

Analysis III - 203(d)

Winter Semester 2024

Session 9: November 14, 2024

Exercise 1 Show that

$$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}. \quad (1)$$

Hint: square the integral and convert it to polar/radial coordinates.

Exercise 2 We check whether the following are distributions or not.

- Show that T is a distribution, where

$$T(\phi) = \int_{-1}^1 \phi(x) dx. \quad (2)$$

- Show that T is a distribution, where

$$T(\phi) = \int_{-\infty}^{\infty} \phi(x) dx. \quad (3)$$

- Show that S is not a distribution, where

$$S(\phi) = \int_0^1 |\phi(x)| dx. \quad (4)$$

Exercise 3 Suppose that $f : \mathbb{R} \rightarrow \mathbb{R}$ is an integrable function. Whenever $a > 0$, show that

$$T(\phi) = \frac{1}{a} \langle f, \phi(a \cdot) \rangle \quad (5)$$

is a distribution.

Exercise 4 Find the distributional derivative of the function

$$f(x) = |x|. \quad (6)$$

Exercise 5 Consider the function with period 2 that satisfies

$$f(x) = \begin{cases} -x & \text{if } -1 < x \leq 0 \\ x & \text{if } 0 < x \leq 1 \end{cases} \quad (7)$$

- Draw a plot of this function from -4 to 4 .
- Is this function differentiable? What is the first distributional derivative of this function?
- What is the second distributional derivative of this function?