

Exercise Sheet 2

Introduction to Partial Differential Equations (W. S. 2024/25)

EPFL, Mathematics section, Dr. Nicola De Nitti

- The exercise series are published every Tuesday morning at 8am on the moodle page of the course. The exercises can be handed in until the following Tuesday at 8am, via moodle.

Exercise 1. Let $\Omega \subset \mathbb{R}^n$ be an open bounded domain. Let $(u_m)_{m \in \mathbb{N}}$ be a sequence of harmonic functions in Ω , converging uniformly to u . Prove that u is harmonic in Ω .

Exercise 2. Prove that the zeros of a harmonic function in $\mathbb{R}^n$ (with $n \geq 2$) cannot be isolated.

Exercise 3. Let $\Omega \subset \mathbb{R}^n$ be an open bounded domain. Given a harmonic function u , prove that

- (i) u^2 is sub-harmonic;
- (ii) $|\nabla u|^2$ is sub-harmonic.

Exercise 4. Let u be harmonic in $\mathbb{R}^n$ (with $n \geq 2$).

- (i) Prove that, if $\int_{\mathbb{R}^n} u^2 < \infty$, then $u = 0$.
- (ii) Prove that, if $\int_{\mathbb{R}^n} |\nabla u|^2 < \infty$, then u is constant.