Numerical Analysis
GC / SIE
Introduction

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EPFL

Outline

- Organization
- Why Numerical Analysis?
- A quick tour through the lecture

Organization

Lectures

- Lecturer:
  Prof. Daniel Kressner
  MATH, EPFL
  MA B2 514
  daniel.kressner@epfl.ch

  Field of research:
  numerical linear algebra and high-performance computing

  Best way to contact me: Send an e-mail!

- Room and date:
  SG 0211, Thursday 8:15 – 10:00.
  - First lecture: 12.09.2024
  - Last lecture: 19.12.2024

Recommended Literature

The material discussed in class room, the lecture notes, and the exercises are sufficient.

- The lecture notes are available on Moodle, in English.
- Repro@EPFL has offered to print these lecture notes on demand. (Let me know if this does not work out.)
- There is also a slightly outdated French version, for your convenience. The official reference is the English version.
- The lectures will follow the lecture notes closely.

Supplementary literature:

- A. Quarteroni, F. Saleri, P. Gervasio, *Scientific Computing with MATLAB and OCTAVE*, 4th edition, Springer, 2014
- A. Quarteroni, F. Saleri, and P. Gervasi. *Calcul Scientifique.* 2nd edition. Springer, 2010.
- J. Rappaz, M. Picasso, *Introduction à l'analyse numérique*, PPUR, 2010.
- M. Picasso, *Analyse Numérique pour Ingénieurs*, online course COURSERA, `https://www.coursera.org/course/analysenumerique`

The library has online versions of Springer books.

Exercises

- Room and Date:
  INF 3, Thursday, 10h15 - 11h00
- Assistants:

Assistant	Office	E-mail
Fabio Matti	MA B2 514	fabio.matti@epfl.ch
Andrea Eric		andrea.eric@epfl.ch
Giulia Dohy		giulia.dohy@epfl.ch
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  The assistants will be happy to answer questions during the exercise sessions and also by e-mail. However, for the benefit of everyone, it would be best to post your questions on the **Ed Discussion Forum** (link on moodle)
- Each exercise session will be a combination of theory questions and PYTHON practice (except for today).
- You will find detailed solution sheets on Moodle about one week later.
- Please take the exercises seriously![1]

[1] At least as seriously as the lectures.

Python

- During this course we will study many algorithms that will allow us to solve mathematical problems with the help of a computer.
- The use of Python is an important part of the exercises and the exam.
- However, this is **NOT** a coding course. You will not be asked to code complex algorithms. A large part of the algorithms will be provided by us.
- Follow the instructions on Moodle concerning Python/Jupyter/Noto.
- **Today's exercise session** is meant as an intro/recap of Python concepts important for this course.

Learning Outcomes

- Choose a numerical method for solving a specific problem.
- Interpret obtained numerical results from a theoretical perspective.
- Estimate numerical errors.
- Prove theoretical properties of numerical methods.
- Implement simple numerical algorithms with Python.
- Apply numerical algorithms to specific problems.
- Describe numerical methods.
- State theoretical properties of mathematical problems and numerical methods.

Fundamental concepts we will talk a lot about:

- **stability** of an algorithm
- **precision** of an algorithm
- **cost** of an algorithm

