CORRECTED

First part: multiple choice questions

For each question, mark the box corresponding to the correct answer. Each question has **exactly one** correct answer.

Question 1 : Consider the series with parameter $x \in \,]0,1[\, \cup \,]1,+\infty[$ defined by

$$\sum_{n=1}^{\infty} \frac{1}{(\log(x))^n}.$$

Then, the series converges if and only if

■ $x \in \,]0, \frac{1}{e}[\, \cup \,]e, +\infty[$

□ $x \in \,]\frac{1}{e}, 1[\, \cup \,]1, e[$

□ $x \in \,]e, +\infty[$

□ $x \in \,]0, \frac{1}{e}[$

Question 2 : Let $(a_n)_{n\geq 1}$ be the sequence $a_n := \dfrac{(5n+1)^n}{n^n 5^n}$. Then,

□ $\lim_{n\to\infty} a_n = +\infty$ □ $\lim_{n\to\infty} a_n = 5$ ■ $\lim_{n\to\infty} a_n = e^{\frac{1}{5}}$ □ $\lim_{n\to\infty} a_n = 0$

Question 3 : Let $a_0 \in \mathbb{R}$ and $(a_n)_{n\geq 0}$ a sequence of real numbers satisfying the following recurrence relation for $n \geq 1$

$$a_n = \frac{a_{n-1}}{2} + \frac{1}{2}.$$

Then:

■ if $a_0 = 0$ the sequence is convergent

□ if $a_0 < 1$ the sequence is decreasing

□ if $a_0 > 1$ the sequence is increasing

□ if $a_0 < 0$, $\lim_{n\to\infty} a_n = -\infty$

Question 4 : Let $(a_n)_{n\geq 1}$ be the sequence defined by $a_n = (-1)^n + \frac{1}{n}$, and let $A = \{a_1, a_2, a_3, \dots\}$. Then :

■ $\inf A = -1$ and $\sup A = 1 + \frac{1}{2}$

□ $\inf A = -1$ and $\sup A = 1$

□ $\inf A = 0$ and $\sup A = 1$

□ $\inf A = 0$ and $\sup A = 1 + \frac{1}{2}$

Question 5 : Let $(u_n)_{n\geq 0}$ be the sequence defined by $u_0 = 1$ and, for $n \geq 1$, $u_n = -\frac{2}{3}u_{n-1} + 2$. Then :

□ $\lim_{n\to\infty} u_n = +\infty$

■ $\lim_{n\to\infty} u_n = \frac{6}{5}$

□ $\lim_{n\to\infty} u_n = 2$

□ $\lim_{n\to\infty} u_n = -\infty$

Question 6 : Let $(a_n)_{n\geq 1}$ be the sequence defined by

$$a_n = (-1)^{n+1} + \left(-\frac{1}{2}\right)^n + \frac{3}{n}.$$

Then :

□ $\liminf_{n\to\infty} a_n = -\frac{1}{4}$ and $\limsup_{n\to\infty} a_n = \frac{3}{2}$

■ $\liminf_{n\to\infty} a_n = -1$ and $\limsup_{n\to\infty} a_n = 1$

□ $\liminf_{n\to\infty} a_n = -1$ and $\limsup_{n\to\infty} a_n = \frac{3}{2}$

□ $\liminf_{n\to\infty} a_n = \frac{3}{4}$ and $\limsup_{n\to\infty} a_n = \frac{7}{2}$

Question 7 : One of the solutions of the equation $z^5 = \left(1+\sqrt{3}\,\mathrm{i}\right)^2$ is

- ☐ $z = \sqrt[5]{2}\left(\cos\left(\frac{16\pi}{15}\right) + \mathrm{i}\sin\left(\frac{16\pi}{15}\right)\right)$
- ☐ $z = \sqrt[5]{4}\left(\cos\left(\frac{16\pi}{15}\right) + \mathrm{i}\sin\left(\frac{16\pi}{15}\right)\right)$
- ■ $z = \sqrt[5]{4}\left(\cos\left(\frac{2\pi}{15}\right) + \mathrm{i}\sin\left(\frac{2\pi}{15}\right)\right)$
- ☐ $z = \sqrt[5]{2}\left(\cos\left(\frac{2\pi}{15}\right) + \mathrm{i}\sin\left(\frac{2\pi}{15}\right)\right)$

Question 8 : Let, for $k \in \mathbb{N}^*$, $a_k = (-1)^k \dfrac{k+2}{k^3}$ and let $s_n = \displaystyle\sum_{k=1}^{n} a_k$. Then :

- ☐ $\lim\limits_{n\to\infty} s_n = -\infty$.
- ☐ the series $\displaystyle\sum_{k=1}^{+\infty} a_k$ converges, but not absolutely.
- ☐ $\lim\limits_{n\to\infty} s_n = +\infty$
- ■ the series $\displaystyle\sum_{k=1}^{+\infty} a_k$ converges absolutely.

Second part: true/false questions

For each question, mark the box (without erasing) TRUE if the statement is **always true** and the box FALSE if it is **not always true** (i.e., it is sometimes false).

Question 9 : Let A and B be two nonempty bounded subsets of $\mathbb{R}$. If $\inf A > \sup B$, then $A \cap B$ is empty.

■ TRUE ☐ FALSE

Question 10 : Let $(a_n)_{n\geq 1}$ a sequence of strictly negative numbers. Then the series $\sum_{n=1}^{\infty} a_n$ converges absolutely if and only if it converges.

■ TRUE ☐ FALSE

Question 11 : Let $z_1, z_2 \in \mathbb{C}$ be such that $\mathrm{Re}\,(z_1 \cdot z_2) = 0$. Then $\mathrm{Re}\,(z_1) \cdot \mathrm{Re}\,(z_2) = 0$.

☐ TRUE ■ FALSE

Question 12 : Let $(a_n)_{n\geq 0}$ be a sequence of non zero real numbers such that $\lim_{n\to\infty} a_n = 2$. Then $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 1$.

■ TRUE ☐ FALSE

Question 13 : Let $f\colon \mathbb{R} \to \mathbb{R}$ be a function such that $\lim_{x\to+\infty} f(x) = +\infty$ and let $(a_n)_{n\geq 0}$ be the sequence defined by $a_0 = 1$ and, for $n \geq 1$, $a_n = f(a_{n-1})$. Then $\lim_{n\to\infty} a_n = +\infty$.

☐ TRUE ■ FALSE