Speculative Betas

HARRISON HONG and DAVID A. SRAER*

ABSTRACT

The risk and return trade-off, the cornerstone of modern asset pricing theory, is often of the wrong sign. Our explanation is that high beta assets are more prone to speculative overpricing than low beta ones. When investors disagree about the prospects of the stock market, high beta assets are more sensitive to this aggregate disagreement and experience a greater divergence of opinion about their payoffs. If their dividends' variance is low enough, these assets experience speculative demand from optimistic investors. Short-sales constraints then result in these high beta assets being over-priced. When aggregate disagreement is low, the Security Market Line is upward sloping due to risk-sharing. When aggregate disagreement is high, expected returns can actually decrease with beta, especially for stocks with low idiosyncratic variance. We confirm our theory using a measure of disagreement about stock market earnings.

JEL classification: G02, G12.

*Hong: Department of Economics, Princeton University, hhong@princeton.edu; Sraer (Corresponding Author): Department of Economics and Haas School of Business, UC Berkeley, sraer@berkeley.edu. Hong acknowledges support from the National Science Foundation through grant SES-0850404. Sraer gratefully acknowledges support from the European Research Council (Grant No. FP7/2007-2013 - 249429) as well as the hospitality of the Toulouse School of Economics. We are especially grateful to Ken Singleton (the editor) and two anonymous referees for helping us improve the paper significantly. For insightful comments, we also thank Harjoat Bhamra, Andrea Frazzini, Augustin Landier, Lasse Pedersen, Ailsa Roell, Jianfeng Yu and seminar participants at MIT Sloan, the Toulouse School of Economics, Norges Bank-Stavanger Microstructure Conference, Norges Bank, Brazilian Finance Society Meetings, Indiana University, Arizona State University, the 2nd Miami behavioral finance conference, CNMV Securities Market Conference, University of Bocconi, University of Lugano, University of Colorado, Brandeis University, Temple University, Hong Kong University of Science and Technology, Duisenberg School Behavioral Finance Conference, Western Finance Association, China International Finance Conference, Harvard Business School, USC, Q-Group, Helsinki Behavioral Finance Conference, BI Bergen, IMF, Singapore Management University, Boston College, McGill University, NYU Five Star Conference, Oxford-Man, Essec, NOVA-Catholica Lisbon, ISCTE, Ohio State University, Rice University, University of Texas at Austin, University of Wisconsin, and EDHEC-Princeton Conference. The authors have no material financial or non-financial interests related to this research, as identified in the Journal of Finance's disclosure policy.

There is compelling evidence that high risk assets often deliver lower expected returns than low risk assets. This is contrary to the risk and return trade-off at the heart of neoclassical asset pricing theory. This high-risk, low-return puzzle literature, which dates back to Black (1972) and Black et al. (1972), shows that low risk stocks, as measured by a stock's co-movement with the stock market or Sharpe (1964)'s Capital Asset Pricing Model (CAPM) beta, have significantly outperformed high risk stocks over the last thirty years.[1] Baker et al. (2011) show that the cumulative performance of stocks since January 1968 actually declines with beta. For instance, a dollar invested in a value-weighted portfolio of the lowest quintile of beta stocks would have yielded \$96.21 (\$15.35 in real terms) at the end of December 2010. A dollar invested in the highest quintile of beta stocks would have yielded around \$26.39 (\$4.21 in real terms). Related, both Baker et al. (2011) and Frazzini and Pedersen (2010) point out that a strategy of shorting high beta stocks and buying low beta stocks generates excess profits as large as famous excess stock return predictability patterns such as the value-growth or price momentum effects.[2]

Black (1972) originally tried to reconcile a flat Security Market Line by relaxing one of the central CAPM assumptions of borrowing at the risk-free rate. He showed that when there are borrowing constraints, risk tolerant investors desiring more portfolio volatility will demand high beta stocks since they cannot simply lever up the tangency portfolio. However, borrowing constraints can only deliver a flatter Security Market Line relative to the CAPM but not a downward sloping one. Investors would not bid up high beta stock prices to the point of having lower returns than low beta stocks. Indeed, it is difficult to get a downward sloping line even if one introduced noise traders as in Delong et al. (1990) or liquidity shocks as in Campbell et al. (1993) since noise trader or fundamental risk in these models lead to higher expected returns.[3]

In contrast to Black (1972), we provide a theory for this high-risk and low-return puzzle even when investors can borrow at the risk-free rate. We show that relaxing instead the other CAPM assumptions of homogeneous expectations and costless short-selling can deliver rich patterns in the Security Market Line, including an inverted-U shape or even a downward sloping line. Our model starts from a CAPM framework, in which firms' cash flows follow a one factor model and there are a finite number of securities so that markets are incomplete. We allow investors to disagree about the market or common factor of firms' cash flows and prohibit *some* investors from short-selling. Investors only disagree about the mean of the common factor of cash flows and there are two groups of investors, buyers such as retail mutual funds who cannot short and arbitrageurs such as hedge funds who can short.

There is substantial evidence in support of both of these assumptions. First, there is time-varying disagreement among professional forecasters' and households' expectations about many macroeconomic state variables such as market earnings, industrial production growth and inflation (Cukierman and Wachtel (1979), Kandel and Pearson (1995), Mankiw et al. (2004), Lamont (2002)). Such aggregate disagreement might emanate from many sources including heterogeneous priors or cognitive biases like overconfidence.[4] Second, short-sales constraints bind for some investors due to institutional reasons as opposed to the physical cost of shorting.[5] For instance, many investors in the stock market such as retail mutual funds, which in 2010 have 20 trillion dollars of assets under management, are prohibited by charter from shorting either directly (Almazan et al. (2004)) or indirectly through the use of derivatives (Koski and Pontiff (1999)). Only a smaller subset of investors, such as hedge funds with 1.8 trillion dollars in asset management, can and does short.

Our main result is that high beta assets are over-priced compared to low beta ones when disagreement about the common factor of firms' cash flows is high. If investors disagree about the mean of the common factor, then their forecasts of the payoffs of high beta stocks will naturally diverge more than their forecasts of low beta ones. In other words, beta amplifies disagreement about the macro-economy. Because of short-sales constraints, high beta stocks, which are more sensitive to aggregate disagreement than low beta ones, are only held in equilibrium by optimists as pessimists are sidelined. This greater divergence of opinion creates overpricing of high beta stocks compared to low beta ones (Miller (1977) and Chen et al. (2002)).[6] Arbitrageurs attempt to correct this mispricing but their limited risk-bearing capacity results only in limited shorting, leading to equilibrium overpricing.[7]

That more disagreement on high beta stocks lead to overpricing of these stocks is far from obvious in an equilibrium model like ours. Optimistic investors can achieve a large exposure to the common factor by buying high beta stocks or levering up low beta ones. If high beta stocks are overpriced, optimistic investors should favor the levering up of low beta assets, which could potentially undo the initial mispricing. The key reason why this is not the case in our model is that when markets are incomplete (which is implicit in all theories of limits of arbitrage (as in Delong et al. (1990) or Shleifer and Vishny (1997)) and most modern asset pricing models (Merton (1987))), idiosyncratic risk matters for investors' portfolios. In our context, while levering up low beta stocks increases the exposure to the common factor, it also magnifies the idiosyncratic risk that investors have to bear. This role of idiosyncratic volatility as a limit of arbitrage is motivated by a number of empirical papers that show that idiosyncratic risk is the biggest impediment to arbitrage (Pontiff (1996), Wurgler and Zhuravskaya (2002)). It leads, in equilibrium, to a situation where levering up low beta stocks ends up being less efficient than buying high beta stocks when speculating on the common factor of firms' cash flows. In other words, higher beta assets are naturally more speculative.

Our model yields the following key testable implications. When macro-disagreement is low, all investors are long and short-sales constraints do not bind. The traditional risk-sharing motive leads high beta assets to attract a lower price or higher expected return. For high enough levels of aggregate disagreement, the relationship between risk and return takes on an inverted U-shape. For assets with a beta below a certain cut-off, expected returns are increasing in beta as there is little disagreement about these stock's cash flows and therefore short-selling constraints do not bind in equilibrium.

But for assets with a beta above an equilibrium cut-off, disagreement about the dividend becomes sufficiently large that the pessimist investors are sidelined. This speculative overpricing effect can dominate the risk-sharing effect and the expected returns of high beta assets can actually be lower than those of low beta ones. As disagreement increases, the cut-off level for beta below which all investors are long falls and the fraction of assets experiencing binding short-sales constraints increases.[8]

We test these predictions using a monthly time-series of disagreement about market earnings. Disagreement about a stock's cash-flow is simply measured by the standard deviation of analysts' forecasts of the long-term growth of Earnings Per Share (EPS), as in Diether et al. (2002). The aggregate disagreement measure is a beta-weighted average of the stock-level disagreement measure for all stocks in our sample, similar in spirit to Yu (2010). The weighting by beta in our proxy for aggregate disagreement is suggested by our theory. After all, stocks with very low beta have by definition almost no sensitivity to aggregate disagreement, and their disagreement should mostly reflect idiosyncratic disagreement. Aggregate disagreement can be high during both down-markets, like the recessions of 1981-1982 and 2007-2008, and up-markets, like the dot-com boom of the late nineties (Figure 1). Panel (c) of Figure 6 shows the 12-months excess returns on 20 β-sorted porftolios (see Section II.B for details on the construction of these portfolios). In months with low aggregate disagreement (defined as the bottom quartile of the aggregate disagreement distribution and denoted by blue dots), we that that returns are in fact increasing with beta. In months with high aggregate disagreement however (defined as the top quartile of the disagreement distribution and denoted by red dots), the risk-return relationship has an inverted-U shape. In these months, the two top and bottom portfolios have average excess return net of the risk-free rate of around 0%, while the portfolios around the median portfolio have average excess returns of around 8%. This inverted U-shape relationship is formally estimated in the context of a standard Fama-MacBeth analysis where the excess return/beta relationship is shown to be strictly more concave when aggregate disagreement is large.[9]

