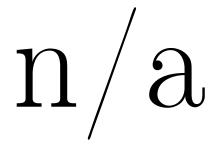


Lecturer: R. Svaldi Analysis I - (n/a) 11th January 2021 3 hours



SCIPER: 999999

Do not turn the page before the start of the exam. This document is double-sided, has 8 pages, the last ones possibly blank. TOTAL: 34 questions. Do not unstaple.

- Place your student card on your table.
- The only papers you are allowed to use are the booklet of the exam and the scratch paper provided by the proctors.
- Using a **calculator** or any electronic device is not permitted during the exam.
- For the **multiple choice** questions, we give :
  - +3 points if your answer is correct,
    - 0 points if you give no answer or more than one,
  - -1 points if your answer is incorrect.
- For the **true/false** questions, we give :
  - +1 points if your answer is correct,
  - 0 points if you give no answer or more than one,
  - -1 points if your answer is incorrect.
- Use a black or dark blue ballpen and clearly erase with correction fluid if necessary.
- If a question is wrong, the teacher may decide to nullify it.

Respectez les consignes suivantes   Read these guidelines   Beachten Sie bitte die unten stehenden Richtlinien											
choisir une répo	ne PAS choisir une réponse   NOT select an answer NICHT Antwort auswählen						Corriger une réponse   Correct an answer Antwort korrigieren				
X	$\checkmark$										
ce qu'il ne faut <u>PAS</u> faire   what should <u>NOT</u> be done   was man <u>NICHT</u> tun sollte											
						•					

## Part 1: this part of the exam contains 23 multiple choice questions.

For each question, mark the box corresponding to the answer that you wish to select as the correct one.

Each question has a unique correct answer.

**Question 1:** Given any function  $f: \mathbb{R} \to \mathbb{R}$  which is twice differentiable on  $\mathbb{R}$  and that admits a point of local minimum at x = 0, then,

f'(0) = 0 and  $f''(0) \neq 0$ 

 $f'(0) \neq 0 \text{ and } f''(0) \neq 0$ 

f'(0) = 0 and f''(0) > 0

f'(0) = 0 and f''(0) < 0

**Question 2:** Let  $f: \mathbb{R} \to \mathbb{R}$  be the function

$$f(x) := \begin{cases} 1 & \text{for } x \le 0, \\ \sqrt{1 - x^2} & \text{for } 0 < x \le 1, \\ 0 & \text{for } x > 1. \end{cases}$$

Then,

- | | f is differentiable at x=0 and f is continuous at x=1
- the left derivative of f exists at x = 0 and f is differentiable at x = 1
- f is continuous at x = 0 and f is differentiable at x = 1
- the right derivative of f exists at x = 0 and f is differentiable at x = 1

## Question 3:

Let  $f: \mathbb{R} \to \mathbb{R}$  be the function  $f(x) := x^3$ . We define the function  $f_1 := f$  and, for any natural number  $n \geq 2$ , the function  $f_n := f \circ f_{n-1}$ . Then, for all natural number  $m \geq 1$ ,

