



Teacher: Michel Bierlaire

Cours: Introduction to optimization and operations research

EPFL

Tuesday 30 January 2024 (9:15 - 12:15)

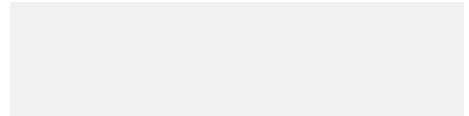
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Dhakane Advait Nilesch

Part 1: multiple choice questions

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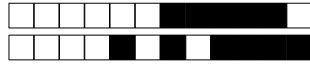


Wait until the beginning of the exam before turning the page. This document is printed double-sided and has 18 multiple choice questions. Each question has exactly one correct answer. Do not remove the staple. The exam contains 12 pages.

Carefully read the following instructions:

- Place your student card on the table.
- You are allowed to have a **4 pages** (8 sides) **handwritten** summary. Any summary which is not handwritten cannot be used and **will be confiscated**. The summary must clearly show your name and today's date. The summary will be collected at the end of the exam.
- Draft paper is available, so **please do not write on the exam**. Calculators and electronic devices are **not allowed**.
- The exam is corrected electronically. Use a **pen (not a pencil) with black or dark blue ink** and avoid altering and erasing your answers if possible. If you must change an answer, use correction tape (avoid correction fluid). Do not draw an empty box.
- The grading scheme for the multiple choice questions is:
 - +1.5 point for a correct answer,
 - 0 points if no answer is given,
 - −0.5 point for a wrong answer.
- If a question contains a mistake, the teacher can remove it from the exam.
- Follow these guidelines for **marking your answers**:

Respectez les consignes suivantes Read these guidelines Beachten Sie bitte die unten stehenden Richtlinien		
choisir une réponse select an answer Antwort auswählen	ne PAS choisir une réponse NOT select an answer NICHT Antwort auswählen	Corriger une réponse Correct an answer Antwort korrigieren
  		 
ce qu'il ne faut PAS faire what should NOT be done was man NICHT tun sollte		
     		



Question 1 : In a variant of the Travelling Salesman Problem, let x_{ij} be a binary variable that indicates whether or not we travel from city i to city j and introduce parameters ω_{ij} that equal 1 if and only if a one-way street allows travel from city i to city j , but not from j to i , and 0 if there are no restrictions from city j to city i . To not completely block connections, it holds that $\omega_{ij} + \omega_{ji} \leq 1$ for each pair (i, j) . Considering this setting, how can the constraints be modeled to ensure that the directionality of one-way streets is respected?

- ☐ For each pair of cities (i, j) , $x_{ij} + x_{ji} \geq \omega_{ij}$.
- ☐ For each pair of cities (i, j) , $x_{ji} \leq 1 - \omega_{ij}$.
- ☐ For each pair of cities (i, j) , $x_{ij} = \omega_{ij}$ and $1 - x_{ji} = 1 - \omega_{ij}$.
- ☐ For each pair of cities (i, j) , $x_{ij} \leq \omega_{ij}$.

Question 2 : Consider the following feasible solution to a linear optimization problem $x = (0, 0, 0, \frac{1}{3})^T$. Based on this information, what can we conclude about the dual problem?

- ☐ The dual is not unbounded.
- ☐ The dual is feasible.
- ☐ The dual is unbounded.
- ☐ The dual is infeasible.

Question 3 : Which one of the following conditions is a necessary condition for the constraint matrix of a transshipment problem to be totally unimodular?

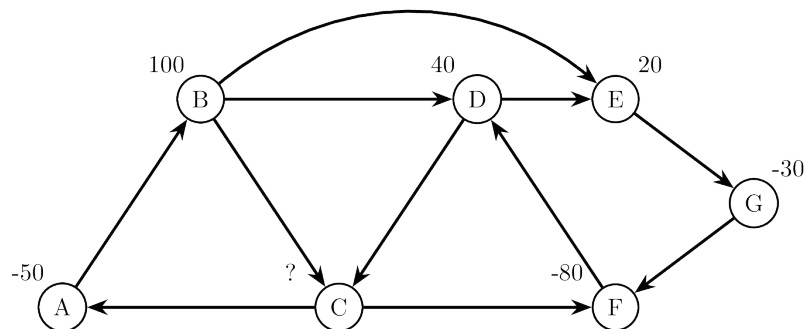
- ☐ The supply/demand values must be integer.
- ☐ The supply/demand values must verify the capacity constraints.
- ☐ Each entry of the incidence matrix must be either 0, 1, or -1.
- ☐ The supply/demand values must be zero.

