

**EPFL**

Ens: Z. Patakfalvi
Analysis I - (n/a)
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65 minutes

n/a

n/a

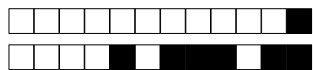
SCIPER : 999999

Do not turn the page before the start of the exam. This document is double-sided, has 4 pages, the last ones possibly blank. Do not unstaple.

- Place your student card on your table.
- **No other paper materials** are allowed to be used during the exam.
- Using a **calculator** or any electronic device is not permitted during the exam.
- For the **multiple choice** questions, we give :
 - +3 points if your answer is correct,
 - 0 points if you give no answer or more than one,
 - 1 points if your answer is incorrect.
- For the **true/false** questions, we give :
 - +1 points if your answer is correct,
 - 0 points if you give no answer or more than one,
 - 1 points if your answer is incorrect.
- Use a **black or dark blue ballpen** and clearly erase with **correction fluid** if necessary.
- If a question is wrong, the teacher may decide to nullify it.

This exam is printed on recycled paper.

Respectez les consignes suivantes Observe this guidelines Beachten Sie bitte die unten stehenden Richtlinien		
choisir une réponse select an answer Antwort auswählen	ne PAS choisir une réponse NOT select an answer NICHT Antwort auswählen	Corriger une réponse Correct an answer Antwort korrigieren
  		 
ce qu'il ne faut PAS faire what should NOT be done was man NICHT tun sollte		
     		

**First part: multiple choice questions**

For each question, mark the box corresponding to the correct answer. Each question has **exactly one** correct answer.

Question 1 : Let s be a real number, and let $(b_n)_{n \geq 1}$ be the sequence defined by $b_n = \frac{1}{n^s}$ if n is even, and $b_n = \frac{1}{n^{2s}}$ if n is odd. Then the series $\sum_{n=1}^{\infty} b_n$ converges if and only if

- ☐ $s > 2$ ☐ $s > 1$ ☐ $s > 0$ ☐ $s > \frac{1}{2}$

Question 2 : Let A be the subset of $\mathbb{R}$ defined by $A = \left\{x > 0 : \cos\left(\frac{1}{x}\right) > 0\right\}$. Then

- ☐ $\inf A = 0$ ☐ $\sup A = 0$ ☐ $\sup A = \frac{\pi}{2}$ ☐ $\inf A = \frac{2}{\pi}$

Question 3 : Let $(a_n)_{n \geq 0}$ be the sequence defined by $a_0 = \frac{3}{2}$, and $a_{n+1} = \frac{1}{2} + \frac{1}{2}\sqrt{8a_n - 7}$, for every $n \geq 0$. Then:

- ☐ $\lim_{n \rightarrow \infty} a_n = +\infty$ ☐ $\lim_{n \rightarrow \infty} a_n = 1$
☐ the sequence is divergent ☐ $\lim_{n \rightarrow \infty} a_n = 2$

Question 4 : Let $S \subseteq \mathbb{C}$ be the set of solutions of the equation $\bar{z}^2 = z^2$ among the complex numbers. Then:

- ☐ $S = \{z \in \mathbb{C} : \operatorname{Re}(z) = 0 \text{ or } \operatorname{Im}(z) = 0\}$ ☐ $S = \{-1, +1, -i, +i\}$
☐ $S = \emptyset$ ☐ $S = \mathbb{R}$

Question 5 : Let $m \in \mathbb{R}$ be a real number, and let $f : \mathbb{R} \rightarrow \mathbb{R}$ be the function defined by

$$f(x) = \begin{cases} \frac{\sin^2(x)}{\ln(1+2x^2)} & \text{if } x < 0, \\ m & \text{if } x = 0, \\ \frac{x+1}{x^2+3x+1} & \text{if } x > 0. \end{cases}$$

- ☐ If $m = \frac{1}{2}$, then f is continuous at $x = 0$.
☐ If $m = \frac{1}{2}$, then f is left continuous but not right continuous at $x = 0$.
☐ If $m = 1$, then f is continuous at $x = 0$.
☐ If $m = \frac{1}{3}$, then f is right continuous but not left continuous at $x = 0$.

Question 6 : Set $\lambda := -\frac{1}{6}$. Find the convergent one out of the following series:

- ☐ $\sum_{n=1}^{\infty} \left(\frac{\lambda+1}{\lambda-1}\right)^n$ ☐ $\sum_{n=1}^{\infty} \left(\frac{1}{1-\lambda^2}\right)^n$ ☐ $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\lambda^n}$ ☐ $\sum_{n=1}^{\infty} \frac{1}{n^{1+\lambda}}$



Question 7 : Let $(x_n)_{n \geq 1}$ be the sequence defined by $x_n = \sqrt[n]{7}$ if n is even and by $x_n = \frac{1}{n^7}$ if n is odd. Then:

☐ $\limsup_{n \rightarrow \infty} x_n = 0$, and $\liminf_{n \rightarrow \infty} x_n = 1$

☐ $\limsup_{n \rightarrow \infty} x_n = 1$, and $\liminf_{n \rightarrow \infty} x_n = 0$

☐ $\limsup_{n \rightarrow \infty} x_n = \liminf_{n \rightarrow \infty} x_n = 1$

☐ $\limsup_{n \rightarrow \infty} x_n = \liminf_{n \rightarrow \infty} x_n = 0$

Second part: true/false questions

For each question, mark the box (without erasing) TRUE if the statement is **always true** and the box FALSE if it is **not always true** (i.e., it is sometimes false).

Question 8 : Let $z \neq 0$ be a complex number with argument $\frac{\pi}{4}$. Then, the argument of the complex number $\frac{1}{z^2}$ is $-\frac{\pi}{2}$.

☐ TRUE ☐ FALSE

Question 9 : Let $(x_n)_{n \geq 0}$ be the sequence defined by $x_0 = 2$, and $x_n = x_{n-1} - \frac{1}{n}$, for every $n \geq 1$. Then $(x_n)_{n \geq 0}$ is convergent.

☐ TRUE ☐ FALSE

Question 10 : Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a bijective function. Then f is strictly monotone.

☐ TRUE ☐ FALSE

Question 11 : Let $A \subset \mathbb{R}$ be a bounded set, and let $B := \{x \in \mathbb{R} : x \text{ is an upper bound of } A\}$. Then $\inf B \in B$.

☐ TRUE ☐ FALSE

**Third part, open question**

Answer in the empty space below. Your answer should be carefully justified, and all the steps of your argument should be discussed in details. Leave the check-boxes empty, they are used for the grading.

Question 12: *This question is worth 5 points.*

☐ ₀ ☐ ₁ ☐ ₂ ☐ ₃ ☐ ₄ ☐ ₅

Do not write here.

Consider the recursive sequence given by $x_n = \sqrt{\frac{3}{2}x_{n-1} - \frac{1}{2}}$ and $x_0 = 2$.

- (a) Show by induction that $x_n \geq 1$ for all $n \in \mathbb{N}$ [2 pts].
- (b) Show by induction that x_n is a decreasing sequence [2 pts].
- (c) Argue that $\lim_{n \rightarrow \infty} x_n$ exists, and compute it [1 pts].