[Insert Figure 1 here]

Our baseline analysis assumes that stocks' cash-flow process is homoskedastic. When we allow for heteroskedasticity, our main asset pricing equation is slightly modified. Intuitively, a large idiosyncratic variance makes optimist investors reluctant to demand too much of a stock. Thus, a stock with a large cash-flow beta – and therefore whose expected cash-flow is high from the optimists' point of view – may nonetheless have little demand from the optimists if the stock has high idiosyncratic variance. In equilibrium, this low demand from optimist will drive down the price and make pessimists long this asset. As a result, such a stock may not experience the same speculative over-pricing as a stock with a similar cash-flow beta but a lower idiosyncratic variance. In other words, stocks experience overpricing only when the ratio of their cash-flow beta to idiosyncratic variance is high enough. Below a certain cutoff in this ratio, stocks are priced as in the CAPM and the partial Security Market Line (the relationship between expected returns and β for stocks below this cutoff) is upward sloping and independent of aggregate disagreement. Above the cutoff, the partial Security Market Line has a slope that strictly decreases with aggregate disagreement. We confirm this additional prediction in the data.

Our findings are consistent with Diether et al. (2002) and Yu (2010), who find that dispersion of earnings forecasts predicts low returns in the cross-section and for the market return in the time-series respectively, as predicted in models with disagreement and short-sales constraints. Our particular focus is on the theory and the empirics of the Security Market Line as a function of aggregate disagreement. Importantly, we show below that the patterns observed in the data is not simply a function of high beta stocks performing badly during down markets nor is it a function of high disagreement stocks underperforming.

Finally, in an overlapping-generations (OLG) extension of our static model, we show that these predictions also hold in a dynamic setting where disagreement follows a two-state markov chain. Investors anticipate that high beta assets are more likely to experience binding short-sales constraints in the future and hence have a potentially higher resale price than low beta ones relative to fundamentals (Harrison and Kreps (1978), Morris (1996), Scheinkman and Xiong (2003) and Hong et al. (2006)). Since disagreement is persistent, this pushes up the price of high beta assets in both the low and high disagreement states. At the same time, since the price of high beta assets vary more with aggregate disagreement, these stocks carry an extra risk-premium. We use this dynamic model to show that a basic simulation of the model can yield economically significant flattenings of the Security Market Line using reasonable levels of disagreement and risk-aversion among investors.

Our paper proceeds as follows. We present the model in Section I. We describe the data we use in our empirical analysis in Section II. We present the empirical analysis in Section III. We conclude in Section IV. All proofs are in Appendix A.

I. Model

A. Static Setting

We consider an economy populated with a continuum of investors of mass 1. There are two periods, $t = 0, 1$. There are N risky assets and the risk-free rate is exogenously set at r. Risky asset i delivers a dividend $\tilde{d}_i$ at date 1, which is given by:

$$\forall i \in \{1, \dots, N\}, \quad \tilde{d}_i = d + b_i \tilde{z} + \tilde{\epsilon}_i.$$

The common factor in stock i's dividend is $\tilde{z}$, with $\mathbb{E}[\tilde{z}] = 0$ and $\text{Var}[\tilde{z}] = \sigma_z^2$. The idiosyncratic component in stock i's dividend is $\tilde{\epsilon}_i$, with $\mathbb{E}[\tilde{\epsilon}_i] = 0$ and $\text{Var}[\tilde{\epsilon}_i] = \sigma_\epsilon^2$. By definition, for all $i \in [1, N]$, $\text{Cov}\,(\tilde{z}, \tilde{\epsilon}_i) = 0$. b_i is the cash-flow beta of asset i and is assumed to be strictly positive. Each asset i is in supply $s_i = \frac{1}{N}$ and we assume w.l.o.g. that:[10]

$$0 < b_1 < b_2 < \dots < b_N.$$

Assets in the economy are indexed by their cash-flow betas, which are increasing in i. The share-weighted average b in the economy is set to 1 ($\sum_{i=1}^N \frac{b_i}{N} = 1$).

Investors are divided into two groups. A fraction α of them hold heterogenous beliefs and cannot short. We call these buyers mutual funds (MF), who are in practice prohibited from shorting by charter. These investors are divided in two groups of mass $\frac{1}{2}$, A and B, who disagree about the mean value of the aggregate shock $\tilde{z}$. Group A believes that $\mathbb{E}^A[\tilde{z}] = \lambda$ while group B believes that $\mathbb{E}^B[\tilde{z}] = -\lambda$. We assume w.l.o.g. that $\lambda > 0$ so that investors in group A are the optimists and investors in group B the pessimists.

A fraction $1-\alpha$ of investors hold homogeneous and correct beliefs and are not subject to the short-sales constraint. We index these investors by a (for "arbitrageurs"). For concreteness, one might interpret these buyers as hedge funds (HF), who can generally short at little cost. That these investors have homogenous beliefs is simply assumed for expositional convenience. Heterogeneous priors for unconstrained investors wash out in the aggregate and have thus no impact on equilibrium asset prices in our model.

Investors maximize their date-1 wealth and have mean-variance preferences:

$$U(\tilde{W}^k) = \mathbb{E}^k[\tilde{W}^k] - \frac{1}{2\gamma} Var(\tilde{W}^k)$$

where $k \in \{a, A, B\}$ and γ is the investors' risk tolerance. Investors in group A or B maximize under the constraint that their holding of stocks have to be greater than 0.

B. Equilibrium

The following theorem characterizes the equilibrium.

Theorem 1. *Let* $\theta = \frac{\frac{\alpha}{2}}{1-\frac{\alpha}{2}}$ *and define* $(u_i)_{i\in[0,N+1]}$ *such that* $u_{N+1} = 0$, $u_i = \frac{1}{\gamma N b_i}\left(\sigma_\epsilon^2 + \sigma_z^2\left(\sum_{j<i} b_j^2\right)\right) + \frac{\sigma_z^2}{\gamma}\left(\sum_{j\geq i}\frac{b_j}{N}\right)$ *for* $i \in [1, N]$ *and* $u_0 = \infty$. u *is a strictly decreasing sequence. Let* $\bar{i} = \min\{k \in [0, N+1] \mid \lambda > u_k\}$. *There exists a unique equilibrium, in which asset prices are given by:*

$$P_i(1+r) = \begin{cases} d - \frac{1}{\gamma}\left(b_i\sigma_z^2 + \frac{\sigma_\epsilon^2}{N}\right) & \text{for } i < \bar{i} \\ d - \frac{1}{\gamma}\left(b_i\sigma_z^2 + \frac{\sigma_\epsilon^2}{N}\right) + \underbrace{\frac{\theta}{\gamma}\left(b_i\sigma_z^2\omega(\lambda) - \frac{\sigma_\epsilon^2}{N}\right)}_{\pi^i = speculative\ premium} & \text{for } i \geq \bar{i} \end{cases}, \tag{1}$$

where $\omega(\lambda) = \frac{\lambda\gamma - \frac{\sigma_z^2}{N}\left(\sum_{i\geq\bar{i}} b_i\right)}{\sigma_z^2\left(1+\sigma_z^2\left(\sum_{i<\bar{i}}\frac{b_i^2}{\sigma_\epsilon^2}\right)\right)}$.

Proof. See Appendix A ∎

The main intuition underlying the equilibrium is that there is more disagreement among investors about the expected dividends of high b_i assets relative to low b_i assets. Above a certain level of b_i ($b_i \geq b_{\bar{i}}$), investors sufficiently disagree that the pessimists, that is, investors in group B, would like to optimally short these stocks. However, this is impossible because of the short-sales constraint. These high b stocks thus experience a speculative premium since their price reflects disproportionately the belief of the optimists, that is, investors in group A. As aggregate disagreement grows, the cash flow beta of the marginal asset — the asset above which group B investors are sidelined — decreases and there is a larger fraction of assets experiencing overpricing.[11]

We can derive a number of comparative static results regarding this equilibrium. The first ones relates to overpricing. When short-sales constraints are binding, that is, for assets $i \geq \bar{i}$, the difference between the equilibrium price and the price that would prevail in the absence of short-sales constraints (i.e., when MFs can short without restriction or when $\alpha = 0$) is given by:

$$\pi^i = \frac{\theta}{\gamma}\left(b_i \sigma_z^2 \omega(\lambda) - \frac{\sigma_\epsilon^2}{N}\right). \tag{2}$$

This term, which we call the speculative premium, captures the degree of overpricing due to the short-sales constraints. The following corollary explores how this speculative premium varies with aggregate disagreement, cash-flow betas, and the fraction of short-sales constrained agents.

