Worksheet #1

Topology I - point set topology

September 10, 2024

Problem 1. *1 Let A, B and C be arbitrary subsets of an arbitrary set U. Are the statements below true or false? If true, provide proof, and if false, provide a counter-example.*

(i) $(A \backslash B) \cap (A \backslash C) = A \backslash (B \cup C)$

(ii) $A \cup B = A \cup C \Rightarrow B = C$

Problem 2. *Prove a function $f : A \to B$ is bijective if and only if $f(X^c) = (f(X))^c$ for any $X \subset A$.*

Problem 3. *If $f : X \to Y$ is any function and $A \subset X, B \subset Y$ are arbitrary subsets, prove $f^{-1}(Y \backslash B) = X \backslash f^{-1}(B)$.*

Problem 4. *Prove the set of all functions $f : \mathbb{N} \to \mathbb{N}$ is uncountable.*

Problem 5. *Let A be any collection of disjoint open intervals in $\mathbb{R}$. Prove A is countable.*

Problem 6. *Prove the following are equivalence relations and describe the quotient.*

(i) The relation $\sim$ on the set $\mathbb{C}$ given by

$$(x + iy) \sim (u + iv) \iff x = u$$

(ii) The relation $*$ on the set $\mathbb{Q}$ given by

$$p * q \iff p - q \in \mathbb{Z}$$

(iii) The relation $\star$ on the set $\mathbb{R}^2$ given by

$$(x_1, y_1) \star (x_2, y_2) \iff x_1 y_1 = x_2 y_2$$

Problem 7. *Show the relation on the set $\mathbb{N}$ given by "$n \mid m \iff \exists k \in \mathbb{Z}$ such that $m = k \cdot n$" is an order relation.*