Numerical Analysis
GC / SIE
Linear Systems

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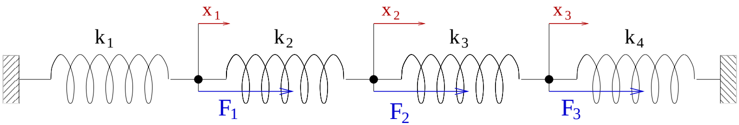
A simple example

Let us consider a configuration of springs under some forces. We want to compute the elongation of each spring.



The equilibrium of forces at every node gives

Linear system of 3 equations and 3 unknowns: $K\mathbf{x} = \mathbf{F}$

After some work, it is possible to obtain a solution by hand.

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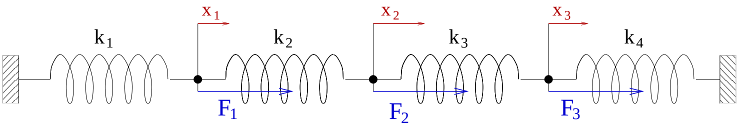

The equilibrium of forces at every node gives

$$\begin{cases} k_1 x_1 + k_2(x_1 - x_2) = F_1 \\ k_2(x_2 - x_1) + k_3(x_2 - x_3) = F_2 \\ k_3(x_3 - x_2) + k_4 x_3 = F_3 \end{cases}$$

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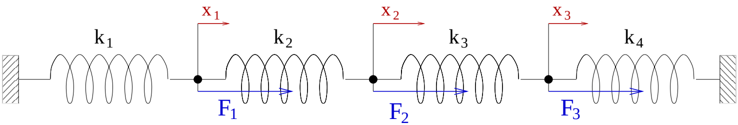

The equilibrium of forces at every node gives

$$\Rightarrow \quad \begin{cases} (k_1 + k_2)x_1 - k_2 x_2 = F_1 \\ -k_2 x_1 + (k_2 + k_3)x_2 - k_3 x_3 = F_2 \\ - k_3 x_2 + (k_3 + k_4)x_3 = F_3 \end{cases}$$

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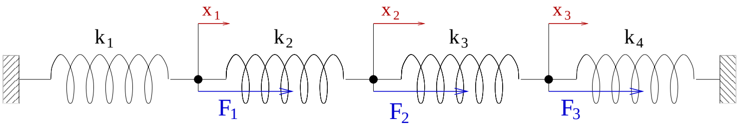

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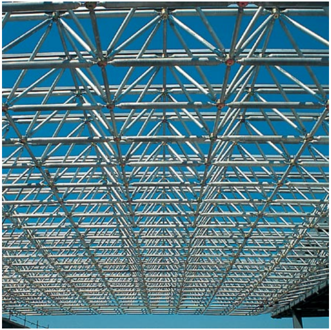
A more complex application



By imposing the equilibrium of forces at each node, we get a large linear system of equations (static case) or a large system of ordinary differential equations system (dynamic case). Hopeless to solve by hand. Need to use numerical algorithms!

- Numerical solution of linear systems.
- Discretization of differential equations (dynamic case).

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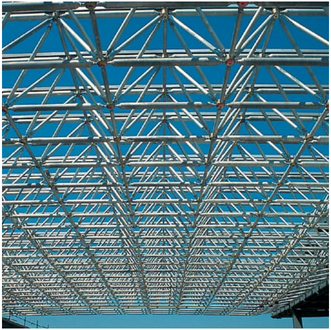

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More complex applications – structural analysis

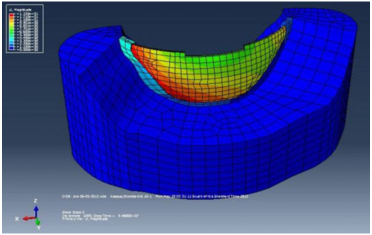

Photo credit: CoMSIRU, University of Cape Town

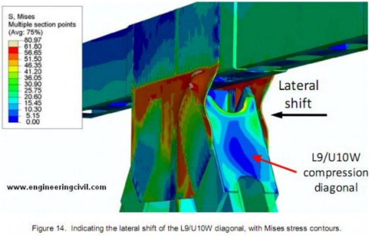

Photo credit: www.engineeringcivil.com

The structure is divided in many small “cubes” (finite elements). By imposing the equilibrium of forces at each one of them a large linear system is obtained (it may have million of unknowns).

Dealing with such an application relies on the following ingredients:

- Approximation / Interpolation.
- Discretizations of differential equations.
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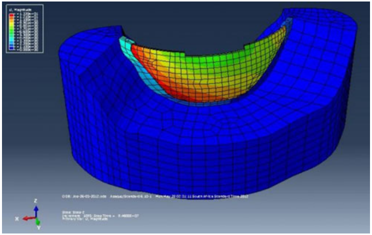

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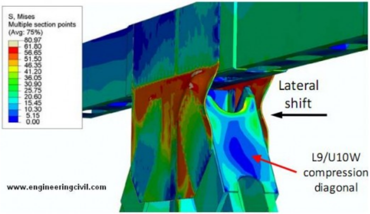

Figure 14. Indicating the lateral shift of the L9/U10W diagonal, with Mises stress contours.

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