EPFL

Ens. : TEACHER
Analysis I - SECTION
November 21st 2020
Duration : 70 minutes

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Student One

SCIPER : **111111**

Do not turn the page before the start of the exam. This document is double-sided, has XX questions on 4 pages, the last pages possibly blank. Do not unstaple.

- Place your student card on your table.
- **No other paper materials** are allowed to be used during the exam.
- Using a **calculator** or any electronic device is not permitted during the exam.
- For the **multiple choice** questions, we give :
  $+3$ points if your answer is correct,
  0 points if you give no answer or more than one,
  -1 points if your answer is incorrect.
- For the **true/false** questions, we give :
  $+1$ points if your answer is correct,
  0 points if you give no answer or more than one,
  -1 points if your answer is incorrect.
- Use a **black or dark blue ballpen** and clearly erase with **correction fluid** if necessary.
- If a question is wrong, the teacher may decide to nullify it.

Respectez les consignes suivantes | Observe this guidelines | Beachten Sie bitte die unten stehenden Richtlinien

choisir une réponse | select an answer
Antwort auswählen

ne PAS choisir une réponse | NOT select an answer
NICHT Antwort auswählen

Corriger une réponse | Correct an answer
Antwort korrigieren

ce qu'il ne faut **PAS** faire | what should **NOT** be done | was man **NICHT** tun sollte

Pour votre examen, imprimez de préférence les documents compilés à l'aide de auto-multiple-choice.

Common part: 8 multiple choice questions

For each question, mark the box corresponding to the correct answer. Each question has **exactly one** correct answer.

Question 1 Let $f\colon [1,+\infty[\to \mathbb{R}$ be the function defined by $f(x) = \sin\left(\mathrm{Arctg}(\sqrt{x}\,)\right)$. Then the range of f is

- ☐ $[-1,1]$.
- ☐ $\left[0, \frac{\sqrt{2}}{2}\right[$.
- ☐ $\left[\frac{\sqrt{2}}{2}, 1\right[$.
- ☐ $]0,1]$.

Question 2 Let (a_n) be the sequence of real numbers defined as $a_n = \dfrac{\sqrt[4n]{5}-4}{\sqrt[5n]{4}-5}$ for all $n \in \mathbb{N}\setminus\{0\}$. Then,

- ☐ the sequence converges and $\lim\limits_{n\to+\infty} = \frac{5}{4}$.
- ☐ the sequence converges and $\lim\limits_{n\to+\infty} = \frac{3}{4}$.
- ☐ lthe sequence converges and $\lim\limits_{n\to+\infty} = \frac{4}{5}$.
- ☐ the sequence is divergent.

Question 3 Set $A = \left\{ x \in \mathbb{R}_+^* \setminus \{1\} \;:\; \dfrac{1}{\mathrm{Log}(x)} < 1 \right\}$. Then,

- ☐ $\mathrm{Inf}\, A = 0$.
- ☐ $\mathrm{Inf}\, A = \mathrm{e}$.
- ☐ A is not bounded from below.
- ☐ $\mathrm{Sup}\, A = \mathrm{e}$.

Question 4 The imaginary part of $\left(-1 + i\sqrt{3}\,\right)^5$ is

- ☐ $16\sqrt{3}$.
- ☐ $-16\sqrt{3}$.
- ☐ $32\sqrt{3}$.
- ☐ $32\sqrt{3}\, i$.

Question 5 Define $a_n = \left(\sqrt{n+2} - \sqrt{n+1}\,\right)\sin\left(\frac{1}{n}\right)$, for every $n \in \mathbb{N}^*$. Then,

- ☐ the series $\sum\limits_{n=1}^{\infty} a_n$ is divergent.
- ☐ both the series $\sum\limits_{n=1}^{\infty} a_n$ and the series $\sum\limits_{n=1}^{\infty} (-1)^n a_n$ are convergent.
- ☐ the series $\sum\limits_{n=1}^{\infty} (-1)^n a_n$ is divergent.
- ☐ the series $\sum\limits_{n=1}^{\infty} a_n$ is convergent, but it is not absolutely convergent.

Question 6 Out of the functions $f, g, h : \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = \begin{cases} \sqrt{x}\,\sin\left(\frac{1}{x}\right) & \text{if } x > 0 \\ -\sqrt{-x} & \text{if } x \leq 0 \end{cases}, \qquad g(x) = \begin{cases} x\,\mathrm{sh}\left(\frac{1}{x}\right) & \text{if } x > 0 \\ 0 & \text{if } x \leq 0 \end{cases},$$

$$h(x) = \begin{cases} \sqrt{x}\,\mathrm{Arctg}\left(\frac{1}{x}\right) & \text{si } x > 0 \\ x\,\mathrm{Log}(|x|) & \text{si } x < 0 \\ 0 & \text{si } x = 0 \end{cases},$$

find those that are continuous in $x = 0$:

- ☐ g and h.
- ☐ f and h.
- ☐ f and g.
- ☐ all three.

Question 7 The limit $\lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{5n+\sqrt{3n-\sqrt{2n}}}}$

- [ ] does not exist.
- [ ] exists, and it is $\frac{1}{\sqrt{5}}$.
- [ ] exists, and it is $\frac{1}{\sqrt{6}}$.
- [ ] exists, and it is $\frac{1}{\sqrt{5+\sqrt{3-\sqrt{2}}}}$.

Question 8 The series $\sum_{n=1}^{\infty} \frac{1}{\sqrt[5]{n^{\frac{2}{\alpha}}\left(n^{2\alpha}+1\right)}}$ converges if

- [ ] $\frac{1}{2} < \alpha < 1$.
- [ ] $1 < \alpha < 2$.
- [ ] $\alpha = \frac{1}{2}$.
- [ ] $0 < \alpha < \frac{1}{2}$.

Common part: 5 true/false questions

For each question, mark the box (without erasing) TRUE if the statement is **always true** and the box FALSE if it is **not always true** (i.e., it is sometimes false).

Question 9 For all $\omega \in \mathbb{C}$, $\omega \neq 0$, there exist infinitely many complex numbers $z \in \mathbb{C}$ such that $\operatorname{Im}(\omega z) = 0$.

☐ VRAI ☐ FAUX

Question 10 Let $f : \mathbb{R} \to \mathbb{R}$ be the function defined by

$$f(x) = \begin{cases} x^2 & \text{if } x \in \mathbb{Q} \\ x & \text{if } x \notin \mathbb{Q} \end{cases}.$$

Then f is continuous at exactly two points.

☐ VRAI ☐ FAUX

Question 11 Let $f : \mathbb{R} \to \mathbb{R}$ be an bounded and increasing function, and for all $n \in \mathbb{N}$, let a_n be the real number defined by $a_n = f(n)$. Then $(a_n)_{n \geq 0}$ is a Cauchy sequence.

☐ VRAI ☐ FAUX

Question 12 Let $(a_n)_{n \geq 0}$ be a sequence of positive real numbers. If $\sum_{n=0}^{\infty} a_n$ converges, then $\sum_{n=0}^{\infty} (-1)^n a_n$ converges.

☐ VRAI ☐ FAUX

Question 13 A strictly increasing function $f : [0,1] \to [0,1]$ is always bijective.

☐ VRAI ☐ FAUX