**MATH-111(en)** FALL 2024
**Linear Algebra** Annina Iseli

# MINI SOLUTIONS for Homework 4

## Ex 4.4 (Some matrix products)

Let

$$A = \begin{pmatrix} 4 & -5 & 3 \\ 5 & 7 & -2 \\ -3 & 2 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} 7 & 0 & -1 \\ -1 & 5 & 2 \end{pmatrix}, \quad C = \begin{pmatrix} -1 & 5 \\ 4 & -3 \\ 1 & 0 \end{pmatrix}.$$

Compute $AC$, $BC$ and $CB$.

**Solution:**

$$AC = \begin{pmatrix} -21 & 35 \\ 21 & 4 \\ 10 & -21 \end{pmatrix}, \quad BC = \begin{pmatrix} -8 & 35 \\ 23 & -20 \end{pmatrix} \quad \text{and} \quad CB = \begin{pmatrix} -12 & 25 & 11 \\ 31 & -15 & -10 \\ 7 & 0 & -1 \end{pmatrix}.$$

## Ex 4.5 (When do these matrices commute?)

Consider the matrices

$$A = \begin{pmatrix} 3 & -4 \\ -5 & 1 \end{pmatrix} \quad \text{and } B = \begin{pmatrix} 7 & 4 \\ 5 & k \end{pmatrix}.$$

For which values of $k$ does the equality $AB = BA$ hold?

**Solution:**
$AB = BA$ if and only if $k = 9$.

## Ex 4.6 (More matrix products)

Consider the matrices:

$$A = \begin{pmatrix} 7 & 0 \\ -1 & 5 \\ -1 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 4 \\ -4 & 0 \end{pmatrix}, \quad C = \begin{pmatrix} 7 \\ -3 \end{pmatrix}, \quad D = \begin{pmatrix} 8 & 2 \end{pmatrix}.$$

If they are defined, compute the matrices

$$AB, CA, CD, DC, DBC, BDB, A^T A \text{ and } AA^T.$$

For those that are not defined, explain why.

**Solution:**

$$AB = \begin{pmatrix} 7 & 28 \\ -21 & -4 \\ -9 & -4 \end{pmatrix}, \; CD = \begin{pmatrix} 56 & 14 \\ -24 & -6 \end{pmatrix}, \; DC = [50], \; DBC = [-96].$$

$$A^T A = \begin{pmatrix} 51 & -7 \\ -7 & 29 \end{pmatrix}, \qquad AA^T = \begin{pmatrix} 49 & -7 & -7 \\ -7 & 26 & 11 \\ -7 & 11 & 5 \end{pmatrix}.$$

**Ex 4.7 (Multiplication by diagonal matrices)**
Consider the matrices

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 5 \end{pmatrix} \quad \text{and} \quad D = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{pmatrix}.$$

1. Compute $AD$ and $DA$ and explain how the rows and columns of $A$ change when one multiplies $A$ by $D$ from the right and from the left.

2. Find all the diagonal matrices $M$ of dimension $3 \times 3$ such that $AM = MA$.

**Solution:**

1. $$AD = \begin{pmatrix} 2 & 3 & 4 \\ 2 & 6 & 12 \\ 2 & 12 & 20 \end{pmatrix} \quad \text{and} \quad DA = \begin{pmatrix} 2 & 2 & 2 \\ 3 & 6 & 9 \\ 4 & 16 & 20 \end{pmatrix}.$$

2. $M = \lambda I_3$, $\lambda \in \mathbb{R}$ where $I_3$ is the $3 \times 3$ identity matrix.

**Ex 4.8 (Upper triangular matrices)**

1. Compute $\begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix}$ for $a, b \in \mathbb{R}$.

2. Let $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$, and compute the following matrices: $A^8, A^T A, AA^T$.

3. Find a matrix $B$ such that $\begin{pmatrix} 1 & 5 \\ 0 & 1 \end{pmatrix} \cdot B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

**Solution:**

1. $$\begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & a+b \\ 0 & 1 \end{pmatrix}$$

2. $$A^8 = \begin{pmatrix} 1 & 8 \\ 0 & 1 \end{pmatrix}, \qquad A^T A = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}, \qquad AA^T \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}.$$

3. $$B = \begin{pmatrix} 1 & -5 \\ 0 & 1 \end{pmatrix}.$$

**Ex 4.10 (A matrix equation)**
Find a solution $X$ for the matrix equation

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} X = \begin{pmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \\ 3 & 5 & 7 \end{pmatrix}.$$

**Solution:**

$$X = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 2 & 3 & 4 \end{pmatrix}$$

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