

Series 1

Keywords: Linear systems, associated matrices, elementary operations, pivot columns, Gauss-Jordan algorithm

Question 1

On a two-pan balance, we place marbles, cubes, and pyramids (all objects of the same shape have the same weight). The balance is balanced when there are 7 marbles on the left and 6 cubes and 2 pyramids on the right, as well as with 4 pyramids on the left and 1 cube and 1 marble on the right.

- i) Write a linear system that represents this information and solve it.
- ii) If the weight of one of these objects is known, can the weights of the other two be determined?

Question 2 Find all three-digit numbers that increase by 270 when the first two leftmost digits are swapped and decrease by 99 when the extreme digits are swapped.

Question 3

Consider the linear system

$$\begin{cases} x_1 - 5x_2 = 0 \\ -x_1 - 2x_2 = 0 \\ 3x_1 - 3x_2 = 1 \end{cases}$$

- i) Is the system compatible?
- ii) Give a geometric interpretation of the result.

Question 4

Considering the variables in the order x, y, u, v, w , what is the augmented matrix of the system on the side?

$$\begin{cases} w + 2x - y = 4 \\ -y + x = 3 \\ w + 3x - 2y = 7 \\ 2u + 4v + w + 7x = 7 \end{cases}$$

☐ $\left(\begin{array}{ccccc|c} 0 & 0 & 1 & 2 & -1 & 4 \\ 0 & 0 & 0 & -1 & 1 & 3 \\ 0 & 0 & 1 & 3 & -2 & 7 \\ 2 & 4 & 1 & 0 & 7 & 7 \end{array} \right)$

☐ $\left(\begin{array}{ccccc|c} 2 & 0 & 1 & 2 & -1 & 4 \\ 0 & 0 & 0 & 1 & -1 & 3 \\ 0 & 0 & 1 & 3 & -2 & 7 \\ 2 & 4 & 1 & 7 & 0 & 7 \end{array} \right)$

☐ $\left(\begin{array}{ccccc|c} 2 & -1 & 0 & 0 & 1 & 4 \\ 1 & -1 & 0 & 0 & 0 & 3 \\ 3 & -2 & 0 & 0 & 1 & 7 \\ 7 & 0 & 2 & 4 & 1 & 7 \end{array} \right)$

☐ $\left(\begin{array}{ccccc|c} 1 & 2 & -1 & 0 & 0 & 4 \\ -1 & 1 & 0 & 0 & 0 & 3 \\ 1 & 3 & -2 & 0 & 0 & 7 \\ 2 & 4 & 1 & 7 & 0 & 7 \end{array} \right)$

Question 5

Using the Gauss-Jordan elimination algorithm, solve the following system:

$$\begin{cases} w + 2x - y = 4 \\ -y + x = 3 \\ w + 3x - 2y = 7 \\ 2u + 4v + w + 7x = 7 \end{cases}$$

Question 6 Let $a \in \mathbb{R}$. Using the Gauss-Jordan elimination algorithm, determine the values of the parameter a for which the system

$$\begin{cases} ax + (1 - a)y + (1 - a)z = a^2, \\ ax + (1 + a)y + (1 + a)z = a - a^2, \\ x + y + z = 1 - a \end{cases}$$

- a) has no solution,
- b) has infinitely many solutions,
- c) has a unique solution.

Then, solve the system in cases b) and c).

Question 7

Which of the columns of the following matrix is *not* a pivot column?

$$\begin{pmatrix} 0 & 1 & -1 & 3 \\ 1 & 0 & 2 & 3 \\ 1 & 2 & 0 & 3 \\ 1 & 2 & 0 & 6 \end{pmatrix}$$

☐ the fourth

☐ the third

☐ the first

☐ the second

Question 8 The following linear system, where a is a real parameter,

$$\begin{cases} 2x + 2y + 2z &= 1 \\ 2y + 2z &= 1 - 2a \\ 2x + 4ay + 2z &= 1 \\ 4x + 4ay + 2z &= 1 + 2a \end{cases}$$

☐ has no solution when $a \neq \frac{1}{2}$

☐ has infinitely many solutions when $a = \frac{1}{2}$

☐ has no solution when $a = \frac{1}{2}$

☐ has a unique solution when $a = \frac{1}{2}$

Question 9

1) A linear system can have exactly 217 solutions.

☐ False

☐ True

2) Two systems that have the same solution sets have identical associated matrices.

☐ False

☐ True

3) A matrix in reduced echelon form of size 4×5 has 5 pivot columns.

☐ True

☐ False

4) A matrix of size 6×18 has at most 6 pivot columns.

☐ True

☐ False