

Note: several exercises are extracted from [B.Dacorogna and C.Tanteri, *Analyse avancée pour ingénieurs* (2018)]. Their corrections can be found there.

Exercise 1 (Ex 2.1 page 17).

1. Let $\Gamma = \{(x, y) \in \mathbb{R}^2 : y = f(x), x \in [a, b]\}$. Show that:

$$\text{length}(\Gamma) = \int_a^b \sqrt{1 + (f'(t))^2} dt.$$

2. Deduce the length of the curve:

$$\Gamma = \{(x, y) \in \mathbb{R}^2 : y = \cosh x, x \in [0, 1]\}.$$

3. Let $\Gamma = \{(x, y) \in \mathbb{R}^2 : x(t) = r(t) \cos t; y(t) = r(t) \sin t, t \in [a, b]\}$.
Calculate the length of Γ in terms of r .

Exercise 2 (Ex 2.4 page 18).

Calculate $\int_{\Gamma} f dl$ when $f(x, y, z) = x^2 + y^2 + \sqrt{2}z$, and:

$$\Gamma = \left\{ \gamma(t) = \left(\cos t, \sin t, \frac{1}{2}t^2 \right) : t \in [0, 1] \right\}.$$

$$\text{Hint: } \int \sqrt{x^2 + a^2} dx = \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \ln(x + \sqrt{x^2 + a^2}).$$

Exercise 3 (Ex 2.2 page 17).

We consider $F(x, y) = (xy, y^2 - x)$ and:

$$\Gamma_1 = \{(t, t) : t \in [0, 1]\}, \Gamma_2 = \{(t, e^t) : t \in [0, 1]\}, \Gamma_3 = \{(\sqrt{t}, t^2) : t \in [1, 2]\}.$$

Calculate $\int_{\Gamma_i} F \cdot dl$ for $i = 1, 2, 3$.

Exercise 4 (Ex 2.3 page 17).

Calculate $\int_{\Gamma} F \cdot dl$ when:

1. $\Gamma = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 = 1, z = 0\}$, $F(x, y, z) = (x, z, y)$;
2. $\Gamma = \{(x, y, z) \in \mathbb{R}^3 : y = e^x, z = x, x \in [0, 1]\}$, $F(x, y, z) = (x, y, z)$.

Exercise 5 (Ex 2.6 page 18).

Let $F(x, y) = (x + y, -x)$ and $\Gamma = \{(x, y) \in \mathbb{R}^2 : y^2 + 4x^4 - 4x^2 = 0, x \geq 0\}$ parameterized by $\gamma(t) = (\sin t, \sin(2t))$ with $t \in [0, \pi]$.

1. Show that γ is indeed a parameterization of Γ .
2. Calculate $\int_{\Gamma} F \cdot dl$.

Exercise 6 (Ex 2.5 page 18).

Let Γ be a simple regular curve in $\mathbb{R}^3$, joining A and B .

Using Newton's Law (Force = mass $\times$ acceleration), calculate the necessary work to move a particle of constant mass from A to B along Γ .