135

$A = \{\text{il pleut}\}$ $\quad (A, A^c)$ partition

$\boxed{X = \text{v.a.} = \text{quantité de pluie}}$

$$\mathbb{E}[X] = \mathbb{E}[X|A]\overbrace{\Pr(A)}^{0{,}2} + \underbrace{\mathbb{E}[X|A^c]}_{=0}\underbrace{\Pr(A^c)}_{0{,}8}$$

$$\mathbb{E}[X|A] = \mathbb{E}[Z], \quad Z \sim \exp(\lambda)$$

$$= \int_0^\infty \lambda e^{-\lambda x} x\,dx = \int_0^\infty y e^{-y} \frac{dy}{\lambda}$$

$y = \lambda x$

$dx = \frac{dy}{\lambda}$

$$= \frac{1}{\lambda} \int_0^\infty y e^{-y} \, dy =$$

$u(y) = y$ $\quad$ $v'(y) = e^{-y}$

$u'(y) = 1$ $\quad$ $v(y) = -e^{-y}$

$$\frac{1}{\lambda}\left[\Big| u(y)v(y) \Big|_0^\infty - \int_0^\infty \underbrace{1}_{u'} \underbrace{(-e^{-y})}_{v} \, dy \right]$$

$\underbrace{u(y)v(y)}_{-ye^{-y}}$

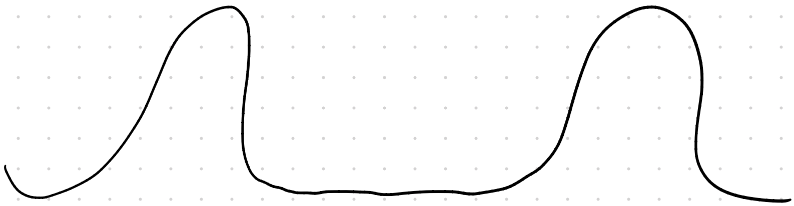
$$= \frac{1}{\lambda}\left[0 + \int_0^\infty e^{-y} \, dy \right] = \frac{1}{\lambda}$$

$$\mathbb{E}[Z] = \frac{1}{\lambda}$$

$$\mathbb{E}[X] = 0,2\, \mathbb{E}[Z] = \frac{1}{5\lambda}$$

$$\lambda = \frac{1}{20} \longrightarrow \mathbb{E}[X] = 4$$

(136)



>0

(138) $U \sim U[a,b)$ $F_U(u) = \frac{u-a}{b-a}$

a) Si $u \in (a,b)$

$x_p = F_U^{-1}(p) \longleftrightarrow F_U(x_p) = p$

$\longleftrightarrow \frac{x_p - a}{b-a} = p \longleftrightarrow x_p = a + p(b-a)$

b) $F(x) = \int_1^x \alpha y^{-1-\alpha}\, dy = 1 - \frac{1}{x^\alpha}$ $(x \geq 1)$

$F(x_p) = p \longleftrightarrow p = 1 - \frac{1}{x_p^\alpha} \longleftrightarrow \frac{1}{x_p^\alpha} = 1-p$

$x_p = (1-p)^{-\frac{1}{\alpha}} = \left(\frac{1}{1-p}\right)^{\frac{1}{\alpha}}$

142

$\mu = \mathbb{E}[X_1]$

$\mu - \epsilon \quad \mu + \epsilon$

143 $\Phi(x) = \Pr(Z \leq x)$

$Z \sim N(0,1)$

$Z_n \longrightarrow Z$

146 $\overline{X}_n \overset{app}{\sim} N(\mu, \frac{\sigma^2}{n})$

$S_n = n\overline{X}_n \overset{app}{\sim} N(n\mu, \sigma^2 n)$

$= N(\mathbb{E}[S_n], var(S_n))$

$\overline{X}_n \overset{app}{\sim} N(\mathbb{E}[\overline{X}_n], Var(\overline{X}_n))$

197 $Pr(\bar{X}_n \leq r)$

$= Pr\left(\sum_{i=1}^{n} X_i \leq rn\right)$

$X = \sum_{i=1}^{690} Y_i$ avec

$Y_i \overset{iid}{\sim} Poiss(\lambda)$

Y_i = nombre d'erreurs à la page i

TCL $X \overset{app}{\sim} N(E[X], var X)$

$= N(n\lambda, n\lambda)$

$Pr(X \leq 50) = Pr\left(\frac{X - n\lambda}{\sqrt{n\lambda}} \leq \frac{50 - n\lambda}{\sqrt{n\lambda}}\right)$

$$\approx \Phi\left(\frac{50-n\lambda}{\sqrt{n\lambda}}\right)$$

$n = 640 \quad \lambda = 0.1$

$$= \Phi\left(\frac{-14}{8}\right) = \Phi(-1.75)$$

$$\approx 0.04$$

14g $\quad Z_n = \sqrt{n}\,\frac{\bar{X}_n - \mu}{\sigma}$

$$g(\bar{X}_n) \to g(\mu)$$

$$\sqrt{n}\,\frac{g(\bar{X}_n) - g(\mu)}{\sigma}$$

$Z_n \overset{app}{\sim} N(0,1) \quad g'(\mu) Z_n \overset{app}{\sim} N(0, g'(\mu)$

$$g\left(\frac{Y}{n}\right) \underset{\sim}{\text{app}} \frac{g'(\mu)}{\sqrt{n}} Y + g(\theta)$$