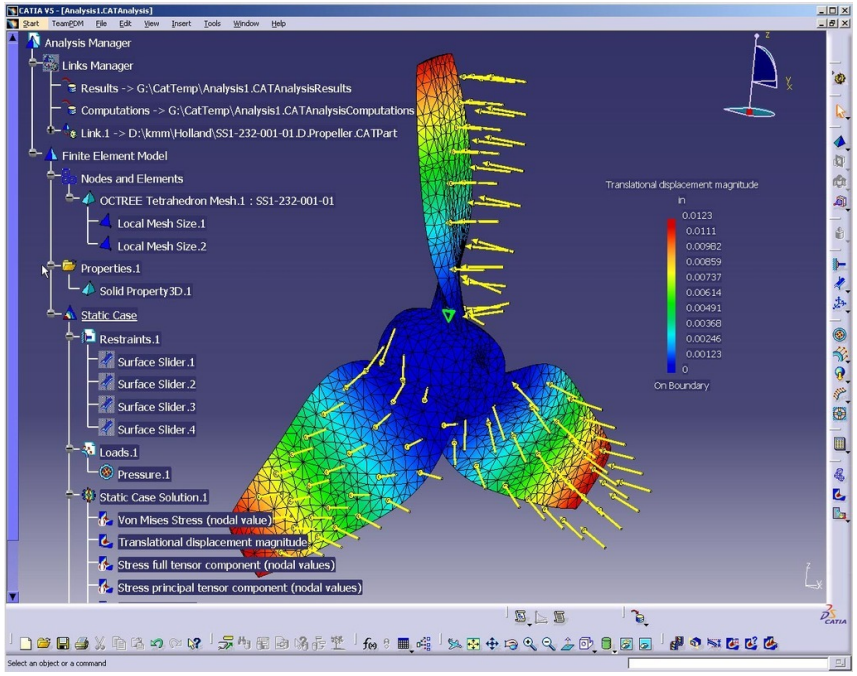
Exam

- The exam will test the learning outcomes, consisting of a combination of mathematical/algorithmic questions and Python questions.
- We will run a mock exam towards the end of the semester. Details to be announced!
- We will closely monitor the Ed Discussion Forum during the week(s) before the exam.

What is numerical analysis?

Numerical Simulation

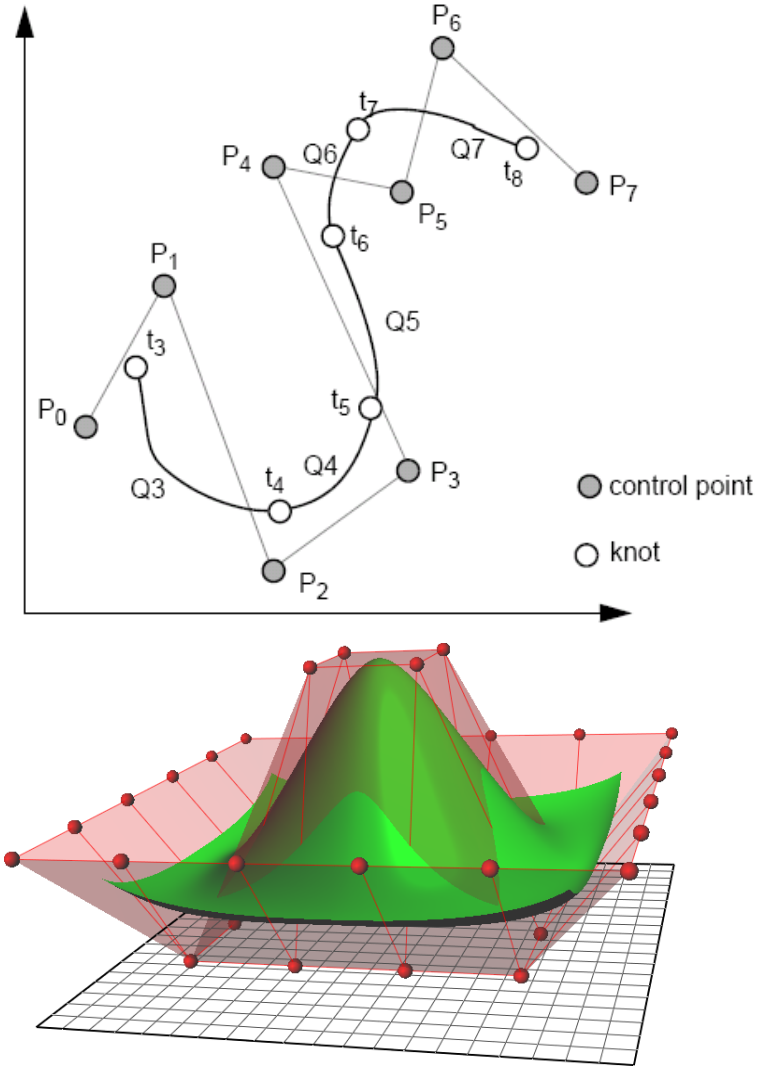
- Numerical simulation has become ubiquitous in civil and environmental engineering.
- The structural analysis of a 3*D* structure, reservoir simulation, simulation of sedimentation, . . . are all massive computational tasks!