$$f_m(x) = mx^3$$

$$f_m(x) = x^{(3m)}$$

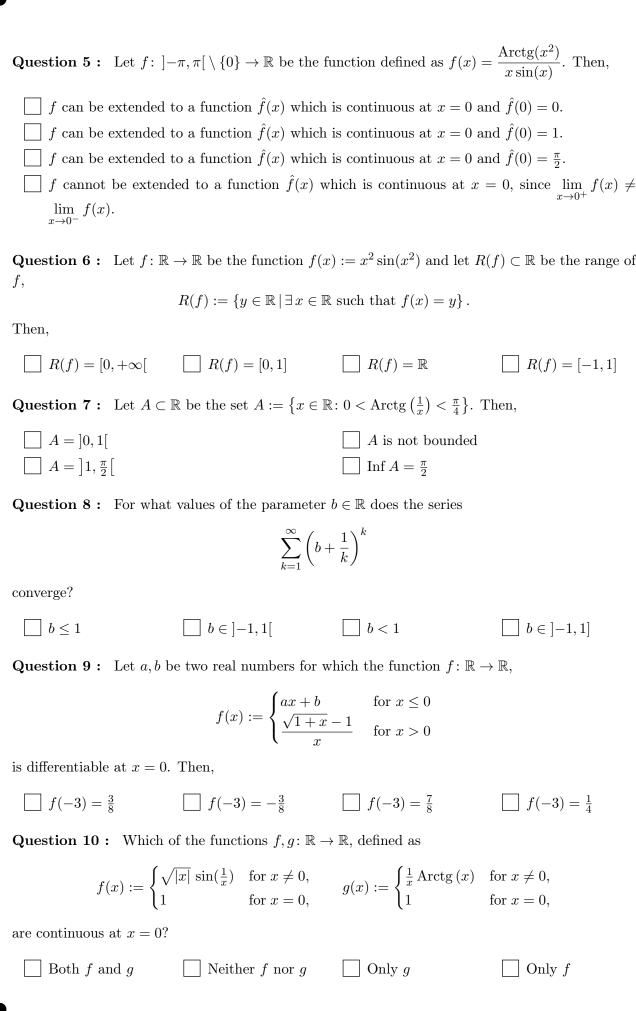
$$f_m(x) = (3x)^m$$

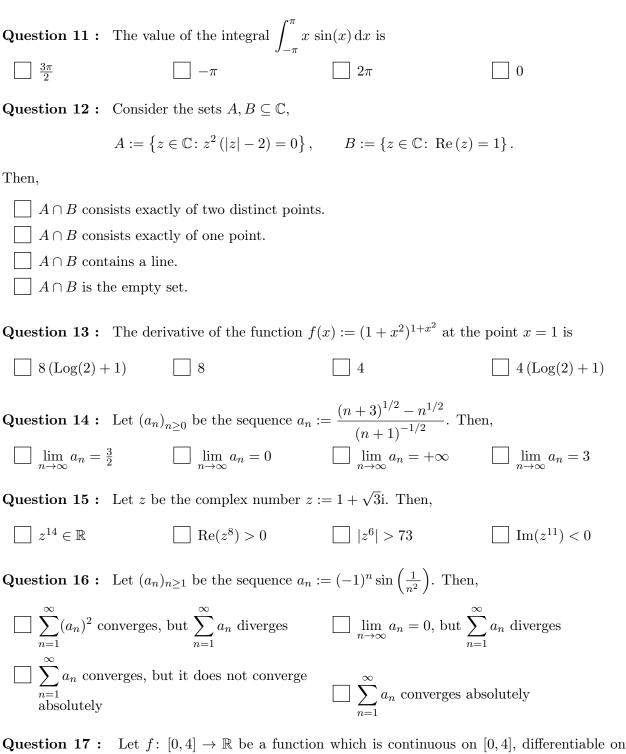
**Question 4:** Let  $c \in \mathbb{R}$ , and let  $(a_n)_{n \geq 1}$  be the sequence

$$a_n := \left\{ \sum_{k=0}^n \frac{2^k}{k!} \text{ for } n \text{ even}, \left( \sum_{k=0}^n \frac{1}{k!} \right)^c \text{ for } n \text{ odd.} \right\}$$

Then,

- the sequence  $(a_n)_{n\geq 1}$  converges for exactly one value of c
- $\lim_{m \to \infty} a_{2m+1} = c$
- The sequence  $(a_n)_{n\geq 1}$  diverges for any value of c





