

Ens.: TEACHER
Analysis I - SECTION
November 21st 2020
Duration: 70 minutes

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Student One

 $\mathrm{SCIPER}\colon 111111$

Do not turn the page before the start of the exam. This document is double-sided, has XX questions on 4 pages, the last pages possibly blank. Do not unstaple.

- Place your student card on your table.
- No other paper materials are allowed to be used during the exam.
- Using a calculator or any electronic device is not permitted during the exam.
- For the **multiple choice** questions, we give :
 - +3 points if your answer is correct,
 - 0 points if you give no answer or more than one,
 - -1 points if your answer is incorrect.
- For the **true/false** questions, we give :
 - +1 points if your answer is correct,
 - 0 points if you give no answer or more than one,
 - -1 points if your answer is incorrect.
- Use a black or dark blue ballpen and clearly erase with correction fluid if necessary.
- If a question is wrong, the teacher may decide to nullify it.

Respectez les consignes suivantes Observe this guidelines Beachten Sie bitte die unten stehenden Richtlinien							
choisir une réponse select an answer Antwort auswählen		ne PAS choisir une réponse NOT select an answer NICHT Antwort auswählen				Corriger une réponse Correct an answer Antwort korrigieren	
X							
ce qu'il ne faut <u>PAS</u> faire what should <u>NOT</u> be done was man <u>NICHT</u> tun sollte							
				•			

Common part: 8 multiple choice questions

For each question, mark the box corresponding to the correct answer. Each question has exactly one correct answer.

Let $f: [1, +\infty] \to \mathbb{R}$ be the function defined by $f(x) = \sin(\operatorname{Arctg}(\sqrt{x}))$. Then the range of Question 1 f is

- [-1,1].

Let (a_n) be the sequence of real numbers defined as $a_n = \frac{\sqrt[4n]{5} - 4}{\sqrt[5n]{4} - 5}$ for all $n \in \mathbb{N} \setminus \{0\}$. Then, Question 2

- \Box the sequence converges and $\lim_{n\to+\infty} = \frac{5}{4}$.
- lthe sequence converges and $\lim_{n\to+\infty} = \frac{4}{5}$.
- the sequence is divergent.

Set $A = \left\{ x \in \mathbb{R}_+^* \setminus \{1\} : \frac{1}{\text{Log}(x)} < 1 \right\}$. Then, Question 3

A is not bounded from below.

Question 4

- $16\sqrt{3}$.

- $32\sqrt{3} i$

Define $a_n = (\sqrt{n+2} - \sqrt{n+1}) \sin(\frac{1}{n})$, for every $n \in \mathbb{N}^*$. Then, Question 5

- \square the series $\sum_{n=0}^{\infty} a_n$ is divergent.
- both the series $\sum_{n=1}^{\infty} a_n$ and the series $\sum_{n=1}^{\infty} (-1)^n a_n$ are convergent.
- \square the series $\sum_{n=0}^{\infty} (-1)^n a_n$ is divergent.
- \Box the series $\sum_{n=1}^{\infty} a_n$ is convergent, but it is not absolutely convergent.

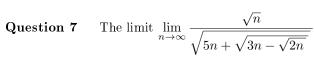
Question 6 Out of the functions $f, g, h : \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = \left\{ \begin{array}{ll} \sqrt{x} \sin(\frac{1}{x}) & \text{if } x > 0 \\ -\sqrt{-x} & \text{if } x \le 0 \end{array} \right., \qquad g(x) = \left\{ \begin{array}{ll} x \sin(\frac{1}{x}) & \text{if } x > 0 \\ 0 & \text{if } x \le 0 \end{array} \right.,$$

$$h(x) = \begin{cases} \sqrt{x} \operatorname{Arctg}\left(\frac{1}{x}\right) & \text{si } x > 0\\ x \operatorname{Log}(|x|) & \text{si } x < 0\\ 0 & \text{si } x = 0 \end{cases},$$

find those that are continuous in x = 0:

- g and h.
- $\int f$ and h.
- $\bigcap f$ and g.
- all three.



does not exist.

exists, and it is $\frac{1}{\sqrt{6}}$.

The series $\sum_{n=1}^{\infty} \frac{1}{\sqrt[5]{n^{\frac{2}{\alpha}}(n^{2\alpha}+1)}}$ converges if Question 8



Common part: 5 true/false questions

For each question, mark the box (without erasing) TRUE if the statement is **always true** and the box FALSE if it is **not always true** (i.e., it is sometimes false).

Question 9 For all $\omega \in \mathbb{C}$, $\omega \neq 0$, there exist infinitely many complex numbers $z \in \mathbb{C}$ such that $\operatorname{Im}(\omega z) = 0$.

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Question 10 Let $f: \mathbb{R} \to \mathbb{R}$ be the function defined by

$$f(x) = \begin{cases} x^2 & \text{if } x \in \mathbb{Q} \\ x & \text{if } x \notin \mathbb{Q} \end{cases}.$$

Then f is continuous at exactly two points.

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Question 11 Let $f: \mathbb{R} \to \mathbb{R}$ be an bounded and increasing function, and for all $n \in \mathbb{N}$, let a_n be the real number defined by $a_n = f(n)$. Then $(a_n)_{n \geq 0}$ is a Cauchy sequence.

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Question 12 Let $(a_n)_{n\geq 0}$ be a sequence of positive real numbers. If $\sum_{n=0}^{\infty} a_n$ converges, then $\sum_{n=0}^{\infty} (-1)^n a_n$ converges.

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Question 13 A strictly increasing function $f:[0,1] \to [0,1]$ is always bijective.

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