- Get to know algorithms behind commercial software packages.
- Goal of this lecture: Understand most important ingredients of modern numerical methods and get a feeling for numerical computations.
- Understand limitations and what can go wrong!
- Will pay attention to reliability, accuracy, and efficiency.

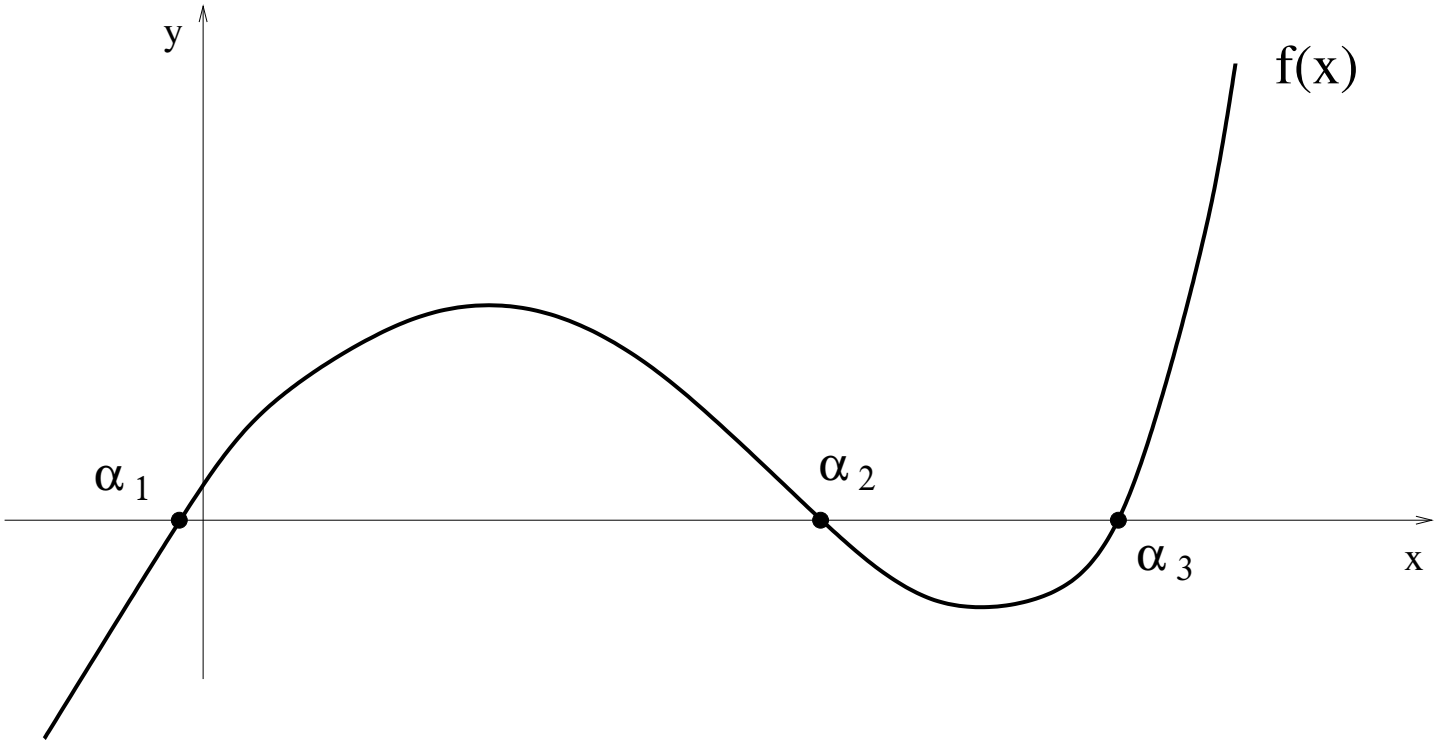
Other applications of numerical methods

- B-splines, NURBS, . . ., to represent smooth curves and surfaces. Contained in every vector drawing program and CAD package.
- data fitting
- shape optimization
- active damping
- environmental impact simulation
- crowd behavior
- . . .