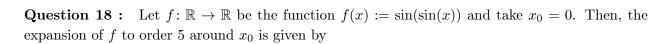
**Question 17:** Let  $f: [0,4] \to \mathbb{R}$  be a function which is continuous on [0,4], differentiable on [0,4] and such that  $f'(x) \ge 2$  for all  $x \in ]0,4[$ . Then,

$$0 \le f(3) - f(2) \le 1$$

$$f(4) - f(1) \le 4$$

$$f(4) - f(0) \le 1$$

$$f(2) - f(0) \ge 4$$



$$f(x) = x - \frac{1}{3}x^3 + \frac{1}{120}x^5 + x^5\varepsilon(x)$$

Question 19: Let  $f: ]-1,1[ \to \mathbb{R}$  be the function  $f(t) := \frac{1}{4+3t}$ , and take  $t_0 = 0$ . Then the expansion of f to order 2 around  $t_0$  is given by

$$f(t) = \frac{1}{4} - \frac{3}{16}t + \frac{9}{32}t^2 + t^2\varepsilon(t)$$

$$f(t) = \frac{1}{4} + \frac{3}{16}t - \frac{9}{64}t^2 + t^2\varepsilon(t)$$

$$f(t) = \frac{1}{4} - \frac{3}{16}t + \frac{9}{64}t^2 + t^2\varepsilon(t)$$

$$f(t) = \frac{1}{4} - \frac{3}{16}t + \frac{9}{128}t^2 + t^2\varepsilon(t)$$

Question 20: The generalised integral  $\int_{1}^{2^{-}} \frac{x+1}{\sqrt{2-x}} dx$ 

converges and its value is  $\frac{8}{3}$ 

- converges and its value is 4
- converges and its value is  $\frac{16}{3}$
- diverges

Question 21: Let  $I \subset \mathbb{R}$  be the domain of convergence of the power series  $\sum_{n=0}^{\infty} \sqrt{n} (x+1)^n$ . Then,

$$I = ]-2,0[$$

$$I = ]-1,1[$$

$$I = \mathbb{R}$$

Question 22: The value of the integral  $\int_0^1 x \sqrt{x^2 + 1} dx$  is

$$3(2\sqrt{2}-1)$$

Question 23: Let  $(a_n)_{n\geq 1}$  be the sequence  $a_n:=\frac{(5n+1)^n}{n^n5^n}$ . Then,

## Part 2: this part of the exam contains 11 true/false questions.

For each question, mark the box next to "TRUE" if you think the statement of the question is correct or mark the box next to "FALSE" if you think the statement of the question is incorrect.

**Question 24:** Let  $f: ]0,1[ \to \mathbb{R}$  be a monotone function which is non-constant and differentiable over ]0,1[. Then, either  $f'(x) \ge 0$  for all  $x \in ]0,1[$  or  $f'(x) \le 0$  for all  $x \in ]0,1[$ .

TRUE FALSE

**Question 25:** For all given  $y \in \mathbb{R}$ ,  $y \neq 0$ , the equation  $z^4 = iy$ , in the indeterminate z, has exactly 4 distinct solutions in  $\mathbb{C}$ .

TRUE FALSE

Question 26: Let  $(a_k)_{k\geq 0}$  be a sequence of real numbers such that for all  $k\geq 0$ ,  $a_k\geq 0$ , and  $\lim_{k\to\infty}\left|\frac{a_{k+1}}{a_k}\right|=0$ . Then the power series  $\sum_{k=0}^{\infty}a_k\,x^k$  converges for all  $x\in\mathbb{R}$ .

TRUE FALSE

**Question 27:** Let  $A \subset \mathbb{R}$  be a subset of the real numbers. If  $\operatorname{Inf} A \in A$  and  $\operatorname{Sup} A \in A$ , then A is a closed interval.

TRUE FALSE

**Question 28:** Let  $g,h: ]-1,1[\to \mathbb{R}$  be two functions which are differentiable on ]-1,1[ Assume that g(0)=h(0)=0, and  $h'(x)\neq 0$  for all  $x\in ]-1,1[$ . If  $\lim_{x\to 0}\frac{g'(x)}{h'(x)}$  does not exist, then also  $\lim_{x\to 0}\frac{g(x)}{h(x)}$  does not exist.

TRUE FALSE

Question 29: Let  $f: \mathbb{N} \to \mathbb{R}$  be a function such that for all  $n \geq 1$ , f(n) > n. Then the series  $\sum_{n=1}^{\infty} \frac{1}{f(n)}$  converges.

TRUE FALSE