Question 4 : A manager in a company composed of 100 workers has to dispatch 100 tasks, such that each worker has one task to complete. Each worker has different skills and efficiency in performing each task. The goal of the manager is to minimize the total cost of completing all tasks. To which instance of the transshipment problem seen in class does this problem correspond? Select the **correct** answer.

- ☐ The shortest path problem.
- ☐ The maximum flow problem.
- ☐ The transportation problem.
- ☐ The assignment problem.



Question 5 : Consider the following transshipment network:



What is the value of the missing divergence at node C? Select the **correct** answer.

- ☐ 5
- ☐ -10
- ☐ 0
- ☐ 10

Question 6 : Which of the following statements is **wrong**?

- ☐ Any graph G that is connected and without cycles is a tree.
- ☐ A tree with n arcs has $m = n + 1$ nodes.
- ☐ Adding any arc to a tree disconnects it.
- ☐ Adding any arc to a tree creates a cycle.

Question 7 : In the context of linear optimization, a basic feasible solution (BFS) is considered degenerate if:

- ☐ All the decision variables are non-zero.
- ☐ There are more constraints than variables in the linear optimization problem.
- ☐ At least one of the basic variables is equal to zero.
- ☐ The objective function value is negative.

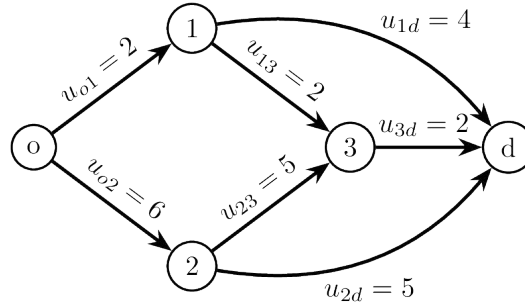
Question 8 : The big issue when dealing with combinatorial (integer) optimization problems is the lack of optimality conditions combined with the fact that the time necessary to enumerate all

- ☐ constraints
- ☐ optimal solutions
- ☐ variables
- ☐ feasible solutions

grows exponentially with the size of the problem.



Question 9 : Here is a network. Next to each arc we find its capacity constraint. The objective is to transport the maximum quantity of flow from node o to node d.



Which of the mathematical models correctly defines the associated problem? Select the **correct** answer.

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$$\begin{aligned}
 \max \quad & u_{o1}x_{o1} + u_{o2}x_{o2} + u_{13}x_{13} + u_{23}x_{23} + \\
 & u_{1d}x_{1d} + u_{2d}x_{2d} + u_{3d}x_{3d} \\
 \text{s.t.} \quad & x_{o1} + x_{o2} - x_{do} = 0 \\
 & x_{1d} + x_{13} - x_{o1} = 0 \\
 & x_{23} + x_{2d} - x_{o2} = 0 \\
 & x_{3d} - x_{13} - x_{23} = 0 \\
 & x_{do} - x_{1d} - x_{2d} - x_{3d} = 0 \\
 & x_{ij} \geq 0, \forall ij \in \mathcal{A} \cup (d, o)
 \end{aligned}$$

$$\begin{aligned}
 \max \quad & x_{o1} + x_{o2} + x_{13} + x_{23} + x_{1d} + \\
 & x_{2d} + x_{3d} \\
 \text{s.t.} \quad & x_{o1} + x_{o2} - x_{do} = 0 \\
 & x_{1d} + x_{13} - x_{o1} = 0 \\
 & x_{23} + x_{2d} - x_{o2} = 0 \\
 & x_{3d} - x_{13} - x_{23} = 0 \\
 & x_{do} - x_{1d} - x_{2d} - x_{3d} = 0 \\
 & x_{ij} \leq u_{ij}, \forall ij \in \mathcal{A} \\
 & x_{ij} \geq 0, \forall ij \in \mathcal{A} \cup (d, o)
 \end{aligned}$$