Corollary 1. *Assets with high cash-flow betas, that is, $i \geq \bar{i}$, are over-priced (relative to the benchmark with no short-sales constraints or when $\alpha = 0$) and the amount of overpricing, defined as the difference between the price and the benchmark price in the absence of short-sales constraints, is increasing with disagreement λ, with cash-flow betas b_i and with the fraction of short-sales constrained investors α. Furthermore, an increase in aggregate disagreement λ leads to a larger increase in mispricing for assets with larger cash flow betas.*

Proof. See Appendix B □

The second comparative static we consider relates to the holdings observed in equilibrium. Remember that HFs (i.e. investors in group a) are not restricted in their ability to short. Intuitively, HFs short assets with large mispricing, that is, high b assets. As aggregate disagreement increases, mispricing increases, so that HFs end up shorting more. Since an increase in aggregate disagreement leads to a larger relative increase in mispricing for higher b stocks, the corresponding increase in shorting is also larger for high b stocks. In other words, there is a weakly increasing relationship between shorting by HFs and b. Provided that λ is large enough, this relationship becomes strictly steeper as aggregate disagreement increases. We summarize these comparative statics in the following corollary:

Corollary 2. *Group A investors are long all assets. Group B investors are long assets $i < \bar{i} - 1$ and have 0 holdings of assets $i \geq \bar{i}$. There exists $\hat{\lambda} > 0$ such that provided that $\lambda > \hat{\lambda}$, there exists $\tilde{i} \in [\bar{i}, N]$ such that (1) group a investors short high cash-flow beta assets, that is, assets $i \geq \tilde{i}$ (2) the \$ amount of stocks being shorted in equilibrium increases with aggregate disagreement λ and (3) the sensitivity of shorting to aggregate disagreement is higher for high cash-flow beta assets.*

Proof. See Appendix C □

C. Beta and Expected Return

We now restate the equilibrium in terms of expected excess returns and relate them to the familiar market β from the CAPM. We note $\tilde{r}_i^e$ the excess return per share for asset i and $\tilde{R}_m^e$ the excess return per share for the market portfolio. The market portfolio is simply defined as the portfolio of all assets in the market. The value of the market portfolio is $P_m = \sum_{j=1}^N s_j P_j = \sum_{j=1}^N \frac{P_j}{N}$ since we have normalized the supply of stocks to $\frac{1}{N}$. Then, by definition:

$$\tilde{R}_i^e = d + b_i \tilde{z} + \tilde{\epsilon}_i - (1+r)P_i \quad \text{and} \quad \tilde{R}_m^e = \sum_{i=1}^N s_i \tilde{R}_i^e = \sum_{i=1}^N \frac{1}{N} \tilde{R}_i^e = d + \tilde{z} + \sum_{i=1}^N \frac{\tilde{\epsilon}_i}{N} - (1+r)P_m.$$

Define $\beta_i = \frac{\text{Cov}\left(\tilde{R}_i^e, \tilde{R}_m^e\right)}{Var(\tilde{R}_m^e)} = \frac{b_i \sigma_z^2 + \frac{\sigma_\epsilon^2}{N}}{\sigma_z^2 + \frac{\sigma_\epsilon^2}{N}}$ to be the stock market beta of stock i. By definition, the expected excess return per share on stock i is simply given by:

$$\mathbb{E}[\tilde{R}_i^e] = d - (1+r)P_i.$$

Using Theorem 1, we can express this expected excess return per share on stock i as:[12]

$$\mathbb{E}[\tilde{R}_i^e] = \begin{cases} \beta_i \dfrac{\sigma_z^2 + \frac{\sigma_\epsilon^2}{N}}{\gamma} & \text{for } i < \bar{i} \\ \beta_i \dfrac{\sigma_z^2 + \frac{\sigma_\epsilon^2}{N}}{\gamma} \left(1 - \theta\omega(\lambda)\right) + \theta \dfrac{\sigma_\epsilon^2}{\gamma N} \left(1 + \omega(\lambda)\right) & \text{for } i \geq \bar{i} \end{cases}, \tag{3}$$

This representation follows directly from Theorem 1: we simply express the price of asset i as a function of the market beta of asset i, β_i. For $\alpha = 0$ (so that $\theta = 0$), investors have homogenous beliefs so that λ does not affect the expected returns of the assets. In fact, when $\alpha = 0$, there are no binding short-sales constraints, so that $\bar{i} = N + 1$ and we can simply rewrite for all $i \in [1, N]$: $\mathbb{E}[\tilde{R}_i^e] = \beta_i \mathbb{E}[\tilde{R}_m^e]$, that is, the standard CAPM formula. However, when a fraction $\alpha > 0$ of investors are short-sales constrained and aggregate disagreement is large enough, $\bar{i} \leq N$ and the expected return per share for assets $i \geq \bar{i}$ depend on aggregate disagreement λ.

More precisely, the Security Market Line is then piecewise linear. For low beta assets ($\beta_i < \beta_{\bar{i}}$), expected excess returns are solely driven by risk-sharing: higher β assets are more exposed to market risk and thus command a higher expected return. When β crosses a certain threshold ($\beta \geq \beta_{\bar{i}}$), however, expected excess returns are also driven by speculation, in the sense that pessimists are sidelined from these high beta stocks: over this part of the Security Market Line, higher beta assets, which are more exposed to aggregate disagreement, command a larger speculative premium and thus receive smaller expected returns than what they would absent disagreement. Note that provided that λ is large enough, the Security Market Line can even be downward sloping over the interval $[\beta_{\bar{i}}, \beta_N]$, that is, for speculative assets.

We illustrate the role of aggregate disagreement on the shape of the Security Market Line in Figure 2: the Security Market Line is plotted for three possible levels of λ, $\lambda^0 < \lambda^1 < \lambda 2$. The Security Market Line is simply the 45 degree line when $\lambda = \lambda^0 = 0$ as seen in the top panel of Figure 2(a)). λ^1 is assumed to be large enough that a strictly positive fraction of assets experience binding short-sales constraints and hence speculative mispricing (assets above $\bar{i}$): expected returns are increasing with beta but at a lower pace for assets above the endogenous marginal asset $\bar{i}$ (2(b)). When $\lambda = \lambda^2 > \lambda^1$ (Figure 2(c)), the slope of the Security Market Line for assets $i \geq \bar{i}$ is negative, that is, the Security Market Line has an inverted-U shape.

[Insert Figure 2 here]

In our empirical analysis below, we approach this relationship between expected excess returns and β by looking at the concavity of the Security Market Line and how this concavity is related to our empiricaly proxy for aggregate disagreement. More precisely, we estimate every month a cross-sectional regression of realized excess returns of 20 β-sorted portfolios on the portfolio β and the portfolio β^2. The time-series of the coefficient estimate on β^2 represents a time-series of returns which essentially goes long the low and high beta portfolios and short the portfolios around the median beta portfolio. Essentially, the Security Market Line described in equation 3 predicts that this square portfolio should experience lower return when aggregate disagreement is high – or in other words that the Security Market Line should be more concave when aggregate disagreement is high. This is our first main empirical prediction.

Prediction 1. *In low disagreement months, the Security Market Line is upward sloping. In high disagreement months, the Security Market Line has a kink-shape: its slope is strictly positive for low β assets, but strictly lower (and potentially negative) for high β assets. The Security Market Line should be more concave following months with high aggregate disagreement; equivalently, a portfolio long low and high beta assets and short medium beta assets should experience a lower performance following months of high aggregate disagreement.*

A consequence of the previous analysis is that the slope of the Security Market Line should also decrease following a month of high aggregate disagreement. However, this is a weaker prediction of the model since it does not exploit the specificity of our model, namely the kink in the security market line, which, as we will see in Section III is an important feature of the data.

Corollary 3. *Let $\hat{\mu}$ be the coefficient estimate of a cross-sectional regression of realized returns $\left(\tilde{R}_i^e\right)_{i\in[1,N]}$ on $(\beta_i)_{i\in[1,N]}$ and a constant. The coefficient $\hat{\mu}$ decreases with aggregate disagreement λ and with the fraction of short-sales constrained agents in the economy α. Furthermore, the negative effect of aggregate disagreement λ on $\hat{\mu}$ is larger when there are fewer arbitrageurs in the economy (i.e., when α increases).*

Proof. See Appendix E □

D. Discussion of Assumptions

Our theory relies on two fundamental ingredients, disagreement and short-sales constraint. Both are important. In the absence of disagreement, all investors share the same portfolio and since there is a strictly positive supply of assets, this portfolio is long only. Thus, the short-sales constraint is irrelevant – it never binds – and the standard CAPM results apply. In the absence of short-sales constraints, the disagreement of group A and group B investors washes out in the market clearing condition and prices simply reflect the average belief, which we have assumed to be correct.

The model also relies on important simplifying assumptions. First of them is that, in our framework, investors disagree only on the expectation of the aggregate factor, $\tilde{z}$. A more general setting would allow investors to also disagree on the idiosyncratic component of stocks dividend $\tilde{\epsilon}_i$. If the idiosyncratic disagreement on a stock is not systematically related to this stock's cash-flow beta, then our analysis is left unchanged since whatever mispricing is created by idiosyncratic disagreement, it does not affect the shape of the Security Market Line in a systematic fashion. If idiosyncratic disagreement is positively correlated with stocks' cash-flow beta, then the impact of aggregate disagreement on the Security Market Line becomes even stronger. This is because there are now two sources of overpricing linked systematically with b_i: one coming from aggregate disagreement, the other coming from this additional idiosyncratic disagreement.

As we show in Section II.C below, the overall disagreement on the earnings growth of high beta stocks is much larger than the disagreement on low beta stocks, *even* in months with low aggregate disagreement. This suggests that, idiosyncratic disagreement is in fact larger for high beta stocks. We also believe that this conforms to standard intuition on the characteristics of high and low beta stocks.

