CATALOG

Lecturer: R. Svaldi
Analysis I - (n/a)
11th January 2021
3 hours

n/a

n/a

SCIPER: **999999**

Do not turn the page before the start of the exam. This document is double-sided, has 8 pages, the last ones possibly blank. TOTAL: 34 questions. Do not unstaple.

- Place your student card on your table.
- **The only papers you are allowed to use are the booklet of the exam and the scratch paper provided by the proctors.**
- Using a **calculator** or any electronic device is not permitted during the exam.
- For the **multiple choice** questions, we give :
  - $+3$ points if your answer is correct,
  - 0 points if you give no answer or more than one,
  - -1 points if your answer is incorrect.
- For the **true/false** questions, we give :
  - $+1$ points if your answer is correct,
  - 0 points if you give no answer or more than one,
  - -1 points if your answer is incorrect.
- Use a **black or dark blue ballpen** and clearly erase with **correction fluid** if necessary.
- If a question is wrong, the teacher may decide to nullify it.

Respectez les consignes suivantes | Read these guidelines | Beachten Sie bitte die unten stehenden Richtlinien

choisir une réponse | select an answer Antwort auswählen	ne PAS choisir une réponse | NOT select an answer NICHT Antwort auswählen	Corriger une réponse | Correct an answer Antwort korrigieren

ce qu'il ne faut **PAS** faire | what should **NOT** be done | was man **NICHT** tun sollte

Part 1: this part of the exam contains 23 multiple choice questions.

For each question, mark the box corresponding to the answer that you wish to select as the correct one.
Each question has a unique correct answer.

Question [QCM-complexes-A] **:** Let z be the complex number $z := 1 + \sqrt{3}\mathrm{i}$. Then,

☐ $|z^6| > 73$ ☐ $z^{14} \in \mathbb{R}$ ☐ $\mathrm{Re}(z^8) > 0$ ■ $\mathrm{Im}(z^{11}) < 0$

Question [QCM-complexes-B] **:** Consider the sets $A, B \subseteq \mathbb{C}$,

$$A := \left\{z \in \mathbb{C}\colon\, z^2\,(|z| - 2) = 0\right\}, \qquad B := \{z \in \mathbb{C}\colon\, \mathrm{Re}\,(z) = 1\}.$$

Then,

■ $A \cap B$ consists exactly of two distinct points.
☐ $A \cap B$ is the empty set.
☐ $A \cap B$ consists exactly of one point.
☐ $A \cap B$ contains a line.

Question [QCM-contin-deriv-C1-B] **:** Let a, b be two real numbers for which the function $f\colon \mathbb{R} \to \mathbb{R}$,

$$f(x) := \begin{cases} ax + b & \text{for } x \leq 0 \\ \dfrac{\sqrt{1+x} - 1}{x} & \text{for } x > 0 \end{cases}$$

is differentiable at $x = 0$. Then,

☐ $f(-3) = \frac{1}{4}$ ☐ $f(-3) = -\frac{3}{8}$ ■ $f(-3) = \frac{7}{8}$ ☐ $f(-3) = \frac{3}{8}$

Question [QCM-contin-vs-derivab-A] **:** Let $f\colon \mathbb{R} \to \mathbb{R}$ be the function

$$f(x) := \begin{cases} 1 & \text{for } x \leq 0\,, \\ \sqrt{1 - x^2} & \text{for } 0 < x \leq 1\,, \\ 0 & \text{for } x > 1\,. \end{cases}$$

Then,

☐ f is continuous at $x = 0$ and f is differentiable at $x = 1$
☐ the left derivative of f exists at $x = 0$ and f is differentiable at $x = 1$
■ f is differentiable at $x = 0$ and f is continuous at $x = 1$
☐ the right derivative of f exists at $x = 0$ and f is differentiable at $x = 1$

Question [QCM-derivee-B] **:** The derivative of the function $f(x) := (1 + x^2)^{1+x^2}$ at the point $x = 1$ is

■ $8\,(\log(2) + 1)$ ☐ $4\,(\log(2) + 1)$ ☐ 4 ☐ 8

Question [`QCM-dev-limite-A`] **:** Let $f\colon\ ]-1,1[\ \to \mathbb{R}$ be the function $f(t) := \dfrac{1}{4+3t}$, and take $t_0 = 0$. Then the expansion of f to order 2 around t_0 is given by