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$$\begin{aligned}
 \max \quad & x_{o1} + x_{o2} + x_{13} + x_{23} + x_{1d} + \\
 & x_{2d} + x_{3d} \\
 \text{s.t.} \quad & x_{1d} + x_{13} - x_{o1} = 0 \\
 & x_{23} + x_{2d} - x_{o2} = 0 \\
 & x_{3d} - x_{13} - x_{23} = 0 \\
 & x_{ij} \leq u_{ij}, \forall ij \in \mathcal{A} \\
 & x_{ij} \geq 0, \forall ij \in \mathcal{A} \cup (d, o)
 \end{aligned}$$

$$\begin{aligned}
 \max \quad & x_{do} \\
 \text{s.t.} \quad & x_{o1} + x_{o2} - x_{do} = 0 \\
 & x_{1d} + x_{13} - x_{o1} = 0 \\
 & x_{23} + x_{2d} - x_{o2} = 0 \\
 & x_{3d} - x_{13} - x_{23} = 0 \\
 & x_{do} - x_{1d} - x_{2d} - x_{3d} = 0 \\
 & x_{ij} \leq u_{ij}, \forall ij \in \mathcal{A} \\
 & x_{ij} \geq 0, \forall ij \in \mathcal{A} \cup (d, o)
 \end{aligned}$$

Question 10 :

Consider the function $f(x) = x_1^2 + (a+1)x_1x_2 + 4x_2^2 + 2ax_1 + x_2$ and choose the **correct** statement:

- ☐ For $a < 0$ the function f is not convex.
- ☐ There exists a unique $a \in \mathbb{R}$ for which f is strictly convex.
- ☐ There exists $a \in \mathbb{R}$ such that function f is convex, but not strictly convex.
- ☐ The function f is convex $\forall a > 0$.

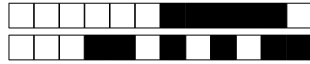
**Question 11 :**

Consider a function $f(x) = x_1^2 + 2x_1x_2 + x_2^2$. Which of the following statements is **correct**?

- ☐ The Newton method converges to the minimum of the function in one iteration regardless of the initial point.
- ☐ The Newton method converges to the minimum of the function in multiple iterations if the starting point is not too far from the solution.
- ☐ The Newton method converges to the minimum of the function in one iteration if the starting point is not too far from the solution.
- ☐ The Newton method converges to the minimum of the function in multiple iterations regardless of the initial point.

Question 12 : Consider a linear optimization problem with n variables and m constraints. A feasible basic solution (x^*) is degenerate if:

- ☐ Less than n constraints are active at x^* .
- ☐ More than n constraints are active at x^* .
- ☐ Less than m constraints are active at x^* .
- ☐ More than m constraints are active at x^* .



Question 13 : Consider the following optimization problem.

$$\begin{aligned} \max & x_1 + 2x_2 + 4x_3 \\ & 2x_1 + x_3 \geq 10 \\ & x_1 + 4x_2 \leq 16 \\ & x_3 - x_2 \leq 5 \\ & x_1 \geq 0 \\ & x_2 \geq 0 \\ & 1 \leq x_3 \leq 4 \end{aligned}$$

Which of the following linear models correspond to the model in standard form?



$$\begin{aligned} \min & -x_1 - 2x_2 - 4x_3' - 4 \\ & 2x_1 + x_3' - e_1 = 9 \\ & x_1 + 4x_2 + e_2 = 16 \\ & x_3' - x_2 + e_3 = 4 \\ & x_3' + e_4 = 3 \\ & x_1, x_2, x_3', e_1, e_2, e_3, e_4 \geq 0 \end{aligned}$$



$$\begin{aligned} \max & -x_1 - 2x_2 - 4x_3' \\ & 2x_1 + x_3' - e_1 = 9 \\ & x_1 + 4x_2 + e_2 = 16 \\ & x_3' - x_2 + e_3 = 4 \\ & x_3' + e_4 = 3 \\ & x_1, x_2, x_3', e_1, e_2, e_3, e_4 \geq 0 \end{aligned}$$



$$\begin{aligned} \min & -x_1 - 2x_2 - 4x_3' + 4 \\ & 2x_1 + x_3' - e_1 = 11 \\ & x_1 + 4x_2 + e_2 = 16 \\ & x_3' - x_2 + e_3 = 6 \\ & x_3' + e_4 = 5 \\ & x_1, x_2, x_3', e_1, e_2, e_3, e_4 \geq 0 \end{aligned}$$



$$\begin{aligned} \min & -x_1 - 2x_2 - 4x_3 \\ & 2x_1 + x_3 - e_1 = 10 \\ & x_1 + 4x_2 + e_2 = 16 \\ & x_3 - x_2 + e_3 = 5 \\ & x_1, x_2, x_3, e_1, e_2, e_3 \geq 0 \end{aligned}$$