The second key assumption in the model is that investors only disagree on the first moment of the aggregate factor $\tilde{z}$ and not on the second moment σ_z^2. From a theoretical viewpoint, this is not so different. In the same way that β scales disagreement regarding $\bar{z}$, β would scale disagreement about σ_z^2. In other words, label the group that underestimates σ_z^2 as the optimists and the group that overestimate σ_z^2 as the pessimists. Optimists are more optimistic about the utility derived from holding a high β asset than a low β asset and symmetrically, the pessimists are more pessimistic about the utility derived from holding a high β asset than a low β asset. Again, high β assets are more sensitive to disagreement about the variance of the aggregate factor σ_z^2 than low β assets. As in our model, this would naturally lead to high β stocks being overpriced when this disagreement about σ_z^2 is large. However, while empirical proxies for disagreement about the mean of the aggregate factor can be constructed, it is not clear how one would proxy for disagreement about its variance.

The third key assumption imposed in the model is that the dividend process is homoskedastic. In the next section, we relax this assumption and allow the dividend process of different assets to have heterogeneous levels of idiosyncratic volatility.

E. Heteroskedastic Idiosyncratic Variance

Our results in Theorem 1 in the static case have been derived under the assumption that the idiosyncratic shocks to the dividend process were homoskedastic, that is, $\forall i \in [1, N], \quad \sigma_i^2 = \sigma_\epsilon^2$. This assumption is easily relaxed. Once dividends can be heteroskedastic, assets need to be ranked in ascending order of $\frac{b_i}{\sigma_i^2}$, which is equivalent to ranking them in ascending order of $\frac{\beta_i}{\sigma_i^2}$. In Appendix A, we show that the unique equilibrium then features a marginal asset $\bar{i}$, such that:

$$\mathbb{E}[\tilde{R}_i^e] = \begin{cases} \beta_i \dfrac{\sigma_z^2 + \sum_{j=1}^N \frac{\sigma_j^2}{N^2}}{\gamma} \quad \text{for } \dfrac{\beta_i}{\sigma_i^2} < \dfrac{\beta_{\bar{i}}}{\sigma_{\bar{i}}^2} \\ \beta_i \dfrac{\sigma_z^2 + \sum_{j=1}^N \frac{\sigma_j^2}{N^2}}{\gamma} \left(1 - \theta\omega(\lambda)\right) + \theta \dfrac{\sigma_i^2}{\gamma N} \left(1 + \omega(\lambda)\right) \quad \text{for } \dfrac{\beta_i}{\sigma_i^2} \geq \dfrac{\beta_{\bar{i}}}{\sigma_{\bar{i}}^2} \end{cases}, \tag{4}$$

Intuitively, consider a stock with a high cash-flow beta. Relative to pessimists, optimist investors believe this stock is likely to have a high dividend. If the stock has a low idiosyncratic variance (σ_i^2), this will lead to a high demand from optimists for this stock. In equilibrium, this will drive out the pessimists from the stock and lead to speculative overpricing. However, if the stock has a high idiosyncratic variance, optimists will be reluctant to demand large quantities of this stock, despite their optimistic valuation. As a result, the pessimists may be required to be long the stock in equilibrium, so that the stock will be fairly priced. Thus, the equilibrium features a cutoff in the ratio of cash-flow beta to idiosyncratic variance.

In particular, the pricing formula in Equation (4) says that for stocks i with β_i/σ_i^2 below the cut-off $\beta_{\bar{i}}/\sigma_{\bar{i}}^2$ (i.e. what we define as **non-speculative stocks**), the slope of the partial Security Market Line (the relationship between expected returns and β for assets below the cutoff) does not depend on aggregate disagreement. For stocks i with a ratio β_i/σ_i^2 above this cut-off (i.e. what we define as **speculative stocks**), the partial Security Market Line is still linear in β but its slope is strictly decreasing with aggregate disagreement. The asset pricing equation (4) also predicts that for these mispriced assets, idiosyncratic variance is priced and that the price of idiosyncratic risk increases with aggregate disagreement. This is related to the previous intuition: all else equal, an asset with a high idiosyncratic variance will receive a smaller demand by optimists, which in equilibrium will drive down its price and drive up it expected return. This leads to our second main empirical prediction of the paper.

Prediction 2. *Define speculative assets as assets with a high ratio of β_i/σ_i^2. Then, the slope of the* *relationship between expected returns and β for these assets decreases strictly with aggregate disagreement. Conversely, for non-speculative assets – assets with a low ratio of β_i/σ_i^2 – the relationship between expected returns and β is independent of aggregate disagreement.*

F. Infinite Number of Assets

We analyze the case where markets become complete and N goes to infinity. To simplify the discussion, we assume that whatever N, the number of assets, assets in the cross-section always have cash-flow betas that are bounded in $[\underline{b}, \bar{b}]$. We adapt our previous notation to define b_i^N the cash-flow beta of asset i when the cross-section has N assets, with $i \leq N$. Our assumption is that for all $N \in \mathbb{N}$ and $i \leq N$, $0 < \underline{b} < b_i^N < \bar{b} < \infty$. With this assumption, we show that in the limit case where $N \to \infty$, asset returns always admit a linear CAPM representation. In particular, the slope of the security market line is independent of λ as long as $\lambda \leq \frac{\sigma_z^2}{\gamma}$ and is strictly decreasing with λ when $\lambda > \frac{\sigma_z^2}{\gamma}$.

Since $u_i^N = \frac{1}{\gamma N b_i^N}\left(\sigma_\epsilon^2 + \sigma_z^2\left(\sum_{j<i}(b_j^N)^2\right)\right) + \frac{\sigma_z^2}{\gamma}\left(\sum_{j\geq i}\frac{b_j^N}{N}\right)$ and the b_i are bounded, it is direct to see that when $N \to \infty$: $u_1^N \to \frac{\sigma_z^2}{\gamma}$ and $u_N^N \to l\frac{\sigma_z^2}{\gamma}$, where $l = \lim_{N\to\infty}\sum_{j<N}\frac{(b_j^N)2}{N b_N^N}$ and $l < 1$ since for all $j < N$, $b_j^N < b_N^N$.

Our first result is that if λ is small enough (i.e. $\lambda\gamma < l\sigma_z^2 = \gamma \lim u_N^N$), then at the limit $N \to \infty$, no asset will experience binding short-sales constraints, so that asset returns will be independent of λ and the standard CAPM formula will apply: $\mathbb{E}[\tilde{R}_i^e] = \beta_i \mathbb{E}[\tilde{R}_m^e]$, with $\mathbb{E}[\tilde{R}_m^e]$ independent of λ.

Our second result is that, provided that λ is large enough ($\lambda\gamma > \sigma_z^2 = \gamma \lim u_1^N$), then at the limit, all assets will experience binding short-sales constraints. In this case, expected returns at the limit are given by:

$$\mathbb{E}[\tilde{R}_i^e] = \beta_i\left((1+\theta)\frac{\sigma_z^2}{\gamma} - \lambda\theta\right) = \beta_i \mathbb{E}[\tilde{R}_m^e(\lambda)].$$

The Security Market Line is linear as in the previous case, but its slope is now strictly decreasing with aggregate disagreement λ. In particular, if $\lambda\gamma > \frac{1+\theta}{\theta}\sigma_z^2$, then the Security Market Line is strictly decreasing.

The final case occurs when $\sigma_z^2 > \lambda\gamma > l\sigma_z^2$. For any finite i , we know that $\lim u_i^N = \frac{\sigma_z^2}{\gamma}$. Thus, the marginal asset has to be in the limit such that $\lim \bar{i}^N = \infty$. But then, we know that $\omega(\lambda) \to 0$, so that the speculative premium at the limit is also 0. Thus, at the limit, asset returns will be independent of λ and the standard CAPM formula will again apply: $\mathbb{E}[\tilde{R}_i^e] = \beta_i \mathbb{E}[\tilde{R}_m^e]$, with $\mathbb{E}[\tilde{R}_m^e]$ independent of λ.

G. Dynamics

G1. Set-up

We consider now a dynamic extension of the previous model, where we also allow for heteroskedasticity in dividend shocks. This extension is done in the context of an overlapping generation framework. Time is infinite, $t = 0, 1, \ldots \infty$. Each period t, a new generation of investors of mass 1 is born and invest in the stock market to consume the proceeds at date $t+1$. Thus at date t, the new generation is buying assets from the current old generation (born at date $t-1$), which has to sell its entire portfolio in order to consume. Each generation is composed of 2 groups of investors: arbitrageurs, or Hedge Funds, in proportion $1-\alpha$, and Mutual Funds in proportion α. Investors have mean-variance preferences with risk tolerance parameter γ. There are N assets, whose dividend process is given by:

$$\tilde{d}_t^i = d + b^i \tilde{z}_t + \tilde{\epsilon}_t^i,$$

where $\mathbb{E}[\tilde{z}] = 0$, $\text{Var}[\tilde{z}] = \sigma_z^2$, $\mathbb{E}[\tilde{\epsilon}^i] = 0$, $\text{Var}[\tilde{\epsilon}^i] = \sigma_i^2$ and $\frac{1}{N}\sum_{i=1}^N b_i = 1$. We normalize the assets to be ranked in ascending order of $\frac{b_i}{\sigma_i^2}$:

$$0 < \frac{b_1}{\sigma_1^2} < \frac{b_2}{\sigma_2^2} < \cdots < \frac{b_N}{\sigma_N^2}$$

The timeline of the model appears on Figure 3. Mutual funds born at date t hold heterogeneous beliefs about the expected value of $\tilde{z}_{t+1}$. Specifically, there are two groups of mutual funds: investors in group A – the optimist MFs – hold expectations about $\tilde{z}_{t+1}$ such that $\mathbb{E}_t^A[\tilde{z}_{t+1}] = \tilde{\lambda}_t$ and investors in group B – the pessimist MFs – hold expectations about $\tilde{z}_{t+1}$ such that $\mathbb{E}^B[\tilde{z}_{t+1}] = -\tilde{\lambda}_t$.[13] Finally, we assume that $\tilde{\lambda}_t \in \{0, \lambda > 0\}$ is a two-state Markov process with persistence $\rho \in]1/2, 1[$.

