EPFL - Fall 2024	Dr. Pablo Antolin
Analysis III - 202 (c) GM EL MX	**Exercises**
Series 9	November 12, 2024

Note: several exercises are extracted from [B.Dacorogna and C.Tanteri, *Analyse avancée pour ingénieurs* (2018)]. Their corrections can be found there.

Tip: To verify the Stokes theorem, proceed as follows:

1. *Sketch the surface* Σ, *then compute* $\operatorname{curl} F(x,y,z)$.
2. *Provide a parameterization* $\sigma : \overline{A} \to \Sigma$ *of the surface* Σ *and compute its normal vector. Add this vector to your sketch.*
3. *Express*
$$\iint_\Sigma \operatorname{curl} F \cdot ds$$
as a double integral where the bounds and the function to be integrated are explicitly indicated.
4. *Write* $\partial\Sigma$ *as the union of simple regular curves; for each of them, provide a parameterization and indicate the circulation direction induced by the parameterization of* Σ *and the positive orientation of* ∂A.
5. *Express*
$$\int_{\partial\Sigma} F \cdot dl$$
as a sum of integrals where the bounds and the functions to be integrated are explicitly indicated.
6. *Verify the conclusion of the Stokes theorem for* Σ *and* F.

Exercise 1 (Ex 7.2 page 89)**.**
Verify Stokes' theorem for

$\Sigma = \{(x,y,z) \in \mathbb{R}^3 : x^2 + y^2 = z^4,\ 0 \le z \le 1\}$ and $F(x,y,z) = (x^2 y, z, x)$.

Exercise 2 (Ex 7.5 page 89)**.**
Verify Stokes' theorem for

$\Sigma = \left\{(x,y,z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 4,\ x \ge 0,\ y \ge 0,\ 1 \le z \le \sqrt{3}\right\}$

and $F(x,y,z) = \big(0, z^2, 0\big)$.

Exercise 3 (Ex 7.7 page 89).
Verify the Stokes theorem for $F(x, y, z) = (0, x^2, 0)$ and Σ the triangle of vertices $(1, 0, 0), (2, 2, 0)$ and $(1, 1, 0)$.

Exercise 4 (Ex 7.6 page 89).
Verify the Stokes theorem for $F(x, y, z) = \big(0, 0, y + z^2\big)$ and

$$\Sigma = \Big\{(x, y, z) \in \mathbb{R}^3 : x^2+y^2+z^2 = 4; x, y, z \geq 0; 0 \leq \arccos\frac{z}{2} \leq \arctan\frac{y}{x} \leq \frac{\pi}{2}\Big\}.$$