EPFL

Probability and Statistics for SIC

2014–2015, Spring semester

Probability and Statistics: Test

13 April 2015

Duration: The test starts at 16:15 and ends at 18:00.

Family name:

Forename:

No. SCIPER:

Exercise	Points	Indicative marks
1		/7 points
2		/9 points
3		/10 points
4		/12 points
Total :		/38 points

REMARKS:

- Personal documents may not be brought into the test. There is no *formulaire*.
- A simple calculator may be used.
- Answers may be given in English or in French.
- Explain your reasoning! An unjustified answer will be treated as incorrect.
- Write your answers in the exam scripts. If you need more space, use the blank pages at the end of the script, or ask for another blank page and staple it into the script.
- The assistants will reply to questions only if there is a typo. If you find a question unclear, explain how you understand it when giving your solution.

Exercise 1. You throw a fair coin, and count 0 if it shows tails and 1 if it shows heads. If it shows tails, then you throw a fair six-sided die whose faces are numbered $1, \ldots, 6$. If the die is not thrown, its score is taken to be 0.

(a) Give the sample space for this random experiment.

(b) Find the probability that the total score on the coin and die is odd.

(c) Find the probability that the total score equals 3 or less.

(d) If the total score equals 3 or less, find the probability that the coin showed heads.

Exercise 2. To win the Euromillions lottery, you must correctly select five distinct numbers in the range $1, \dots, 50$ and two distinct stars labelled $1, \dots, 11$. All combinations of numbers and stars are equally likely, and the outcomes of different draws are independent.

(a) What is the probability of winning the lottery on the next draw?

(b) If 80 million people play Euromillions independently on each draw, give the expected number of winners at the next draw.

(c) Give approximate probabilities that there will be no winner, one winner, and more than one winner at the next draw.

(d) What is the distribution of the number of draws until the next winner?

Exercise 3. Two large jobs are submitted simultaneously to a computer with two cores. The run times for the jobs are independent random variates uniformly distributed from zero to three hours. Let X denote the longest run time.

(a) Show that the probability density function of X is

$$f(x) = \begin{cases} 2x/9, & 0 < x < 3, \\ 0, & \text{elsewhere.} \end{cases}$$

(b) Find the expected waiting time until both processors are free.

(c) When I check after two hours, both jobs are done. What is the probability that the longest run time was less than one hour?

(d) The computer is still running after two hours. At that time, what is the expected remaining waiting time until both cores are free?

Exercise 4. The download time in hours for a software upgrade is random with probability density function $10\exp(-10x)$, where $x > 0$.

(a) Find the probability that the download time exceeds t hours, and give the conditional probability that it exceeds $t + x$ hours, given that it exceeds t hours. Comment.

Frustrated after waiting 10 minutes, I decide to try downloading from another server, without stopping the first download. The joint probability density function of the remaining time for the first download, X, and the time for the new download, Y, may be written in the form

$$f_{X,Y}(x, y) = \begin{cases} c\exp(-10x - 20y), & x, y > 0, \\ 0, & \text{otherwise.} \end{cases}$$

(b) Give the marginal density of Y. Are X and Y independent?

(c) Find the probability that $Y < X$.