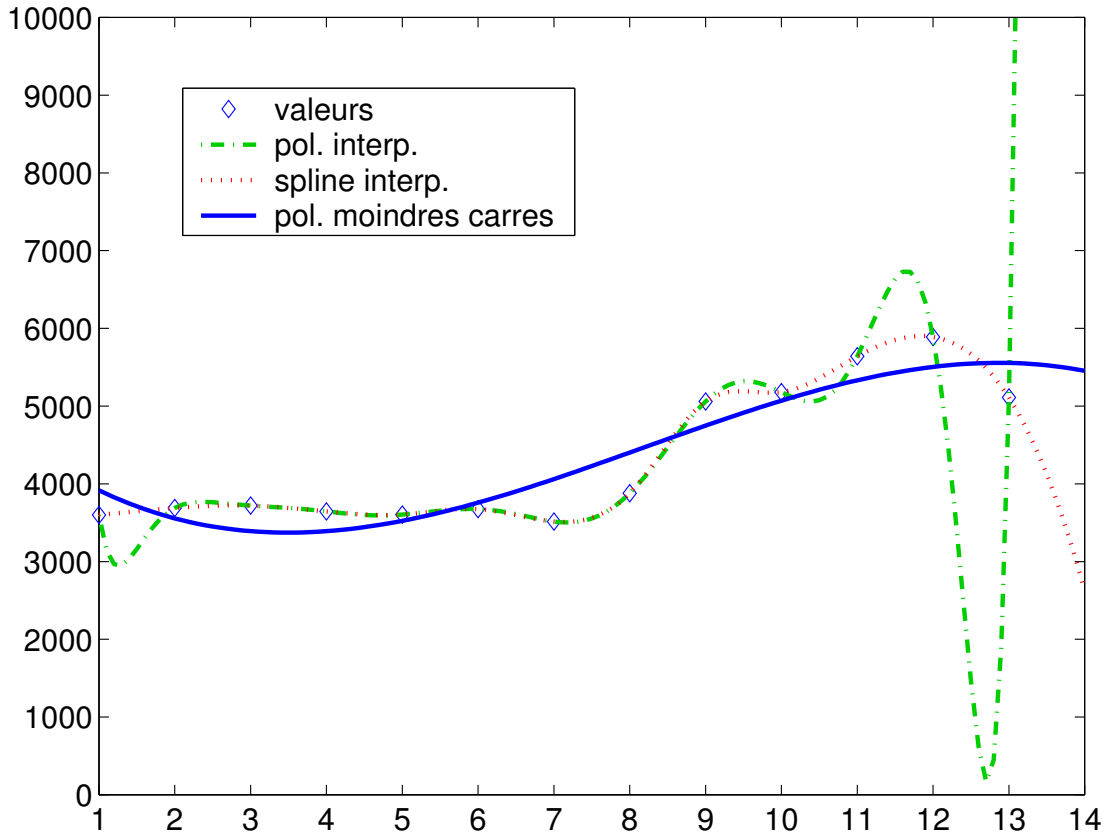


A quick tour through the lecture

Nonlinear Equations

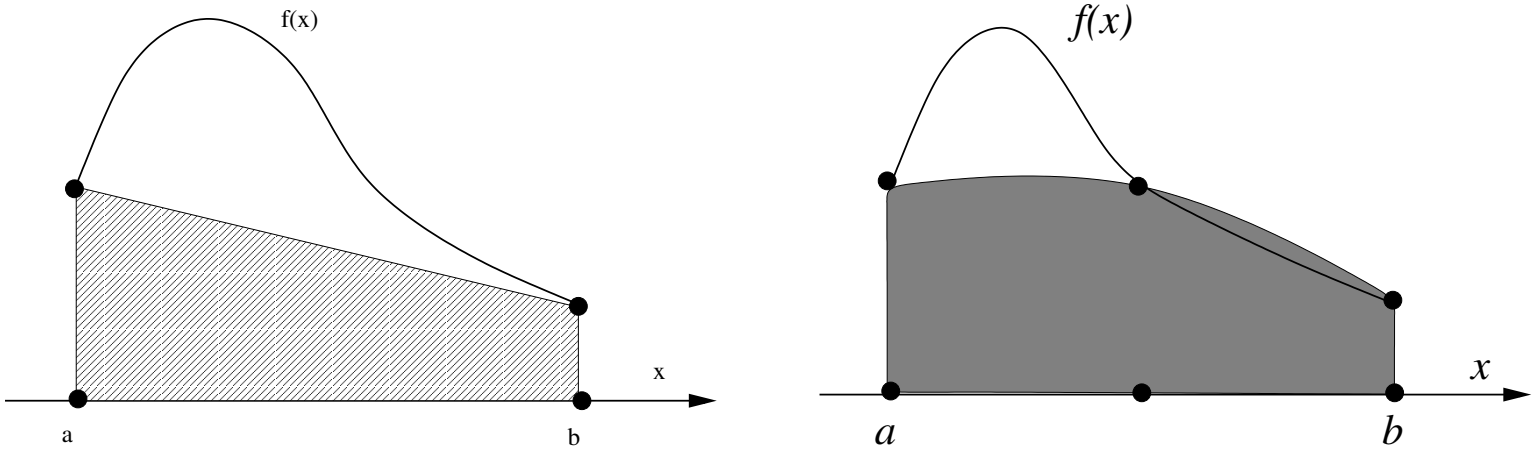


Curve Fitting



Differentiation and Integration

