Numerical Analysis
GC / SIE
Curve Fitting

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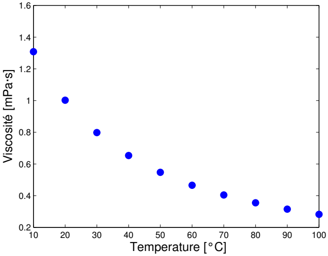
Example 1: Water viscosity

Temperature [°C]	Viscosity [mPa·s]
10	1.308
20	1.002
30	0.7978
40	0.6531
50	0.5471
60	0.4658
70	0.4044
80	0.3550
90	0.3150
100	0.2822

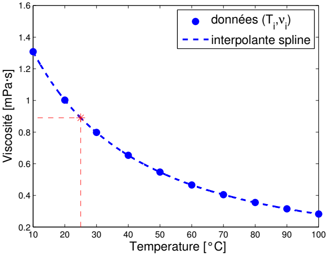
Source: Wikipedia



But we want to calculate the water viscosity at 25 °C.

Data interpolation

Goal: Find a function $\nu = p(T)$ that correctly describes the relation between viscosity and temperature.



Data interpolation problem: Find a function $p(T)$ (from a class of functions) such that

$$\nu_i = p(T_i)$$

where (ν_i, T_i) are the values from the table.

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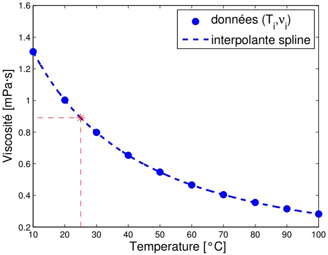

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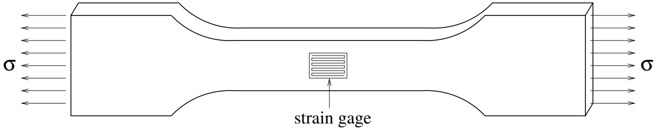
Example 2: Dog bone tensile test

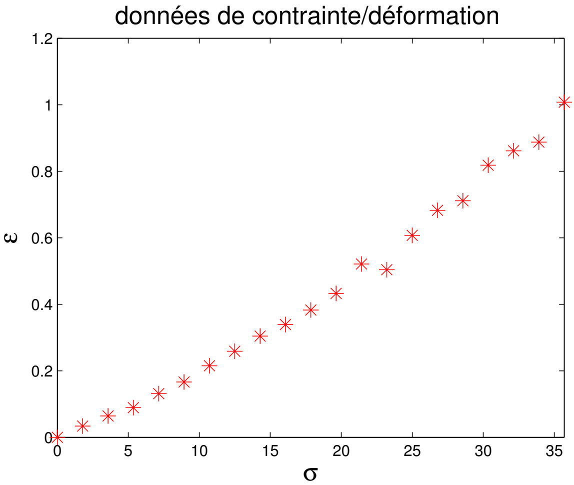


- We apply a stress σ and measure the corresponding strain ε with a strain gauge.
- We apply increasingly larger stresses σ_i, $i = 1, \ldots, n$, equidistant between 0 and σ_{max}, and measure the corresponding strains ε_i.
- Measurement error is not negligible. *Conceptual model*

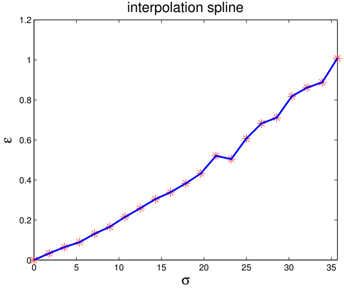
$$\varepsilon_i = f(\sigma_i) + \eta_i$$

where η_i describes measurement error.

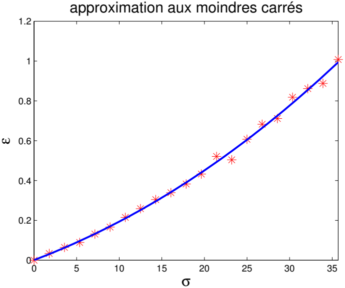
Measured data (with measurement error)



Measured data (with measurement error)



spline interpolation



least-squares approximation

Data approximation with least-squares

- We look for an approximation $p(\sigma)$ of a "true" (unknown) function $f(\sigma)$ that "best" approximates the data $(\sigma_i, \varepsilon_i)$
- In this case, finding an *interpolating* function is not a good idea because it will also interpolate the measurement error.

Least-square approximation problem: We look for a function p, from a given class of functions $\mathcal{W}$, such that the squared distance between $p(\sigma_i)$ and the corresponding measurements ε_i will be *minimal*.

$$p = \operatorname*{argmin}_{q \in \mathcal{W}} \sum_{i=1}^{n} |\varepsilon_i - q(\sigma_i)|^2$$

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