[Insert Figure 3 here]

Call $P_t^i(\tilde{\lambda})$ the price of asset i at date t when realized aggregate disagreement is $\tilde{\lambda}_t \in \{0, \lambda\}$ and define $\Delta P_t^i = P_t^i(\lambda) - P_t^i(0)$. Let $\mu_i^k(\tilde{\lambda}_t)$ be the number of shares of asset i purchased by investors in group k when realized aggregate disagreement is $\tilde{\lambda}_t \in \{0, \lambda\}$ and let λ_t^k be the realized belief at date t for investors in group $k \in$ {a, A, B}.[14] We first compute the date-t+1 wealth of investors in group $k \in$ {a, A, B}, born at date t and with portfolio holdings $\left(\mu_i^k(\tilde{\lambda}_t)\right)_{i\in[1,N]}$:

$$\tilde{W}_{t+1}^k = \left(\sum_{i\leq N}\mu_i^k(\tilde{\lambda}_t)b_i\right)\tilde{z}_{t+1} + \sum_{i\leq N}\mu_i^k(\tilde{\lambda}_t)\tilde{\epsilon}_{t+1}^i + \sum_{i\leq N}\mu_i^k(\tilde{\lambda}_t)\left(d + P_{t+1}^i(\tilde{\lambda}_{t+1}) - (1+r)P_t^i(\lambda_t)\right)$$

Thus, for investors in group k, their own expected wealth at date $t+1$, and its associated variance are given by:

$$\begin{cases} \mathbb{E}^k[\tilde{W^k}] = \left(\sum_{i\leq N}\mu_i^k(\tilde{\lambda}_t)b_i\right)\lambda_t^k + \sum_{i\leq N}\mu_i^k(\tilde{\lambda}_t)\left(d + \mathbb{E}[P_{t+1}^i(\tilde{\lambda}_{t+1})|\tilde{\lambda}_t] - (1+r)P_t^i(\lambda_t)\right) \\ Var[\tilde{W^k}] = \left(\sum_{i\leq N}\mu_i^k(\tilde{\lambda}_t)b_i\right)^2\sigma_z^2 + \sum_{i\leq N}(\mu_i^k(\tilde{\lambda}_t))^2\sigma_i^2 + \rho(1-\rho)\left(\sum_{i\leq N}\mu_i^k(\tilde{\lambda}_t)\left(\Delta P_{t+1}^i\right)\right)^2 \end{cases}$$

Relative to the static model, there are two notable changes. First, investors value the resale price of their holding at date 1 (the $\mathbb{E}[P_{t+1}^i(\tilde{\lambda}_{t+1})|\tilde{\lambda}_t]$ term in expected wealth). Second, investors now bear the corresponding risk that the resale prices move with aggregate disagreement $\tilde{\lambda}_t$ (this is, in our binomial setting, the $\rho(1-\rho)\left(\sum_{i\leq N}\mu_i^k(\tilde{\lambda}_t)\left(\Delta P_{t+1}^i\right)\right)^2$ term in wealth variance).

G2. Equilibrium

The following Theorem characterizes the equilibrium of this economy:

Theorem 2. *Define $(v_i)_{i\in[0,N+1]}$ such that $v_{N+1}=0$, $v_i = \frac{\sigma_z^2}{N}\left(\sum_{k\geq i}b_k\right) + \frac{1}{N}\frac{\sigma_i^2}{b_i}\left(1+\sigma_z^2\sum_{k<i}\frac{b_k^2}{\sigma_k^2}\right)$ for $i\in[1,N]$ and $v_0=\infty$. v is a strictly decreasing sequence. Let $\bar{i} = \min\{k\in[0,N+1] \mid \lambda > v_k\}$. There exists a unique equilibrium. In this equilibrium, short-sales constraints bind only for the group of pessimist investors (i.e., group B), in the high disagreement states ($\tilde{\lambda}_t = \lambda > 0$) and for assets $i\geq\bar{i}$. The speculative premium on these assets is given by:*

$$\pi^j = \frac{\theta}{\gamma}\left(b_j\frac{\lambda\gamma - \frac{\sigma_z^2}{N}\sum_{k\geq\bar{i}}b_k}{1+\sigma_z^2\left(\sum_{i<\bar{i}}\frac{b_i^2}{\sigma_i^2}\right)} - \frac{\sigma_j^2}{N}\right)$$

Finally, define

$$\Gamma^{\star} = \frac{-(1+r) + (2\rho - 1) + \sqrt{((1+r) - (2\rho - 1))^2 + \frac{4}{N}\frac{\theta\rho(1-\rho)}{\gamma}\sum_{j\geq\bar{i}}\pi^j}}{2\frac{\theta\rho(1-\rho)}{\gamma}} > 0$$

In equilibrium, asset returns are given by:

$$\begin{cases} \mathbb{E}[R^j(\lambda)] = \mathbb{E}[R^j(0)] = \frac{1}{\gamma}\left(b_j\sigma_z^2 + \frac{\sigma_i^2}{N}\right) \quad \textit{for } j < \bar{i} \\ \mathbb{E}[R^j(0)] = \frac{1}{\gamma}\left(b_j\sigma_z^2 + \frac{\sigma_i^2}{N} + \rho(1-\rho)\frac{\Gamma^{\star}}{(1+r) - (2\rho-1) + \frac{\theta\rho(1-\rho)}{\gamma}\Gamma^{\star}}\pi^j\right) \quad \textit{for } j \geq \bar{i} \\ \mathbb{E}[R^j(\lambda)] = \frac{1}{\gamma}\left(b_j\sigma_z^2 + \frac{\sigma_i^2}{N} + \rho(1-\rho)\frac{\Gamma^{\star}}{(1+r) - (2\rho-1) + \frac{\theta\rho(1-\rho)}{\gamma}\Gamma^{\star}}\pi^j\right) - \frac{1+r-(2\rho-1)}{(1+r) - (2\rho-1) + \frac{\theta\rho(1-\rho)}{\gamma}\Gamma^{\star}}\pi^j \quad \textit{for } j \geq \bar{i} \end{cases}$$

Proof. See Appendix F. □

Our characterization of how disagreement affects the Security Market Line in our static model still carries over to this dynamic model with heteroskedasticity. Low b/σ^2 assets (i.e., $j < \bar{i}$) are never shorted since there is not enough disagreement among investors to make the pessimist investors willing to go short, even in the high disagreement states. Thus, the price of these assets is the same in both states of nature and similar to the standard CAPM case. In the high aggregate disagreement state ($\tilde{\lambda} = \lambda > 0$), pessimist investors, that is, investors in group B, want to short high b assets to the extent that these assets are not too risky (i.e., assets j such that $\frac{b_j}{\sigma_j^2} \geq \frac{b_{\bar{i}}}{\sigma_{\bar{i}}^2}$), but are prevented from doing so by the short-sale constraint.[15] This leads to overpricing of these assets relative to the benchmark without disagreement.

A consequence of the previous analysis is that the price of assets $j \geq \bar{i}$ depends on the realization of aggregate disagreement. There is an extra source of risk embedded in these assets: their resale price is more exposed to aggregate disagreement. These assets are thus riskier and command an extra risk premium relative to lower b assets. This extra risk premium takes the following form: $\frac{1}{\gamma}\rho(1-\rho)\frac{\Gamma^{\star}}{(1+r)-(2\rho-1)+\frac{\theta\rho(1-\rho)}{\gamma}\Gamma^{\star}}\pi^j$. Relative to a benchmark without disagreement (and where expected returns are always equal to $\frac{1}{\gamma}\left(b_j\sigma_z^2 + \frac{\sigma_i^2}{N}\right)$), high b assets have higher expected returns in low disagreement states (because of the extra risk-premium). In high disagreement states, holding σ^2 constant, the expected returns of high b assets are strictly lower than in low disagreement states, since the large disagreement about next-period dividends lead to overpricing. Thus, in high disagreement states, the slope of the relationship between expected returns and cash-flow betas holding σ^2 constant is smaller for assets with a high ratio of cash-flow beta to idiosyncratic variance (i.e. assets $j \geq \bar{i}$) than assets $j <\geq> \bar{i}$. Whether the expected returns of high b assets are lower or higher than in the benchmark without disagreement depends on the relative size of the extra risk premium and the speculative premium. In the data, however, aggregate disagreement is persistent, that is, ρ is close to 1. A first-order Taylor expansion of $\Gamma^\star$ around $\rho = 1$ gives that $\Gamma^\star \approx \sum_{j \geq \bar{i}} \frac{\pi_j}{N}$ so that in the vicinity of $\rho = 1$, $\mathbb{E}[R^j(\lambda)] < \frac{1}{\gamma}\left(b_j \sigma_z^2 + \frac{\sigma_j^2}{N}\right)$. Intuitively, when aggregate disagreement is persistent, the resale price risk is very limited, since there is only a small probability that the price of high b assets will change next period. Thus, the speculative premium term dominates and expected returns of high b assets are lower than under the no-disagreement benchmark. We summarize these findings in the following proposition:

Corollary 4.

(i) In low disagreement states ($\tilde{\lambda} = 0$), conditional on σ_i^2, expected returns $\mathbb{E}R_j^e$ are strictly increasing with cash-flow beta b_j. Because of resale price risk, the slope of the return/cash-flow beta relationship is higher for assets $j \geq \bar{i}$ than for assets $j < \bar{i}$.

