Important Distributions

The eight most important distributions we have discussed so far are listed below, each with its PMF/PDF. Each of these distributions is important because it has a natural, useful story, so understanding these stories (and recognizing equivalent stories) is crucial. It is also important to know how these distributions are related to each other. For example, Bern(p) is the same as Bin($1, p$), and Bin(n, p) is approximately Pois(λ) if n is large, p is small, and $\lambda = np$ is moderate.

Name	**Param.**	**PMF or PDF**
Bernoulli	p	$P(X = 1) = p$, $P(X = 0) = 1 - p$
Binomial	n, p	$\binom{n}{k}p^k(1-p)^{n-k}$, for $k \in \{0, 1, \dots, n\}$
Geometric	p	$(1-p)^k p$, for $k \in \{0, 1, 2, \dots\}$
Negative Binomial	r, p	$\binom{r+n-1}{n}p^r(1-p)^n$, $n \in \{0, 1, 2, \dots\}$
Hypergeometric	w, b, n	$\frac{\binom{w}{k}\binom{b}{n-k}}{\binom{w+b}{n}}$, for $k \in \{0, 1, \dots, n\}$
Poisson	λ	$\frac{e^{-\lambda}\lambda^k}{k!}$, for $k \in \{0, 1, 2, \dots\}$
Exponential	λ	$\lambda e^{-\lambda x}$ for $x > 0$
Uniform	$a < b$	$\frac{1}{b-a}$, for $x \in (a, b)$
Normal	μ, σ^2	$\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

Some Useful Formulas

De Morgan's Laws

$$(A_1 \cup A_2 \cup \dots \cup A_n)^c = A_1^c \cap A_2^c \cap \dots \cap A_n^c$$
$$(A_1 \cap A_2 \cap \dots \cap A_n)^c = A_1^c \cup A_2^c \cup \dots \cup A_n^c$$

Complements

$$P(A^c) = 1 - P(A)$$

Unions

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A_1 \cup A_2 \cup \dots \cup A_n) = \sum_{i=1}^{n} P(A_i), \text{ if the } A_i \text{ are disjoint}$$

$$P(A_1 \cup A_2 \cup \dots \cup A_n) \le \sum_{i=1}^{n} P(A_i)$$

$$P(A_1 \cup A_2 \cup \dots \cup A_n) = \sum_{k=1}^{n} \left((-1)^{k+1} \sum_{i_1 < i_2 < \dots < i_k} P(A_{i_1} \cap A_{i_2} \cap \dots \cap A_{i_k}) \right)$$

(Inclusion-Exclusion)

Intersections

$$P(A \cap B) = P(A)P(B|A) = P(B)P(A|B)$$

$$P(A_1 \cap A_2 \cap \cdots \cap A_n) = P(A_1)P(A_2|A_1)P(A_3|A_1, A_2) \cdots P(A_n|A_1, \ldots, A_{n-1})$$

Law of Total Probability

If $E_1, E_2, \ldots, E_n$ are a partition of the sample space S (i.e., they are disjoint and their union is all of S) and $P(E_j) \neq 0$ for all j, then

$$P(B) = \sum_{j=1}^{n} P(B|E_j)P(E_j)$$