■ $f(t) = \frac{1}{4} - \frac{3}{16}t + \frac{9}{64}t^2 + t^2\varepsilon(t)$

□ $f(t) = \frac{1}{4} - \frac{3}{16}t + \frac{9}{128}t^2 + t^2\varepsilon(t)$

□ $f(t) = \frac{1}{4} + \frac{3}{16}t - \frac{9}{64}t^2 + t^2\varepsilon(t)$

□ $f(t) = \frac{1}{4} - \frac{3}{16}t + \frac{9}{32}t^2 + t^2\varepsilon(t)$

Question [`QCM-dev-limite-B`] **:** Let $f\colon \mathbb{R} \to \mathbb{R}$ be the function $f(x) := \sin(\sin(x))$ and take $x_0 = 0$. Then, the expansion of f to order 5 around x_0 is given by

■ $f(x) = x - \frac{1}{3}x^3 + \frac{1}{10}x^5 + x^5\varepsilon(x)$

□ $f(x) = x - \frac{1}{6}x^3 + \frac{1}{120}x^5 + x^5\varepsilon(x)$

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□ $f(x) = x - \frac{1}{6}x^3 + \frac{1}{10}x^5 + x^5\varepsilon(x)$

Question [`QCM-induction-B1`] **:**
Let $f\colon \mathbb{R} \to \mathbb{R}$ be the function $f(x) := x^3$. We define the function $f_1 := f$ and, for any natural number $n \geq 2$, the function $f_n := f \circ f_{n-1}$. Then, for all natural numbers $m \geq 1$,

■ $f_m(x) = x^{(3^m)}$ □ $f_m(x) = x^{(3m)}$ □ $f_m(x) = (3x)^m$ □ $f_m(x) = mx^3$

Question [`QCM-inf-sup-B`] **:** Let $A \subset \mathbb{R}$ be the set $A := \left\{x \in \mathbb{R}\colon 0 < \arctan\left(\frac{1}{x}\right) < \frac{\pi}{4}\right\}$. Then,

■ A is not bounded

□ $A = \left]1, \frac{\pi}{2}\right[$

□ $\operatorname{Inf} A = \frac{\pi}{2}$

□ $A =]0,1[$

Question [`QCM-int-generalisee-B`] **:** The generalised integral $\displaystyle\int_1^{2^-} \frac{x+1}{\sqrt{2-x}}\,\mathrm{d}x$

■ converges and its value is $\frac{16}{3}$

□ converges and its value is 4

□ converges and its value is $\frac{8}{3}$

□ diverges

Question [`QCM-integrale-second-A`] **:** The value of the integral $\displaystyle\int_{-\pi}^{\pi} x\,\sin(x)\,\mathrm{d}x$ is

■ 2π □ $-\pi$ □ $\frac{3\pi}{2}$ □ 0

Question [`QCM-integrale-second-B`] **:** The value of the integral $\displaystyle\int_0^1 x\sqrt{x^2+1}\,\mathrm{d}x$ is

■ $\dfrac{2\sqrt{2}-1}{3}$ □ $3(2\sqrt{2}-1)$ □ $\dfrac{\sqrt{2}-1}{2}$ □ $\dfrac{\sqrt{2}}{2}$

Question [`QCM-limite-B`] **:** Let $(a_n)_{n\geq 0}$ be the sequence $a_n := \dfrac{(n+3)^{1/2} - n^{1/2}}{(n+1)^{-1/2}}$. Then,

■ $\lim\limits_{n\to\infty} a_n = \frac{3}{2}$ □ $\lim\limits_{n\to\infty} a_n = 0$ □ $\lim\limits_{n\to\infty} a_n = 3$ □ $\lim\limits_{n\to\infty} a_n = +\infty$

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Question [`QCM-limite-prolongmt-A`] **:** Let $f\colon\,]-\pi,\pi[\,\setminus\{0\}\to\mathbb{R}$ be the function defined as $f(x)=\dfrac{\arctan(x^2)}{x\sin(x)}$. Then,

- ■ f can be extended to a function $\hat{f}(x)$ which is continuous at $x=0$ and $\hat{f}(0)=1$.
- ☐ f can be extended to a function $\hat{f}(x)$ which is continuous at $x=0$ and $\hat{f}(0)=0$.
- ☐ f can be extended to a function $\hat{f}(x)$ which is continuous at $x=0$ and $\hat{f}(0)=\frac{\pi}{2}$.
- ☐ f cannot be extended to a function $\hat{f}(x)$ which is continuous at $x=0$, since $\lim\limits_{x\to 0^+} f(x)\neq \lim\limits_{x\to 0^-} f(x)$.

