Analysis III - 203(d)

Winter Semester 2024

Session 11: November 28, 2024

Exercise 1 *Compute the Fourier coefficients of the following functions, which have period $T = 2\pi$ and have the given values over the interval $[0, 2\pi)$:*

- $$f(x) = e^{x-\pi}$$
- $$g(x) = (x-\pi)^3$$
- $$h(x) = \begin{cases} \sin(x) & 0 \le x < \frac{\pi}{2} \\ 0 & \frac{\pi}{2} \le x < \frac{3\pi}{2} \\ \sin(x) & \frac{3\pi}{2} \le x < 2\pi \end{cases}$$

Hint: Recall that $\cos(\frac{\pi}{2} - \theta) = \sin\theta$ *and* $\cos(\frac{3\pi}{2} - \theta) = -\sin\theta$.

Exercise 2 *Compute the Fourier coefficients of the function f with period $T = 2\pi$ and which satisfies*

$$f(x) = \sin(x) \textit{ if } 0 \le x < \pi,$$

and

$$f(x) = f(-x) \textit{ for all } x \in \mathbb{R}.$$

Exercise 3 *Use Dirichlet's theorem to explain whether the Fourier series converges at $x \in [0, T]$ and to which value:*

- $$f(x) = \begin{cases} x & \textit{if } 0 \le x < 1 \\ 2-x & \textit{if } 1 \le x < 2 \end{cases}, \qquad T = 2.$$
- $$f(x) = \begin{cases} \pi & \textit{if } 0 \le x < 1 \\ e^{x-1} & \textit{if } 1 \le x < 2 \\ \sin(x) & \textit{if } 2 \le x < 3 \end{cases}, \qquad T = 3.$$

Here, the functions have the given period T.

Exercise 4 *Give the Fourier series in complex notation, when* f *has period* $T = 2$ *and*

$$f(x) = x \text{ for } 0 \leq x < 2.$$

Exercise 5 *Find the Fourier coefficients of* $\cos(x)^8$ *with period* $T = 2\pi$*. Use Parseval's identity to compute* $\int_0^{2\pi} \cos(x)^{16} dx$*.*

Exercise 6 *Give the Fourier series of the function* f *with period* $T = 2$ *and*

$$f(x) = \cos(x) \text{ for } -1 \leq x < 1.$$

Give the Fourier series in standard form and in complex notation. Compare $F_3 f$ *at the points* $x = -\pi/4, 0, \pi/4$ *with the original function* f*.*