

Note: several exercises are extracted from [B.Dacorogna and C.Tanteri, *Analyse avancée pour ingénieurs* (2018)]. Their corrections can be found there.

Reminder : In order to compute an integral of surface (Like in exercise 2 and parts 4-5. of exercise 3), proceed this way:

1. Sketch the surface Σ .
2. Give a parameterization $\sigma : \bar{A} \rightarrow \Sigma$ of the surface Σ . We will then define the domain $\bar{A}$ and the function σ .
3. Give a normal vector and add it to your sketch.
4. Use this parameterization in order to express the integral in question as a multiple integral where the bounds and the function that we need to integrate are indicated explicitly.
5. Compute this integral.

Exercise 1 (Ex 4.8 page 43 and Ex 4.7 page 42).

Let $\Omega \subset \mathbb{R}^2$ be a regular set where the border $\partial\Omega$ is oriented positively. Let ν be a field of external normal units to Ω . Let F be a vector field such that $F \in C^1(\bar{\Omega}, \mathbb{R}^2)$ and f a scalar field such that $f \in C^2(\bar{\Omega})$. Show that:

1.
$$\iint_{\Omega} \operatorname{div} F(x, y) \, dx dy = \int_{\partial\Omega} (F \cdot \nu) \, dl$$

Indication: Write $F = (F_1, F_2)$ and apply Green's theorem to the vector field $\Phi = (-F_2, F_1)$.
2.
$$\iint_{\Omega} \Delta f(x, y) \, dx dy = \int_{\partial\Omega} (\operatorname{grad} f \cdot \nu) \, dl.$$

Exercise 2 (Ex 5.1 page 56).

Let $f(x, y, z) = xy + z^2$ and

$$\Sigma = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 = z^2 \text{ et } 0 \leq z \leq 1\}.$$

Compute $\iint_{\Sigma} f \, ds$

Exercise 3 (Ex 5.6 page 57).

Let $0 < a < R$. In $\mathbb{R}^3$ we consider the torus Ω obtained with the rotation of the

disk $(x - R)^2 + z^2 \leq a^2$ around the Oz axis and its representation described as:

$$x = (R + r \cos \varphi) \cos \theta, \quad y = (R + r \cos \varphi) \sin \theta, \quad z = r \sin \varphi$$

with $0 < r < a$, $0 \leq \theta < 2\pi$, $0 \leq \varphi < 2\pi$.

1. Sketch Ω and indicate what does r , θ and φ represent.
2. Compute the Jacobian of the transformation that describes Ω in terms of the variables r , θ and φ .
3. Compute the volume of Ω .
4. Write a regular parameterization of the surface of the torus (noted $\partial\Omega$) and compute a normal to $\partial\Omega$.
5. Compute the area of $\partial\Omega$.
6. Compute $\iiint_{\Omega} z^2 dx dy dz$

The exercises below are a revision of analysis II

Exercise 4.

1. Let $D = \{(x, y) \in \mathbb{R}^2 : x \geq 0, y \geq 0 \text{ et } x + y \leq 1\}$.
Compute $\iint_D \sqrt{1 - x - y} dx dy$.
2. Let $D = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 2(x + \sqrt{x^2 + y^2})\}$.
Compute $\iint_D \frac{dx dy}{(x^2 + y^2)^{\frac{3}{4}}}$.
3. Let $D = \{(x, y, z) \in \mathbb{R}^3 : x \geq 0, y \geq 0, z \geq 0, z \leq 1 - y^2 \text{ et } x + y \leq 1\}$.
Compute $\iiint_D z dx dy dz$.

Exercise 5.

Compute the volume of:

1. $D = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 \leq 1 \text{ et } x^2 + y^2 \leq z\}$.
2. $D = \{(x, y, z) \in \mathbb{R}^3 : x + y + z \leq \sqrt{2}, x^2 + y^2 \leq 1, x \geq 0, y \geq 0 \text{ et } z \geq 0\}$.