(ii) In high disagreement states ($\tilde{\lambda} = \lambda > 0$), conditional on σ_i^2, expected returns $\mathbb{E}R_j^e$ are strictly increasing with cash-flow beta b_j for assets $j < \bar{i}$. For assets $j \geq \bar{i}$, the slope of the return/cash-flow beta relationship can be either higher or lower than for assets $j < \bar{i}$. There exists $\rho^\star < 1$ such that for $\rho \geq \rho^\star$, this slope is strictly lower for $b \geq b_{\bar{i}}$ than for $b < b_{\bar{i}}$.

(iii) Conditional on σ_i^2, expected returns $\mathbb{E}R_j^e$ can decrease strictly with cash-flow beta b_j for assets $j \geq \bar{i}$ in high disagreement states, provided ρ is close to 1 and λ is large enough.

(iv) Conditional on σ_i^2, the slope of the returns/cash-flow beta relationship for assets $j \geq \bar{i}$ is strictly lower in high disagreement states ($\tilde{\lambda} = \lambda > 0$) than in low disagreement states ($\tilde{\lambda} = 0$).

Proof. See Appendix G. □

H. Calibration

In this section, we present a simple calibration of the dynamic model presented in the previous section. The objective of this calibration is to see what magnitude of aggregate disagreement is required to obtain a significant distortion in the Security Market Line. We use the following parameters. First, ρ is set to .95. This estimate is obtained by dividing our time-series into high and low aggregate disagreement month (i.e., above and below the median of aggregate disagreement) and computing the probability of transitioning from high to low disagreement ($\mathbb{P} = .05$) and from low to high disagreement ($\mathbb{P} = .05$). We set α to .66 (i.e., $\theta = .5$), which corresponds to the fraction of the stock market held by mutual funds and retail investors, for which the cost of shorting is presumably non-trivial.

The most difficult parameter for us to set is N. We have shown in Section I.F that when N is large, the Security Market Line can only be upward or downward sloping but not inverted U-shaped. However, we argue that a large N is not a good calibration for our model. In the presence of such fixed costs, investors will trade a much smaller number of assets than the overall number of assets in the market. Of course, introducing fixed costs of trading in our model complicate the analysis substantially and we defer a full treatment of this more complex model to further research. In particular, with fixed trading costs, the choice of which asset to trade becomes endogeneous. We believe, however, that the main elements of our analysis would remain unchanged and we highlight here how this endogenous selection of asset may affect our analysis.

When investors face fixed trading costs, there is in equilibrium a segmentation of the market. Optimists would tend to buy, all else equal, the segment of high cash-flow beta assets, as opposed to our current model with no trading costs where they trade all assets. As in our model, the pessimists would only trade on the segment of low beta assets. However, one notable difference with our current setup is that the pessimists would now be the only investors holding these low beta assets. As a consequence, the low beta assets will be underpriced. This effect will reinforce our results as the under-priced low beta securities make the Security Market Line "kinkier".

With these fixed trading costs, arbitrageurs also need to decide on which assets to trade. First, in equilibrium, they need to hold the segment of intermediate cash-flow beta securities. To the extent that mispricing on high and low beta assets is not large – that aggregate disagreement λ is not too large – the risk-premium they receive for holding these intermediate beta assets will compensate more than the arbitrage premium they would receive from shorting the high beta securities. As disagreement λ increases, a fraction of arbitrageurs will start shorting the high beta assets: in this case, arbitrageurs engage in shorting in an amount such that the utility they derive from shorting the high beta stocks is equal to the utility of holding the intermediate beta stocks. Thus, as λ increases, the amount of arbitrage capital devoted to shorting the high beta stocks increases. However, at the same time, optimistic MFs increase their leverage to bet on high beta stocks. This increase in optimistic MFs' demand may well dominate the effect of increased shorting by arbitrageurs.[16]

Beyond trading costs, there exist additional reasons why mutual fund managers invest among a restricted set of stocks. Most mutual fund managers are benchmarked to indices, such as the Morningstar Large Cap Growth Index or the Russell 1000 Growth Index. These indices typically only have a few hundred stocks as constituent members. Hence, because of their index or tracking mandates, most mutual fund managers are forming their portfolios based on a universe of only a few hundred stocks. Retail investors are also trading within a restricted universe of stocks, as it is well-known that these investors typically only consider buying stocks that they are familiar with, such as stocks headquartered near where they live or stocks with high advertising presence (Huberman (2001) or Barber and Odean (2008)). To the extent that the betas of the securities these investors consider are evenly distributed, our model can be directly applied using the average number of stocks held by each investors as the N in our model.

Consistent with N being small for mutual fund investors, Griffin and Xu (2009) shows that from 1980 to 2004, which overlaps with our sample period, the average number of stocks held by mutual funds is between 43 and 119. Consistent with N being even smaller for retail investors, Kumar and Lee (2006) documents, using a dataset from a large US retail broker in the 1990s, that the average retail investor holds a 4-stock portfolio. Fewer than 5% of retail investors hold more than 10 stocks.

As noted in Barber and Odean (2000), in 1996, approximately 47% of equity investments in the United States were held directly by households and 14% by mutual funds, although these shares evolve quite a bit through time. As such, $N = 50$ seems in the relevant range for the universe of stocks typical number of stocks held by long only investors. The calibration we perform below is not very sensitive to small changes to N around this $N = 50$ number.[17] As expected however, when N becomes very large, we get the result we derived theoretically in Section I.F, when solving the complete market case: the Security Market Line can only be upward or downward sloping but not inverted U-shaped as is the case when N is smaller and in the calibration we perform below.

We set the values of b_i such that $b_i = \frac{2i}{N+1}$. Thus, cash-flow betas are bounded between 0 and 2 and have an average value of 1. We implement our calibration in the following way. We set a value for λ. We then find the values for σ_z^2, σ_ϵ^2 and γ such that the model matches the following empirical moments, computed over the 1981-2011 period: (i) average volatility of the monthly market return (.2% monthly) (ii) the average idiosyncratic variance of monthly stock returns (3.5% monthly) and (iii) the average expected excess return of the market (.63% monthly).[18] Finally, our calibration method borrows from Campbell et al. (1993), who also calibrate a CARA model using dollar returns as we do by setting the dividend to have a price of the asset equal to 1. We report four calibrations in Figure 4:

1. $\lambda = 0.008$, which implies $\sigma_\epsilon^2 = .0305$, $\sigma_z^2 = .0014$ and $\gamma = .32$. 38 of the 50 assets are shorted at equilibrium. This level of disagreement corresponds to 20% of σ_z. Figure 4(a) plots the Security Market Line for these parameter values. Figure 4(a) shows that for this level of disagreement, the distortion on the Security Market Line is limited. Even in the high aggregate disagreement state, the Security Market Line is upward sloping with a slope close to its slope in the low aggregate disagreement state.

2. $\lambda = 0.013$, which implies $\sigma_\epsilon^2 = .0305$, $\sigma_z^2 = .0013$ and $\gamma = .31$. 45 of the 50 assets are shorted at equilibrium. This level of disagreement corresponds to 35% of σ_z. Figure 4(b) shows that for this level of disagreement, the distortion on the Security Market Line becomes noticeable. In the high aggregate disagreement state, the Security Market Line is still upward sloping for all β, but with a much smaller slope for assets with $b_i \geq b_{\bar{i}}$. The Security Market Line is kink-shaped in the high aggregate disagreement state.

3. $\lambda = 0.022$, which implies $\sigma_\epsilon^2 = .0305$, $\sigma_z^2 = .0011$ and $\gamma = .27$. 47 of the 50 assets are shorted at equilibrium. This level of disagreement corresponds to 65% of σ_z. Figure 4(c) shows that for this level of disagreement, in the high aggregate disagreement state, the Security Market Line has an inverted-U shape.

4. $\lambda = 0.05$, which implies $\sigma_\epsilon^2 = .0305$, $\sigma_z^2 = .0005$ and $\gamma = .16$. 48 of the 50 assets are shorted at equilibrium. This level of disagreement corresponds to 187% of σ_z. Figure 4(d) shows that for this level of disagreement, in the high aggregate disagreement state, the Security Market Line is downward sloping. Moreover, we also see on Figure 4(d) that assets with a beta greater than .9 have a negative expected excess return in the high aggregate disagreement state.

These calibrations overall support the idea that for reasonable levels of disagreement, the Security Market Line in the high aggregate disagreement state will be significantly distorted relative to the low aggregate disagreement state.

[Insert Figure 4 here]

II. Data and Variables

A. Data Source

The data in this paper are collected from two main sources. U.S. stock return data are from the CRSP tape and the analyst forecasts are from the I/B/E/S analyst forecast database. The I/B/E/S data starts in December 1981.

We start with all available common stocks on CRSP between December 1981 and December 2014. We then exclude penny stocks with a share price below $5 and microcaps, defined every months as stocks in the bottom 2 deciles of the monthly market capitalization distribution using NYSE breakpoints. β's are computed with respect to the value-weighted market returns provided on Ken French's website. Excess returns are in excess of the US Treasury bill rate, which we also download from Ken French's website. We also use stock analyst forecasts of the earnings-per-share (EPS) long-term growth rate (LTG) as the main proxy for investors' beliefs regarding the future prospects of individual stocks. The data are provided in the I/B/E/S database. As explained in detail in Yu (2010), the long-term forecast has several advantages. First, it features prominently in valuation models. Second, it is less affected by a firm's earnings guidance relative to short-term forecasts. Because the long-term forecast is an expected growth rate, it is directly comparable across firms or across time.

B. β-sorted portfolios

We follow the literature in constructing beta portfolios in the following manner. Each month, we use the past twelve months of daily returns to estimate the market beta of each stock in that cross-section. This is done by regressing, at the stock-level, the stock's excess return on the contemporaneous excess market return as well as five lags of the market return to account for the illiquidity of small stocks (Dimson (1979)). Our measure of β is then the sum of these six OLS coefficients.

