

Analysis III - 203(d)

Winter Semester 2024

Session 10: November 21, 2024

Exercise 1

$$\begin{aligned} \operatorname{ReLU}(x) &= \begin{cases} 0 & \text{if } x < 0 \\ x & \text{if } 0 \leq x \end{cases} \\ \operatorname{ReLU}^k(x) &= \begin{cases} 0 & \text{if } x < 0 \\ x^k & \text{if } 0 \leq x \end{cases} \quad \text{where } k \in \mathbb{N}_0 \\ f(x) &= \begin{cases} -x - 2 & \text{if } -2 < x < -1 \\ -x^2 - 2x - 1 & \text{if } -1 < x < 0 \\ x^2 - 2x + 1 & \text{if } 0 < x < 1 \\ 2 - x & \text{if } 1 < x < 2 \\ 0 & \text{otherwise} \end{cases} \\ j(x) &= \begin{cases} x & \text{if } x < 0 \\ 1 + x & \text{if } 0 \leq x \end{cases}. \end{aligned}$$

Here, employ the definition of distributional derivatives and check against the formula for piecewise differentiable functions as obtained during lecture.

Exercise 2 Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function with period 1 and

$$f(x) = x, \quad 0 < x < 1.$$

Find the distributional derivative.

Exercise 3 Compute the Fourier coefficients of the function f that has period $T = 2\pi$ and satisfies

$$f(x) = \begin{cases} 1 & 0 \leq x < \pi \\ 0 & \pi < x \leq 2\pi \end{cases}$$

Exercise 4 Compute the Fourier coefficients of the following functions, which have period $T = 1$ and have the given values over the interval $[0, 1)$:

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$$f(x) = x^2$$

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$$g(x) = (1 - x)x$$

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$$h(x) = |\sin(2\pi x)|$$