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Ens. : TEACHER
Analysis I - SECTION
November 21st 2020
Duration : 70 minutes

Student One

SCIPER: 111111

Do not turn the page before the start of the exam. This document is double-sided, has XX questions on 4 pages, the last pages possibly blank. Do not unstaple.

- Place your student card on your table.
- **No other paper materials** are allowed to be used during the exam.
- Using a **calculator** or any electronic device is not permitted during the exam.
- For the **multiple choice** questions, we give :
 - +3 points if your answer is correct,
 - 0 points if you give no answer or more than one,
 - 1 points if your answer is incorrect.
- For the **true/false** questions, we give :
 - +1 points if your answer is correct,
 - 0 points if you give no answer or more than one,
 - 1 points if your answer is incorrect.
- Use a **black or dark blue ballpen** and clearly erase with **correction fluid** if necessary.
- If a question is wrong, the teacher may decide to nullify it.

Respectez les consignes suivantes Observe this guidelines Beachten Sie bitte die unten stehenden Richtlinien		
choisir une réponse select an answer Antwort auswählen	ne PAS choisir une réponse NOT select an answer NICHT Antwort auswählen	Corriger une réponse Correct an answer Antwort korrigieren
  		 
ce qu'il ne faut PAS faire what should NOT be done was man NICHT tun sollte		
     		

**Common part: 8 multiple choice questions**

For each question, mark the box corresponding to the correct answer. Each question has **exactly one** correct answer.

Question 1 Let $f: [1, +\infty[\rightarrow \mathbb{R}$ be the function defined by $f(x) = \sin(\operatorname{Arctg}(\sqrt{x}))$. Then the range of f is

- ☐ $[-1, 1]$. ☐ $\left[0, \frac{\sqrt{2}}{2}\right]$. ☒ $\left[\frac{\sqrt{2}}{2}, 1\right]$. ☐ $]0, 1]$.

Question 2 Let (a_n) be the sequence of real numbers defined as $a_n = \frac{\sqrt[4n]{5} - 4}{\sqrt[5n]{4} - 5}$ for all $n \in \mathbb{N} \setminus \{0\}$. Then,

- ☐ the sequence converges and $\lim_{n \rightarrow +\infty} = \frac{5}{4}$. ☒ the sequence converges and $\lim_{n \rightarrow +\infty} = \frac{3}{4}$.
☐ the sequence converges and $\lim_{n \rightarrow +\infty} = \frac{4}{5}$. ☐ the sequence is divergent.

Question 3 Set $A = \left\{x \in \mathbb{R}_+^* \setminus \{1\} : \frac{1}{\operatorname{Log}(x)} < 1\right\}$. Then,

- ☒ $\inf A = 0$. ☐ $\inf A = e$.
☐ A is not bounded from below. ☐ $\sup A = e$.

Question 4 The imaginary part of $(-1 + i\sqrt{3})^5$ is

- ☐ $16\sqrt{3}$. ☒ $-16\sqrt{3}$. ☐ $32\sqrt{3}$. ☐ $32\sqrt{3}i$.

Question 5 Define $a_n = (\sqrt{n+2} - \sqrt{n+1}) \sin\left(\frac{1}{n}\right)$, for every $n \in \mathbb{N}^*$. Then,

- ☐ the series $\sum_{n=1}^{\infty} a_n$ is divergent.
☒ both the series $\sum_{n=1}^{\infty} a_n$ and the series $\sum_{n=1}^{\infty} (-1)^n a_n$ are convergent.
☐ the series $\sum_{n=1}^{\infty} (-1)^n a_n$ is divergent.
☐ the series $\sum_{n=1}^{\infty} a_n$ is convergent, but it is not absolutely convergent.

Question 6 Out of the functions $f, g, h: \mathbb{R} \rightarrow \mathbb{R}$ defined by

$$f(x) = \begin{cases} \sqrt{x} \sin\left(\frac{1}{x}\right) & \text{if } x > 0 \\ -\sqrt{-x} & \text{if } x \leq 0 \end{cases}, \quad g(x) = \begin{cases} x \operatorname{sh}\left(\frac{1}{x}\right) & \text{if } x > 0 \\ 0 & \text{if } x \leq 0 \end{cases},$$
$$h(x) = \begin{cases} \sqrt{x} \operatorname{Arctg}\left(\frac{1}{x}\right) & \text{si } x > 0 \\ x \operatorname{Log}(|x|) & \text{si } x < 0 \\ 0 & \text{si } x = 0 \end{cases},$$

find those that are continuous in $x = 0$:

- ☐ g and h . ☒ f and h . ☐ f and g . ☐ all three.



Question 7 The limit $\lim_{n \rightarrow \infty} \frac{\sqrt{n}}{\sqrt{5n + \sqrt{3n - \sqrt{2n}}}}$

☐ does not exist.

☒ exists, and it is $\frac{1}{\sqrt{5}}$.

☐ exists, and it is $\frac{1}{\sqrt{6}}$.

☐ exists, and it is $\frac{1}{\sqrt{5 + \sqrt{3 - \sqrt{2}}}}$.

Question 8 The series $\sum_{n=1}^{\infty} \frac{1}{\sqrt[5]{n^{\frac{2}{\alpha}}(n^{2\alpha} + 1)}}$ converges if

☐ $\frac{1}{2} < \alpha < 1$.

☐ $1 < \alpha < 2$.

☐ $\alpha = \frac{1}{2}$.

☒ $0 < \alpha < \frac{1}{2}$.

PROJET

**Common part: 5 true/false questions**

For each question, mark the box (without erasing) TRUE if the statement is **always true** and the box FALSE if it is **not always true** (i.e., it is sometimes false).

Question 9 For all $\omega \in \mathbb{C}$, $\omega \neq 0$, there exist infinitely many complex numbers $z \in \mathbb{C}$ such that $\text{Im}(\omega z) = 0$.

☒ VRAI ☐ FAUX

Question 10 Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be the function defined by

$$f(x) = \begin{cases} x^2 & \text{if } x \in \mathbb{Q} \\ x & \text{if } x \notin \mathbb{Q} \end{cases}.$$

Then f is continuous at exactly two points.

☒ VRAI ☐ FAUX

Question 11 Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be an bounded and increasing function, and for all $n \in \mathbb{N}$, let a_n be the real number defined by $a_n = f(n)$. Then $(a_n)_{n \geq 0}$ is a Cauchy sequence.

☒ VRAI ☐ FAUX

Question 12 Let $(a_n)_{n \geq 0}$ be a sequence of positive real numbers. If $\sum_{n=0}^{\infty} a_n$ converges, then $\sum_{n=0}^{\infty} (-1)^n a_n$ converges.

☒ VRAI ☐ FAUX

Question 13 A strictly increasing function $f : [0, 1] \rightarrow [0, 1]$ is always bijective.

☐ VRAI ☒ FAUX