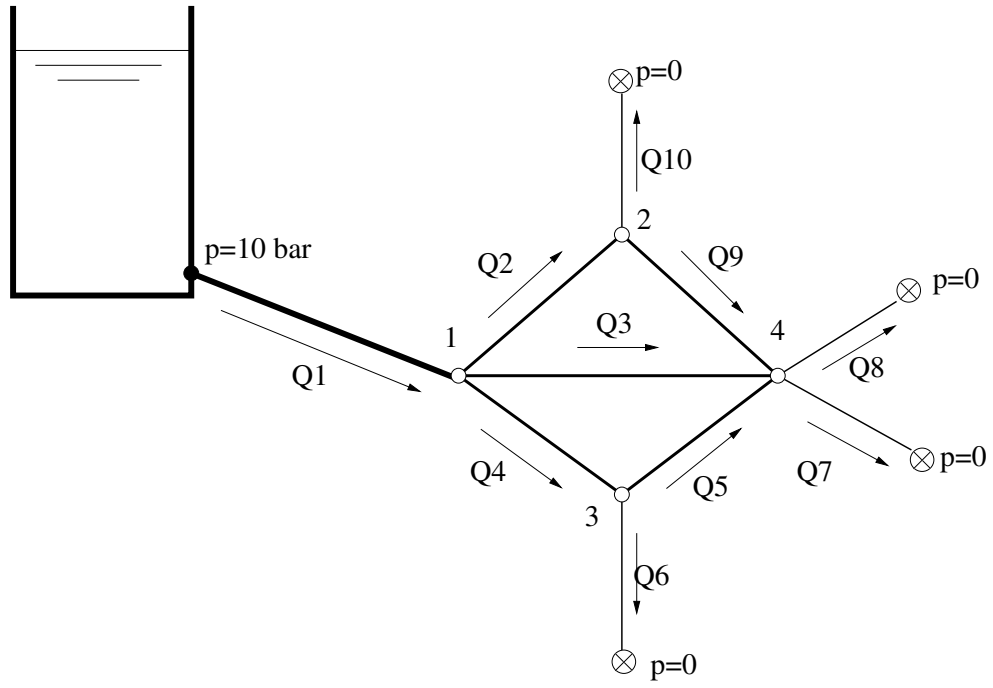
$$\int_0^1 e^{-x^2}\,dx \qquad \int_0^{\frac{\pi}{2}} \sqrt{1+\cos^2 x}\,dx \qquad \int_0^1 \cos x^2\,dx \qquad \int_0^1 \frac{\log x}{\sqrt{1+x^2}}\,dx.$$



