

Séries 2

Keywords: Linear systems, augmented matrices, Gauss-Jordan algorithm, vectors, linear combinaisons.

Question 1

- (1) Write the augmented matrices of the following linear systems.
- (2) Solve these linear systems with elementary operations over the rows of the matrices.

$$\text{a) } \begin{cases} x_1 - 2x_2 = -1 \\ -x_1 + 3x_2 = 3 \end{cases} \quad \text{c) } \begin{cases} 6x_1 - 3x_2 + 2x_3 = 11 \\ -3x_1 + 2x_2 - x_3 = -4 \\ 5x_1 - 3x_2 + 2x_3 = 9 \end{cases}$$

$$\text{b) } \begin{cases} 3x_1 + 2x_2 - x_3 = 12 \\ x_3 + 2x_1 - 4x_2 = -1 \\ x_2 + 2x_3 - 4x_1 = -8 \end{cases} \quad \text{d) } \begin{cases} x_1 - 3x_2 = 5 \\ 5x_3 - x_1 + x_2 = 2 \\ x_2 + x_3 = 0 \end{cases}$$

Question 2

- (1) Put the following matrices into row echelon form, then into reduced row echelon form.
- (2) Suppose these matrices are augmented matrices of linear systems. Determine in each case whether the linear system has exactly one solution, infinitely many solutions, or no solution.

$$A = \left(\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \\ 6 & 7 & 8 & 9 \end{array} \right) \quad B = \left(\begin{array}{c|c} 1 & 3 \\ -4 & 2 \\ -3 & -2 \end{array} \right) \quad C = \left(\begin{array}{ccc|c} 1 & 3 & 5 & 7 \\ 3 & 5 & 7 & 9 \\ 5 & 7 & 9 & 1 \end{array} \right)$$

Question 3

- (1) Verify if the following matrices are in row echelon form (REF) or in reduced row echelon form (RREF).
- (2) Identify the pivot (or leading) variables and the free variables.
- (3) Determine if the corresponding linear systems have exactly one solution, infinitely many solutions, or no solution.

$$A = \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array} \right) \quad B = \left(\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right) \quad C = \left(\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right)$$

$$D = \left(\begin{array}{ccc|c} 0 & 1 & 1 & 1 \\ 0 & 0 & 2 & 2 \end{array} \right) \quad E = \left(\begin{array}{ccc|c} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{array} \right)$$

Question 4 Determine if the following homogeneous linear systems have a non-trivial solution.

$$\text{a) } \begin{cases} 2x_1 - 5x_2 + 8x_3 = 0 \\ -2x_1 - 7x_2 + x_3 = 0 \\ 4x_1 + 2x_2 + 7x_3 = 0 \end{cases} \quad \text{c) } \begin{cases} -7x_1 + 37x_2 + 119x_3 = 0 \\ 5x_1 + 19x_2 + 57x_3 = 0 \end{cases}$$

$$\text{b) } \begin{cases} x_1 - 3x_2 + 7x_3 = 0 \\ -2x_1 + x_2 - 4x_3 = 0 \\ x_1 + 2x_2 + 9x_3 = 0 \end{cases}$$

Question 5

For each of the following linear systems

$$\begin{cases} x + 2y = k \\ 4x + hy = 5 \end{cases} \quad \begin{cases} -3x + hy = 1 \\ 6x + ky = -3 \end{cases}$$

find every values of h and k such that the system

- (1) has no solution,
- (2) has a unique solution
- (3) has infinitely many solutions

Question 6

- (1) If the coefficient matrix of a system with four equations and four unknowns has a pivot in each column, then the system is consistent.

☐ False
☐ True

- (2) If the coefficient matrix of a system with four equations and four unknowns has a pivot in each row, then the system is consistent.

☐ False
☐ True

- (3) If the **augmented** matrix of a system with four equations and four unknowns has a pivot in each row, then the system is consistent.

