153

$$b_{\mu,\sigma^2}(a\tilde{\sigma}_n^2)$$

$$= \sigma^2\left(a\left(\frac{n-1}{n}\right)-1\right)$$

$$Var_{\mu,\sigma^2}(a\tilde{\sigma}_n^2) = var\left(\frac{an\tilde{\sigma}_n^2}{n}\right)$$

$$= \frac{a^2}{n^2} var\left(n\tilde{\sigma}_n^2\right) = \frac{a^2}{n^2} 2\sigma^4(n-1)$$

$$EQM_{\mu,\sigma^2}(a\tilde{\sigma}_n^2) = \frac{a^2}{n^2} 2\sigma^4(n-1) + \sigma^4\left(a\frac{n-1}{n}-1\right)^2$$

Il faut minimiser par rapport à a

$$\frac{\partial}{\partial a} = \sigma^4 \frac{2(n-1)}{n^2} a + \sigma^4 2\left(a\frac{n-1}{n} - 1\right)\frac{n-1}{n}.$$

$$\frac{\partial}{\partial a} = 0 \Leftrightarrow \boxed{a = \frac{n}{n+1}}$$

(algèbre)

$$\frac{\partial^2}{\partial^2 a} > 0 \Rightarrow \text{minimum}$$

Le meilleur estimateur est

$$\frac{1}{n+1}\sum_{i=1}^{n}(X_i - \bar{X}_n)^2$$

pour minimiser le biais $\rightarrow a = \frac{n}{n-1}$

pour minimiser la var $\rightarrow a = 0$

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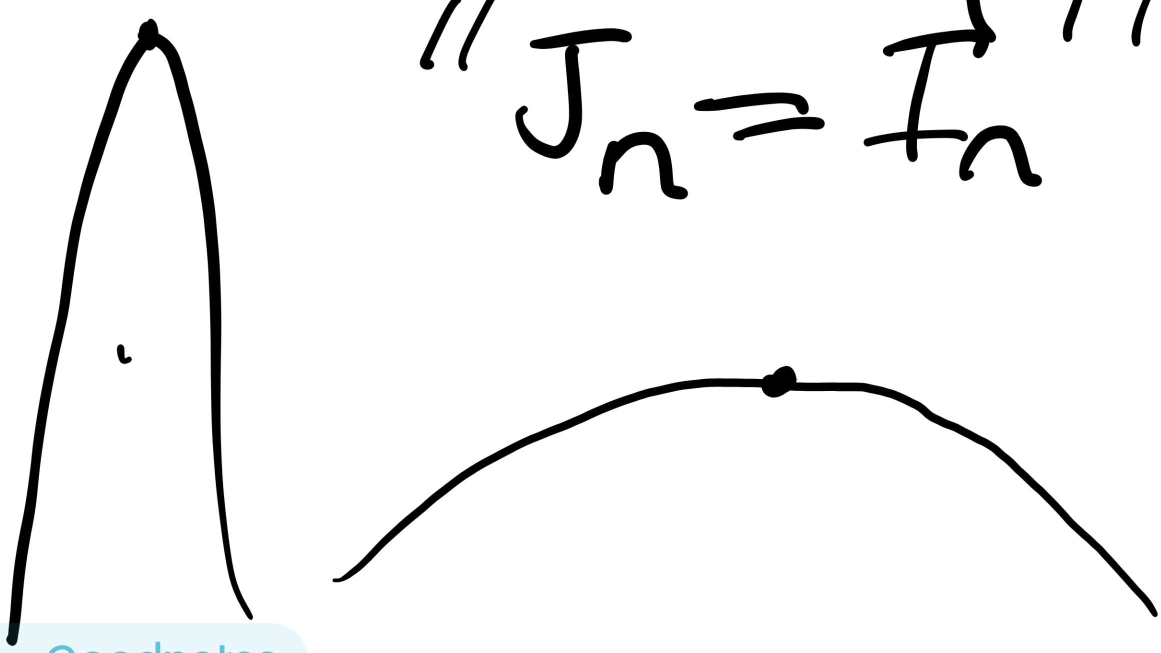
166 méthode delta

avec $g = (\ell')^{-1}$

" $J_n = I_n$ "



167 $\ell(\lambda) = n\log\lambda - \lambda n\overline{X_n}$

$$\ell'(\lambda) = \frac{n}{\lambda} - n\overline{X_n}$$

$$\ell''(\lambda) = -\frac{n}{\lambda^2}$$

$$I_n = -\mathbb{E}_\lambda\left[-\frac{n}{\lambda^2}\right] = \frac{n}{\lambda^2}$$

rien d'aléatoire

$$\hat{\lambda}_{ML} \overset{app}{\sim} \mathcal{N}\left(\lambda, \frac{\lambda^2}{n}\right)$$

(on a vu $\hat{\lambda}_{ML} = \frac{1}{\overline{X_n}}$)

$$J_n = -\ell''(\hat{\lambda}_{ML}) = \frac{n}{\hat{\lambda}_{ML}^2}$$

$$= n\,\overline{X}_n^{\,2}$$

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177 $\Pr\left(z_{\frac{\alpha}{2}} \leq Z_n \leq z_{1-\frac{\alpha}{2}}\right)$

$= 1-\alpha$, où z_β est tel que $\mathbb{P}r(Z_n \leq z_\beta) = \beta$.

Comme la loi de Z_n ne dépend pas de μ,

$$\forall \mu \in \mathbb{R} \quad Pr_{\mu}\left(z_{\frac{\alpha}{2}} \leq Z_n \leq z_{1-\frac{\alpha}{2}}\right)$$

$$= 1 - \alpha$$

Donc

$$1 - \alpha = Pr_{\mu}\left(z_{\frac{\alpha}{2}} \leq \frac{(\overline{X}_n - \mu)\sqrt{n}}{?} \leq z_{1-\frac{\alpha}{2}}\right)$$

$$= Pr_{\mu}\left(-\overline{X}_n + \frac{z_{\frac{\alpha}{2}}}{\sqrt{n}} \leq -\mu \leq -\overline{X}_n + \frac{z_{1-\frac{\alpha}{2}}}{\sqrt{n}}\right)$$

$$= P_\mu\left(\bar{Y}_n - \frac{z_{1-\frac{\alpha}{2}}}{\sqrt{n}} \le \mu \le \bar{Y}_n - \frac{z_{\frac{\alpha}{2}}}{\sqrt{n}}\right)$$

Par symétrie $z_{\frac{\alpha}{2}} = -z_{1-\frac{\alpha}{2}}$

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$$\left[\bar{Y}_n \pm \frac{z_{1-\frac{\alpha}{2}}}{\sqrt{n}}\,\sigma\right] \qquad \alpha = 0{,}05$$

$$= \left[5{,}34 \pm \frac{z_{0{,}975}}{\sqrt{64}=8}\,0{,}12\right]$$

$$\approx \left[5{,}34 \pm \frac{1{,}96}{8}\,0{,}12\right]$$

$$\approx [5{,}37,\ 5{,}37]$$

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175 $\alpha = 0{,}05$

$$t_{n-1,1-\frac{\alpha}{2}} = t_{8;0{,}975}$$

$$= 2{,}306$$

$$\left[\bar{Y}_n \pm \frac{2{,}306}{\sqrt{n}} S_n\right]$$

$$= \left[1040 \pm \frac{2{,}306}{3} 16\right]$$

$$= [1027{,}8,\ 1052{,}2]$$