We then sort stocks every month into 20 β portfolios based on these pre-ranking betas, using only stocks in the NYSE to define the β thresholds. We compute the daily returns on these portfolios, both equal- and value-weighted. Post-ranking β's are then estimated using a similar market model – regressing each portfolio's daily returns on the excess market returns, as well as five lags of the market return, and adding up these six OLS coefficients. These post-ranking β's are computed using the entire sample period (Fama and French (1992)). Table I presents descriptive statistics for the 20 β-sorted portfolios. The 20 β-sorted portfolios do exhibit a significant spread in β, with the post-ranking full sample β of the bottom portfolio equal to .43 and that of the top portfolio equal to 1.78.

[Insert Table I here]

C. Measuring Aggregate Disagreement

Our measure of aggregate disagreement is similar in spirit to Yu (2010). Stock-level disagreement is measured as the dispersion in analyst forecasts of the earnings-per-share (EPS) long-term growth rate (LTG). We then aggregate this stock-level disagreement measure, weighting each stock by its pre-ranking β.[19] Intuitively, our model suggests that there are two components to the overall disagreement on a stock dividend process: (1) a first component coming from the disagreement about the aggregate factor $\tilde{z}$ – the λ in our model and (2) a second component coming from disagreement about the idiosyncratic factor $\tilde{\epsilon}_i$. We are interested in constructing an empirical proxy for the first component only. To that end, disagreement about low β stocks should only play a minor role since disagreement about a low β stock has to come mostly from idiosyncratic disagreement – in the limit, disagreement about a $\beta = 0$ stock can only come from idiosyncratic disagreement. Thus, we weight each stock-level disagreement by the stock's pre-ranking β.[20]

To assess the robustness of our analysis, we use two alternative proxies for aggregate disagreement. The first of these alternative measures is the analyst forecast dispersion of Standard & Poor's (S&P) 500 index annual earnings-per-share (EPS). The problem with this top-down measure is that there are much fewer analysts forecasting this quantity, making it far less attractive when compared to our bottom-up measure. While our preferred measure of aggregate disagreement is constructed using thousands of individual-stock forecasts, there are, on average, only 20 or so analysts in the sample covering the S&P 500 EPS. The second alternative proxy we use is an index of the dispersion of macro-forecasts from the Survey of Professional Forecasters. More precisely, we use the first principal component of the cross-sectional standard deviation of forecasts on GDP, Industrial Production (IP), Corporate Profit and Unemployment from Li and Li

(2014).

To simplify the reading of the tables in the paper, all these time-series measures of aggregate disagreement are standardized to have 0 mean and a variance of 1. Table II presents summary statistics on the time-series variables used in the paper. Figure 1 reports the time-series of our baseline disagreement measure. It peaks during the 1981-1982 recession, the dot-com bubble of the late 90s and the recent recession of 2008. When fundamentals are more uncertain, there is more scope for disagreement among investors. In other words, the aggregate disagreement series is not the same as the business cycle as we see high disagreement in both growth and recession periods.

[Insert Table II here]

In Figure 5, we highlight the role played by aggregate disagreement on the relationship between stock-level disagreement and β. This figure is constructed in the following way. We divide our time-series into high aggregate disagreement months (red dots) and low aggregate disagreement months (blue dots), where high (resp. low) aggregate disagreement months are defined as being in the top (resp. bottom) quartiles of the in-sample distribution of aggregate disagreement. Then, for each of our 20 β-sorted portfolios, we plot the value-weighted average of the stock-level dispersion in analyst earnings forecasts against the post-ranking full sample β of the value-weighted portfolio. Stock-level disagreement increases with β; this relation, moreover, is steeper in months with high aggregate disagreement relative to months with low aggregate disagreement: consistent with the premise of our model, we thus find that β does scale up aggregate disagreement.

[Insert Figure 5 here]

III. Empirical Analysis

A. Aggregate Disagreement and the Concavity of the Security Market Line

A1. Main Analysis

Our empirical analysis examines how the Security Market Line is affected by aggregate disagreement. To this end, we first present in Figure 6 the empirical relationship between β and excess returns. For each of the 20 β portfolios in our sample, we compute the average excess forward return for high (red dots) and low (blue dots) disagreement months (defined as top vs. bottom quartile of aggregate disagreement). Given the persistence in aggregate disagreement, we run this analysis using various horizons: 3-month (top left panel), 6-months (top-right panel), 12 months (bottom-left panel) and 18 months (bottom-right panel). The portfolio returns $r_{P,t}^{(k)}$ for $k = 3, 6, 12, 18$ are value-weighted.

While the relationship between excess forward returns and β is quite noisy at the 3 and 6 months horizon, two striking facts emerge at the 12 and 18 months horizons. First, the average excess return/β relationship is mostly upward sloping for months with low aggregate disagreement, except for the top β portfolio which exhibit a somewhat lower average return. This is overall consistent with our theory whereby low aggregate disagreement means low or even no mispricing and hence a strictly upward sloping Security Market Line. Second, in months of high aggregate disagreement, the excess return/β relationship appears to exhibit the inverted-U shape predicted by the theory.

[Insert Figure 6 here]

To formally test our Prediction 1, we run the following two-stage Fama-McBeth regressions in Table III. Every month, we first estimate the following cross-sectional regression over our 20 β-sorted portfolios:

$$r_{P,t}^{(12)} = \kappa_t + \pi_t \times \beta_P + \phi_t \times (\beta_P)^2 + \epsilon_{P,t}, \quad \text{where} \quad P = 1, ..., 20$$

and $r_{P,t}^{(12)}$ is the 12-months excess return of the P^{th} beta-sorted portfolio and β_P is the full sample post-ranking beta of the P^{th} beta-sorted portfolio.[21] We retrieve from this analysis a time-series of coefficient estimates for κ_t, π_t and ϕ_t. Note that our analysis focuses here on 12 months returns. Since our aggregate disagreement variable is persistent, our results will tend to be stronger over longer horizons (Summers (1986), Campbell and Shiller (1988)). For robustness, we present in Table AII the results from a similar analysis using different horizons.

[Insert Table III here]

The time-series of coefficient estimates ϕ_t is the dependent variable of interest in our analysis. Given the post-ranking β of our β-sorted porftolios, this ϕ_t series corresponds to the excess returns on a portfolio that goes long the two bottom β portfolio (P=1 to 6) as well as the two top β portfolios (P=19 and 20) and short the remaining portfolios. As explained in Section I.C, this portfolio's returns capture, each month, the concavity of the security market line. Prediction 1 is that when aggregate disagreement is higher, this portfolio should have significantly lower returns.

To examine the evidence in support of Prediction 1, we thus regress, in a second-stage, the ϕ_t time-series on $Agg.Disp._{t-1}$ only (column (1)), where $Agg.Disp.$ stands for the monthly β-weighted average of stock level disagreement introduced in Section II.C and is measured in month $t-1$. Noxy-Marx (2014) shows that the returns on defensive equity strategies load significantly on standard risk-factors. Although our portfolio of interest is not a slope portfolio but the square portfolio, we nonetheless follow Noxy-Marx (2014)'s and control in column (2) for the 12-months returns of the Fama and French (1992) factors and Jegadeesh and Titman (1993) momentum factor measured in month t.[22] Column (3) adds the dividend-to-price ratio D/P_{t-1} and the year-on-year inflation rate measured in month $t-1$, $Inflation_{t-1}$, from Cohen et al. (2005). Column (4) finally adds the $TedSpread$ measured in month $t-1$ from Frazzini and Pedersen (2010). Columns (5)-(8) and (9)-(12) are the corresponding columns where the dependent variables are respectively the estimated κ_t and π_t. In these estimations, standard errors are Newey-West adjusted, and allow for 11 lags of serial correlation.

Panel A of Table III shows the results from the second-stage regressions using value-weighted portfolios. A higher $Agg.Disp._{t-1}$ is associated with a smaller ϕ_t, that is, a more concave Security Market Line or equivalently lower average returns of the square portfolio. The t-stats are between -1.9 to -4 depending on the specification. Importantly, the estimate is significant by itself even without any controls, although the inclusion of the D/P ratio and the year-to-year past inflation rate does make the effect of aggregate disagreement on the concavity of the SML more significant.

Interestingly, we see in Table III that a higher return on HML from t to $t+11$ is correlated with a more negative ϕ_t – a more concave Security Market Line. We believe this result is consistent with a simple extension of our model. Note first that our model generates, even in the absence of disagreement, a value-growth effect through risk – high risk stocks have low price and higher expected returns (Berk (1995)). To abstract from this effect, one can simply define the fundamental value of a stock as its expected dividend minus its risk-premium – $F = d - \frac{1}{\gamma}\left(b_i\sigma_z^2 + \frac{\sigma_\epsilon^2}{N}\right)$. This is the fundamental price an investor would expect to pay for the stock based purely on risk-based valuation. As in Daniel et al. (2001), we then define the price-to-fundamental ratio as: $P - F$ and the return on the high-minus-low (HML) portfolio is simply defined as the return of the long-short portfolio that goes long the stock with the lowest price-to-fundamental ratio and short the stock with the highest price-to-fundamental ratio. In our static model, as long as $b_N\omega(\lambda) > \frac{\sigma_\epsilon^2}{N}$, this ratio corresponds to the return on a portfolio short asset N and long any asset $k \in [1, \bar{i} - 1]$. The return on this portfolio is given by: $(b_k - b_N)\frac{\sigma_z^2}{\gamma} + \frac{\theta}{\gamma}\left(b_N\omega(\lambda)\sigma_z^2 - \frac{\sigma_\epsilon^2}{N}\right)$. In particular, the return on this HML portfolio is strictly increasing with λ. Thus, a larger return on the HML portfolio will be associated with a smaller slope of the Security Market Line. Empirically, to the extent that our proxy for aggregate disagreement λ is measured with noise, we should thus expect the return to HML to have a significant and negative correlation with the concavity of the Security Market Line. This is precisely what we observe in Column (2), (3) and (4) of Table III. This result, although not the main point of the paper, is novel in that it connects the failure of the CAPM to HML through time-variation in aggregate disagreement.

