can similarly compute $4!P$, and so on, but $8!P$ requires inverting $599 \pmod{455839}$. The Euclidean algorithm gives that 455839 is divisible by 599 , and we have found a factorization $455839 = 599 \cdot 761$.

The reason that this worked is that the curve $\pmod{599}$ has $640 = 2^7 \cdot 5$ points, while $\pmod{761}$ it has $777 = 3 \cdot 7 \cdot 37$ points. Moreover, 640 and 777 are the smallest positive integers k such that $kP = \infty$ on the curve $\pmod{599}$ and $\pmod{761}$, respectively. Since $8!$ is a multiple of 640 but not a multiple of 777 , we have $8!P = \infty$ on the curve $\pmod{599}$, but not on the curve $\pmod{761}$, hence the repeated addition broke down here, yielding the factorization.