MATH-111(en) FALL 2024
Linear Algebra Annina Iseli

MINI SOLUTIONS for Homework 6

Ex 6.3 (Different methods for computing determinants)
Compute the determinant of each of the following matrices in three ways: Once using cofactor expansion across a row, once using cofactor across a column, and once with row reduction.

$$A = \begin{pmatrix} 2 & 0 & 0 \\ 2 & 1 & 2 \\ 0 & 3 & 4 \end{pmatrix}, \qquad B = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 0 \\ 0 & 8 & 0 \end{pmatrix}$$

Solution:

$$\det(A) = -4, \qquad \det(B) = 96$$

Ex 6.4 (Determinants based on another determinant)
Let

$$A = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$$

and assume that $\det(A) = 7$. Compute the determinants of the following matrices

$$B = \begin{pmatrix} a+d & b+e & c+f \\ d & e & f \\ g & h & i \end{pmatrix}, \quad C = \begin{pmatrix} a & b & c \\ 2d+a & 2e+b & 2f+c \\ g & h & i \end{pmatrix}.$$

Solution:

$$\det(B) = 7, \qquad \det(C) = 14$$

Ex 6.5 (More determinants)
Compute the determinants of the following matrices (You may use your preferred method or try to practice different methods.)

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 0 & 0 \\ 2 & 1 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 1 & 0 & -1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 2 & 3 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 10 & 5 & 10 & 5 \\ 6 & 9 & 0 & -3 \\ 3 & 0 & 0 & 3 \\ 1 & 0 & 1 & 1 \end{pmatrix}$$

Solution:

$$\det(A) = 1, \quad \det(B) = 2, \quad \det(C) = 0.$$

Ex 6.7 (Determinants and volume)

(a) Calculate the volume of the parallelepiped with the following vertices:

$$\begin{pmatrix}0\\0\\0\end{pmatrix}, \quad \begin{pmatrix}1\\4\\0\end{pmatrix}, \quad \begin{pmatrix}-2\\-5\\2\end{pmatrix}, \quad \begin{pmatrix}-1\\2\\-1\end{pmatrix}, \quad \begin{pmatrix}-1\\-1\\2\end{pmatrix}, \quad \begin{pmatrix}0\\6\\-1\end{pmatrix}, \quad \begin{pmatrix}-3\\-3\\1\end{pmatrix}, \quad \begin{pmatrix}-2\\1\\1\end{pmatrix}.$$

(b) Calculate the area of the triangle whose vertices are the points $(1,2),(2,4),(3,3) \in \mathbb{R}^2$.

Solution:

(a) 15.

(b) 3/2.