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Problem Sheet 2 <sup>1</sup>

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**Exercise 1.** To fulfill the requirements for a certain degree, a student can choose to take any 7 out of a list of 20 courses, with the constraint that at least 1 of the 7 courses must be a statistics course. Suppose that 5 of the 20 courses are statistics courses.

1. How many choices are there for which 7 courses to take?
2. Explain intuitively why the answer to part 1. is *not*  $\binom{5}{1} \cdot \binom{19}{6}$ .

**Exercise 2.** A fair die is rolled  $n$  times. What is the probability that at least 1 of the 6 values never appears?

**Exercise 3.** A spam filter is designed by looking at commonly occurring phrases in spam. Suppose that 80% of email is spam. In 10% of the spam emails, the phrase “free money” is used, whereas this phrase is only used in 1% of non-spam emails. A new email has just arrived, which does mention “free money”. What is the probability that it is spam?

**Exercise 4.** A hat contains 100 coins, where 99 are fair but one is double-headed (always landing Heads). A coin is chosen uniformly at random. The chosen coin is flipped 7 times, and it lands Heads all 7 times. Given this information, what is the probability that the chosen coin is double-headed? (Of course, another approach here would be to *look at both sides of the coin*—but this is a metaphorical coin.)

**Exercise 5.** A fair coin is flipped 3 times. The toss results are recorded on separate slips of paper (writing “H” if Heads and “T” if Tails), and the 3 slips of paper are thrown into a hat.

1. Find the probability that all 3 tosses landed Heads, given that at least 2 were Heads.
2. Two of the slips of paper are randomly drawn from the hat and both the letter H. Given this information, what is the probability that all 3 tosses landed Heads?

**Exercise 6.** A bag contains one marble which is either green or blue, with equal probabilities. A green marble is put in the bag (so there are 2 marbles now), and then a random marble is taken out. The marble taken out is green. What is the probability that the remaining marble is also green?

**Exercise 7.** Suppose that there are 5 blood types in the population, named type 1 through type 5, with probabilities  $p_1, p_2, \dots, p_5$ . A crime was committed by two individuals. A suspect, who has blood type 1, has prior probability  $p$  of being guilty. At the crime scene, blood evidence is collected, which shows that one of the criminals has type 1 and the other has type 2.

Find the posterior probability that the suspect is guilty, given the evidence. Does the evidence make it more likely or less likely that the suspect is guilty, or does this depend on the values of the parameters  $p, p_1, p_2, \dots, p_5$ ? If it depends on these values, give a simple criterion for when the evidence makes it more likely that the suspect is guilty.

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<sup>1</sup>Exercises are based on the coursebook Statistics 110: Probability by Joe Blitzstein