Question 14 : Consider the following optimization problem:

$$\begin{aligned} \min_{x \in \mathbb{R}} & 5x + 8, \\ \text{subject to:} & \\ & x^2 \leq 4 \end{aligned}$$

Which of the following is **correct** regarding the existence or non-existence of an optimal solution for this problem?



Any feasible x is optimum.



The problem is unbounded.



The optimal value is 18, and the optimum solution is $x = 2$.



The optimal value is -2 , and the optimum solution is $x = -2$.



Question 15 : A coffee producer blends Brazilian coffee and Colombian coffee to prepare two products: Super Coffee and Deluxe Coffee. We define the following variables: x_1 is the amount of Super Coffee prepared, and x_2 is the amount of Deluxe Coffee prepared. Each kilogram of Super Coffee contains 0.5 kg of Brazilian coffee and 0.5 kg of Colombian coffee. Each kilogram of Deluxe Coffee contains 0.3 kg of Brazilian coffee and 0.7 kg of Colombian coffee. The coffee producer has an availability of 140 kg of Brazilian coffee and 170 kg of Colombian coffee. The profit for each kilogram of Super Coffee is 20 cents and the profit for each kilogram of Deluxe Coffee is 30 cents. The coffee producer wants to determine how many kilograms of each type of blended coffee should be prepared to maximize the profit. Which of the following optimization problems correctly models the problem?

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$$\begin{aligned} \max & 0.2x_1 + 0.3x_2 \\ 0.5x_1 + 0.3x_2 & \leq 140 \\ 0.5x_1 + 0.7x_2 & \leq 170 \\ x_1, x_2 & \geq 0 \end{aligned}$$

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$$\begin{aligned} \max & 0.2x_1 + 0.3x_2 \\ 0.5x_1 + 0.5x_2 & \leq 140 \\ 0.3x_1 + 0.7x_2 & \leq 170 \\ x_1, x_2 & \geq 0 \end{aligned}$$

☐

$$\begin{aligned} \min & 20x_1 + 30x_2 \\ 0.5x_1 + 0.3x_2 & \leq 140 \\ 0.5x_1 + 0.7x_2 & \leq 170 \\ x_1, x_2 & \geq 0 \end{aligned}$$

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$$\begin{aligned} \max & 20x_1 + 30x_2 \\ 0.5x_1 + 0.5x_2 & \leq 140 \\ 0.3x_1 + 0.7x_2 & \leq 170 \\ x_1, x_2 & \geq 0 \end{aligned}$$

Question 16 : Consider a linear minimization problem. Which one of the following statements is always correct?

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At any vertex of the polyhedron which represents the feasible region, all basic directions are feasible.

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Any feasible direction in a feasible basic solution can be written as a linear combination of the basic directions.

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For each feasible basic solution, there exists at least one feasible basic direction.

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If the linear optimization problem contains only equality constraints, the problem cannot be unbounded.

Question 17 : Consider a linear optimization problem $P = \{\min c^T x : Ax = b, x \geq 0\}$ and its dual problem $D = \{\max b^T \lambda : A^T \lambda = c\}$. If D' is the dual of D , then which of the following statements is correct?

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D' is the same problem as D .

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If P is unbounded then D' is infeasible.

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The matrix of D' is totally unimodular.

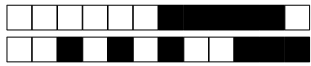
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D' is the same problem as P .

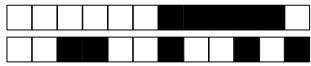


Question 18 : In the simplex method, Bland's rule consists of selecting the non-basic variable with the smallest index and negative reduced cost to enter the tableau first. For what reason is Bland's rule used ?

- ☐ To ensure that the simplex algorithm will terminate in a finite number of iterations.
- ☐ To avoid degenerate vertices.
- ☐ To improve the objective value by the largest possible amount at each iteration.
- ☐ To minimize the total number of iterations that are needed to find the optimal solution.



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