Trapezoidal rule (left) and Simpson rule (right).

Linear Systems

Hydraulic network:



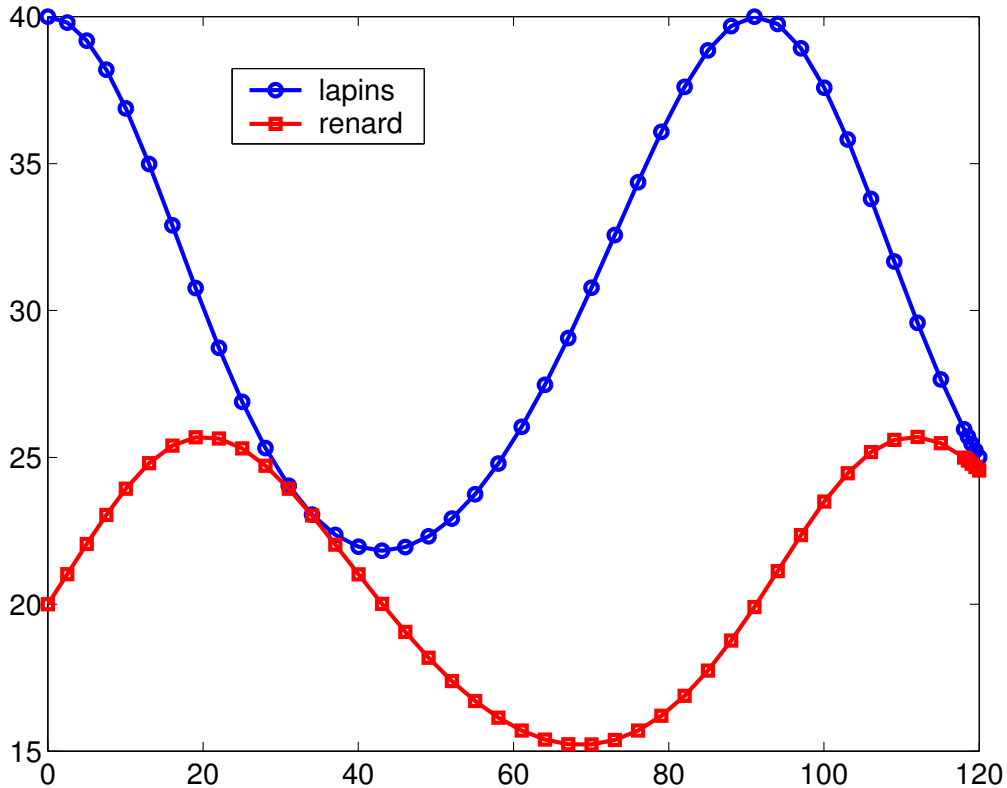
Pressures $\mathbf{p} = [p_1, p_2, p_3, p_4]^T$ at network nodes determined from linear system

$$A\,\mathbf{p} = \mathbf{b}.$$

Ordinary Differential Equations (ODEs)

Lotka-Volterra population model of prey y_1 vs. predators y_2:

$$\begin{aligned} y_1'(t) &= C_1 y_1(t)\left[1 - b_1 y_1(t) - d_2 y_2(t)\right], \\ y_2'(t) &= -C_2 y_2(t)\left[1 - d_1 y_1(t)\right]. \end{aligned}$$



Boundary Vale Problems (BVPs)