Question [`QCM-limite-prolongmt-D`] **:** Which of the functions $f,g\colon\mathbb{R}\to\mathbb{R}$, defined as

$$f(x):=\begin{cases}\sqrt{|x|}\,\sin(\frac{1}{x}) & \text{for } x\neq 0,\\ 1 & \text{for } x=0,\end{cases}\qquad g(x):=\begin{cases}\frac{1}{x}\arctan(x) & \text{for } x\neq 0,\\ 1 & \text{for } x=0,\end{cases}$$

are continuous at $x=0$?

☐ Both f and g ☐ Only f ■ Only g ☐ Neither f nor g

Question [`QCM-minmax-A`] **:** Given any function $f\colon\mathbb{R}\to\mathbb{R}$ which is twice differentiable on $\mathbb{R}$ and that admits a point of local minimum at $x=0$, then,

- ■ $f'(0)=0$ and $f''(0)\geq 0$
- ☐ $f'(0)=0$ and $f''(0)\neq 0$
- ☐ $f'(0)\neq 0$ and $f''(0)\neq 0$
- ☐ $f'(0)=0$ and $f''(0)\leq 0$

Question [`QCM-serie-A`] **:** Let $(a_n)_{n\geq 1}$ be the sequence $a_n:=(-1)^n\sin\left(\frac{1}{n^2}\right)$. Then,

- ■ $\sum\limits_{n=1}^{\infty} a_n$ converges absolutely
- ☐ $\sum\limits_{n=1}^{\infty}(a_n)^2$ converges, but $\sum\limits_{n=1}^{\infty} a_n$ diverges
- ☐ $\sum\limits_{n=1}^{\infty} a_n$ converges, but it does not converge absolutely
- ☐ $\lim\limits_{n\to\infty} a_n=0$, but $\sum\limits_{n=1}^{\infty} a_n$ diverges

Question [`QCM-serie-entiere-A`] **:** Let $I\subset\mathbb{R}$ be the domain of convergence of the power series $\sum\limits_{n=0}^{\infty}\sqrt{n}\,(x+1)^n$. Then,

☐ $I=\mathbb{R}$ ☐ $I=\left[-\frac{5}{2},\frac{1}{2}\right]$ ☐ $I=\,]-1,1[$ ■ $I=\,]-2,0[$

Question [`QCM-serie-parametre-B`] **:** For what values of the parameter $b\in\mathbb{R}$ does the series

$$\sum_{k=1}^{\infty}\left(b+\frac{1}{k}\right)^k$$

converge?

■ $b\in\,]-1,1[$ ☐ $b\in\,]-1,1]$ ☐ $b<1$ ☐ $b\leq 1$

Question [`QCM-suites-convergence-B`] : Let $(a_n)_{n\geq 1}$ be the sequence $a_n := \dfrac{(5n+1)^n}{n^n 5^n}$. Then,

- ■ $\lim\limits_{n\to\infty} a_n = \mathrm{e}^{\frac{1}{5}}$
- □ $\lim\limits_{n\to\infty} a_n = 5$
- □ $\lim\limits_{n\to\infty} a_n = +\infty$
- □ $\lim\limits_{n\to\infty} a_n = 0$

Question [`QCM-suites-convergence-C`] : Let $c \in \mathbb{R}$, and let $(a_n)_{n\geq 1}$ be the sequence

$$a_n := \begin{cases} \sum_{k=0}^{n} \frac{2^k}{k!} & \text{for } n \text{ even,} \\ \left(\sum_{k=0}^{n} \frac{1}{k!} \right)^c & \text{for } n \text{ odd.} \end{cases}$$

Then,

- ■ the sequence $(a_n)_{n\geq 1}$ converges for exactly one value of c
- □ The sequence $(a_n)_{n\geq 1}$ diverges for any value of c
- □ $\lim\limits_{m\to\infty} a_{2m} = +\infty$
- □ $\lim\limits_{m\to\infty} a_{2m+1} = c$

Question [`QCM-theo-accr-finis-B`] : Let $f\colon [0,4] \to \mathbb{R}$ be a function which is continuous on $[0,4]$, differentiable on $]0,4[$ and such that $f'(x) \geq 2$ for all $x \in]0,4[$. Then,

- □ $f(4) - f(1) \leq 4$
- □ $f(4) - f(0) \leq 1$
- ■ $f(2) - f(0) \geq 4$
- □ $0 \leq f(3) - f(2) \leq 1$

Question [`QCM-val-intermed-image-interv-B`] : Let $f\colon \mathbb{R} \to \mathbb{R}$ be the function $f(x) := x^2 \sin(x^2)$ and let $R(f) \subset \mathbb{R}$ be the range of f,

$$R(f) := \{y \in \mathbb{R} \,|\, \exists\, x \in \mathbb{R} \text{ such that } f(x) = y\}\,.$$

Then,

- ■ $R(f) = \mathbb{R}$
- □ $R(f) = [-1,1]$
- □ $R(f) = [0,1]$
- □ $R(f) = [0,+\infty[$

Part 2: this part of the exam contains 11 true/false questions.