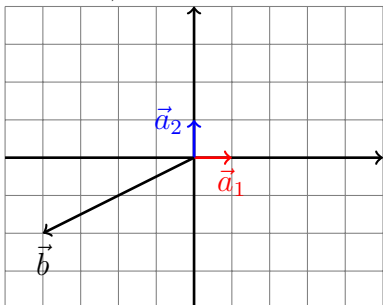
☐ False
☐ True

- (4) If the augmented matrix of a system with four equations and four unknowns has a pivot in each column, then the system is consistent.

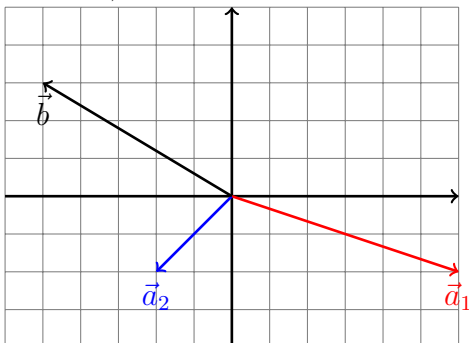
☐ False
☐ True

Question 7 Using the graphs below, find the coefficients of the requested linear combinations. There may be multiple solutions or no solution. In the graphs below, a square represents 1 unit.

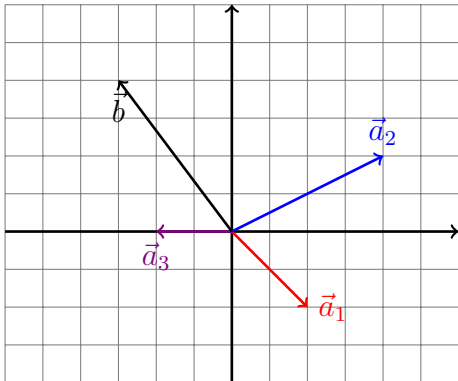
- a) Find λ_1, λ_2 such that $\vec{b} = \lambda_1 \vec{a}_1 + \lambda_2 \vec{a}_2$



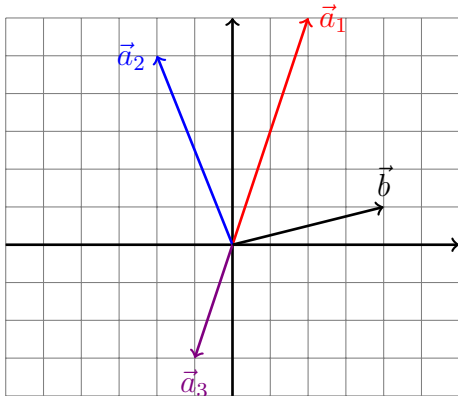
- b) Find λ_1, λ_2 such that $\vec{b} = \lambda_1 \vec{a}_1 + \lambda_2 \vec{a}_2$



- c) Find $\lambda_1, \lambda_2, \lambda_3$ such that $\vec{b} = \lambda_1 \vec{a}_1 + \lambda_2 \vec{a}_2 + \lambda_3 \vec{a}_3$



Question 8 In the graph below, find $\lambda_1, \lambda_2, \lambda_3$ such that $\vec{b} = \lambda_1 \vec{a}_1 + \lambda_2 \vec{a}_2 + \lambda_3 \vec{a}_3$. Can μ_1 and μ_3 be found such that $\vec{b} = \mu_1 \vec{a}_1 + \mu_3 \vec{a}_3$?



Question 9 Let the vectors

$$\vec{a}_1 = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}, \quad \vec{a}_2 = \begin{pmatrix} 5 \\ -13 \\ -3 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} -3 \\ 8 \\ 1 \end{pmatrix}.$$

- (1) Is it possible to write $\vec{b}$ as a linear combination of $\vec{a}_1$ and $\vec{a}_2$?
- (2) Provide a geometric interpretation of the result.

Question 10 Let

$$A = \begin{pmatrix} -3 & 1 \\ 6 & -2 \end{pmatrix}$$

Show that the equation $A\vec{x} = \vec{b}$ is not consistent for every vector $\vec{b}$ in $\mathbb{R}^2$. Find and describe the set of vectors $\vec{b}$ for which $A\vec{x} = \vec{b}$ is consistent.