For each question, mark the box next to "TRUE" if you think the statement of the question is correct or mark the box next to "FALSE" if you think the statement of the question is incorrect.

Question [`TF-complexes-A`] : For all given $y \in \mathbb{R}$, $y \neq 0$, the equation $z^4 = \mathrm{i}y$, in the indeterminate z, has exactly 4 distinct solutions in $\mathbb{C}$.

■ TRUE ☐ FALSE

Question [`TF-derivabilite-discussion-A`] : Let $g, h \colon]-1, 1[\to \mathbb{R}$ be two functions which are differentiable on $]-1, 1[$. Assume that $g(0) = h(0) = 0$, and $h'(x) \neq 0$ for all $x \in]-1, 1[$. If $\lim_{x \to 0} \frac{g'(x)}{h'(x)}$ does not exist, then also $\lim_{x \to 0} \frac{g(x)}{h(x)}$ does not exist.

☐ TRUE ■ FALSE

Question [`TF-dev-limite-A`] : Let $f \colon \mathbb{R} \to \mathbb{R}$ be a function and take $x_0 = 0$. Assume that f admits an expansion to order 2 around x_0 given by $f(x) = a + bx + cx^2 + x^2\varepsilon(x)$. Then, the expansion to order 2 around x_0 for the function $g(x) := f(x)^3$ is given by $g(x) = a^3 + b^3x + c^3x^2 + x^2\varepsilon(x)$.

☐ TRUE ■ FALSE

Question [`TF-fonction-etc-B`] : Let $f \colon]0, 1[\to \mathbb{R}$ be a monotone function which is non-constant and differentiable over $]0, 1[$. Then, either $f'(x) \geq 0$ for all $x \in]0, 1[$ or $f'(x) \leq 0$ for all $x \in]0, 1[$.

■ TRUE ☐ FALSE

Question [`TF-induction-suites-limites-B`] : Let $(a_n)_{n \geq 0}, (b_n)_{n \geq 0}$ be two converging sequences. Assume that $b_n \neq 0$ for all $n \geq 0$. Then, the limit $\lim_{n \to \infty} \frac{a_n}{b_n}$ exists.

☐ TRUE ■ FALSE

Question [`TF-inf-sup-A`] : Let $A \subset \mathbb{R}$ be a subset of the real numbers. If $\operatorname{Inf} A \in A$ and $\operatorname{Sup} A \in A$, then A is a closed interval.

☐ TRUE ■ FALSE

Question [`TF-integrale-A`] : Let $(a_n)_{n\geq 1}$ be a sequence of real numbers such that $\lim_{n\to\infty} a_n = 1$, and let $f\colon \mathbb{R}\to\mathbb{R}$ be a continuous function. Let $(b_n)_{n\geq 1}$ be the sequence

$$b_n = \int_0^{a_n} f(x)\,\mathrm{d}x\,, \qquad n\geq 1\,.$$

Then, the sequence $(b_n)_{n\geq 1}$ converges.

■ TRUE ☐ FALSE

Question [`TF-soussuite-A`] : Let $f\colon \mathbb{R}\to\mathbb{R}$ be a function. If there exists $a\in\mathbb{R}$ such that $\lim_{x\to a} f(x) = +\infty$, then f is not continuous on $\mathbb{R}$.

■ TRUE ☐ FALSE

Question [`TF-serie-B`] : Let $f\colon \mathbb{N}\to\mathbb{R}$ be a function such that for all $n\geq 1$, $f(n) > n$. Then the series $\displaystyle\sum_{n=1}^{\infty}\frac{1}{f(n)}$ converges.

☐ TRUE ■ FALSE

Question [`TF-serie-entiere-A`] : Let $(a_k)_{k\geq 0}$ be a sequence of real numbers such that for all $k\geq 0$, $a_k\neq 0$, and $\displaystyle\lim_{k\to\infty}\left|\frac{a_{k+1}}{a_k}\right| = 0$. Then the power series $\displaystyle\sum_{k=0}^{\infty} a_k\,x^k$ converges for all $x\in\mathbb{R}$.

■ TRUE ☐ FALSE

Question [`TF-soussuite-B`] : Let $(x_n)_{n\geq 0}$ be a sequence of real numbers for which $\lim_{n\to\infty} x_n = +\infty$. Then, the sequence $(x_n)_{n\geq 0}$ does not contain any bounded subsequence.

■ TRUE ☐ FALSE