In contrast, we see that the a larger contemporaneous return on SMB_t corresponds to a more convex SML. Inflation comes in with a negative sign — the higher is inflation, the more concave or flatter the Security Market Line. $TedSpread$ is not significantly related to the concavity of the SML. Panel B of Table III shows the results from a similar analysis using equal-weighted β-sorted portfolios. The results in Panel B are quantitatively similar to those in Panel A, with a higher level of statistical significance. Overall, consistent with Prediction 1, we find that higher level of aggregate disagreement are associated with a more concave security market line in the following months.

A2. Robustness Checks

We present in the Internet Appendix a battery of robustness checks for this result.

In Table AI, we show the analogous results to Table III but where the pre-ranking βs are now estimated by regressing monthly stock returns over the past 3 years on the contemporaneous market returns. The results are quantitatively very close to those obtained in Table III.

In Table AII, we use different horizons for the portfolio returns used in the first-stage regression – namely 1, 3, 6 and 18 months. While the effect of disagreement on the concavity of the SML is insignificant when using a 1 or 3 month horizon (but of the right sign), it is significant when using a 6 or 18-month horizon. Note that once D/P_{t-1} and $Inflation_{t-1}$ are included in the regression, aggregate disagreement becomes significantly and negatively correlated with the concavity of the SML at all horizon. The fact that short-horizon results are weaker is to be expected given the literature on long-horizon predictability associated with persistent predictor variables and the fact that *Agg.Disp.* is persistent.

In Table AIII, we use the alternative measures of aggregate disagreement introduced in Section II.C. In Panel A, aggregate disagreement is constructed using pre-ranking *compressed* β (i.e. $\beta = .5\ \hat{\beta} + .5$) to weight the stock-level disagreement measure. In Panel B, we use $\beta \times$ value-weights to define aggregate disagreement. In Panel C, disagreement is the "top-down" measure of market disagreement used in Yu (2011) and measured as the standard deviation of analyst forecasts of annual S&P 500 earnings, scaled by the average forecast on S&P 500 earnings. In Panel D, disagreement is the first principal component of the monthly cross-sectional standard deviation of forecasts on GDP, IP, Corporate Profit and Unemployment rate in the Survey of Professional Forecasters (SPF) and is taken from Li and Li (2014). All these series are standardized to have mean 0 and variance 1. In all specifications, especially those that include the additional covariates, we get results that are quantitatively close to our baseline results presented in Table III, although of the estimated coefficients are less significant than in our baseline result.[23]

A potential concern with our analysis is that our results are a simple recast of the results in Diether et al. (2002): high beta stocks experience more idiosyncratic disagreement, especially in high aggregate disagreement months so that the effect of aggregate disagreement on the Security Market Line would work entirely through idiosyncratic disagreement. In Table AIV, we show this is not the case. To this end, we replicate the analysis of Table III, but we now control, in our first stage regression, for the logarithm of the average disagreement on the stocks in each of the 20 β-sorted portfolios. Again, our results are virtually unchanged by this additional control in the first-stage regression.

Another empirical concern with the analysis from Table III is that (1) high beta stocks have higher idiosyncratic volatility (2) idiosyncratic stocks have lower returns (Ang et al. (2006)) (3) perhaps especially when aggregate disagreement is high. The next section in the paper, Section III.B tests the asset pricing equation from our model when dividends are allowed to be heteroskedastic. However, we can also simply amend our methodology to include, in the first-stage regression, a control for the median idiosyncratic volatility of stocks in each of the 20 β-sorted portfolios.[24] The results are presented in Table AV. The point estimates are quantitatively similar to those obtained in Table III, although the statistical significance is slightly lower (t-stat ranging from 1.6 to 2.4 in the value-weighted specification, from 2 to 2.6 in the equal-weighted specification). Our main finding is thus robust to controlling directly, in the first-stage regressions, for the idiosyncratic volatility of the stocks included in the 20 β-sorted portfolios.

A3. Disagreement and the Slope of the Security Market Line

Corollary 3 showed that the slope of the SML should decrease with aggregate disagreement. Although we argued in Section I.C that this was a weaker test of our model – since it fails to account for the kinks in the SML predicted in the model – we nonetheless present in Table AVI a test for this prediction. This test is again a two-stage procedure. In the first-stage, we regress each month the excess return on the 20-β sorted portfolios on their post-ranking full sample β:

$$r_{P,t}^{(12)} = \kappa_t + \pi_t \times \beta_P + \epsilon_{P,t}, \quad \text{where} \quad P = 1, ..., 20$$

π_t is here is the variable of interest, that is, the slope of the SML in month t. π_t represents the 12-month excess returns of a "slope" portfolio in month t – a portfolio that goes long the portfolios with above average β and short the portfolio with below average β. Column (1) of Tables AVI shows that by itself, aggregate disagreement in month $t-1$ does predict a significantly flatter SML in the following month. In Column (2), we see that introducing the contemporaneous 4-factor returns in the regression does absorb most of the effect of disagreement on the slope of the SML. However, we also see in this column, as well as in column (3) and (4) that a higher HML 12-month returns in month t is associated with a significantly flatter SML at t.[25] As we explained above, this result is a natural prediction of our model, since aggregate disagreement lead to the mispricing of high beta securities, which then mean-revert. Interestingly, the inclusion of D/P_{t-1} and $Inflation_{t-1}$ in Column (3) and (4) make the effect of aggregate disagreement on the slope of the SML significant again. Consistent with Cohen et al. (2005), a higher level of $Inflation_{t-1}$ leads to a flatter Security Market Line. As in Frazzini and Pedersen (2010), the $TedSpread$ does not significantly explain the average excess returns of the slope portfolio.

B. Heteroskedastic Idiosyncratic Variance

We now turn our attention to our main test for Prediction 2, mainly that the slope of the Security Market Line is more sensitive to aggregate disagreement for stocks with high β_i/σ_i^2 ratio relative to stocks with a low β_i/σ_i^2 ratio.

To test this prediction, we need to ascribe a value for the threshold $\frac{\beta_{\bar{i}}}{\sigma_{\bar{i}}^2}$ defining speculative and non-speculative stocks. Our strategy is to use as a baseline specification a threshold corresponding to the median $\frac{\beta_i}{\sigma_i^2}$ ratio and to then assess the robustness of the results to this particular choice. More precisely, our test for Prediction 2 is based on a 3-stage approach. In the first-stage, we rank stocks each month based on their pre-ranking ratio of β to σ^2 and define as speculative (resp. non speculative) stocks all stocks with a ratio above (resp. below) the NYSE median ratio: $\frac{\hat{\beta}_i}{\hat{\sigma}_i^2}$ $>$ NYSE median $\frac{\hat{\beta}}{\hat{\sigma}^2}$ (resp. $\frac{\hat{\beta}_i}{\hat{\sigma}_i^2} \leq >$NYSE median $\frac{\hat{\beta}}{\hat{\sigma}^2}$). This creates two groups of stocks for each month t: speculative and non-speculative stocks. Within each of these two groups, we then re-rank the stocks in ascending order of their estimated beta at the end of the previous month and assign them to one of 20 beta-sorted portfolios using again NYSE breakpoints. We compute the full sample beta of these 40 value-weighted portfolios (20 beta-sorted portfolios for speculative stocks; 20 beta-sorted portfolios for non-speculative stocks) using the same market model. $\beta_{P,s}$ is the resulting full sample beta, where $P = 1, \ldots, 20$ and s $\in$ {speculative, non speculative}. Table IV presents descriptive statistics for the resulting 40 portfolios. We see in particular that (1) the constructed portfolios generate significant spreads in the post-ranking full sample βs and (2) ex post, the β/σ^2 ratio of the β-sorted portfolios created from speculative stoccks is in fact much higher than the β/σ^2 ratio of the β-sorted porftolios created from non-speculative stocks: the average β/σ^2 ratio for speculative stocks is .61 while it is only .25 for non-speculative stocks.[26]

[Insert Table IV here]

For each of these two groups of portfolios $s \in$ {speculative, non speculative}, we then estimate every month the following cross-sectional regressions, where P is one of the 20 β-sorted portfolios,and t is a month :

$$r^{(12)}_{P,s,t} = \iota_{s,t} + \chi_{s,t} \times \beta_{P,s} + \varrho_{s,t} \times \ln\left(\sigma_{P,s,t-1}\right) + \epsilon_{P,s,t},$$

where $\sigma_{P,s,t-1}$ is the median idiosyncratic volatility of stocks in portfolio (P, s) estimated at the end of month $t-1$ and $r^{(12)}_{P,s,t}$ is the value-weighted 12-months excess return of portfolio (P, s). In contrast to $\beta_{P,s}$, $\sigma^2_{P,s,t-1}$ has a large skew, so that we use the logarithm of $\sigma_{P,s,t-1}$ in the cross-sectional regressions to limit the effect of outliers on the regression estimates. We retrieve a time-series of monthly estimated coefficients: $\iota_{s,t}$, $\chi_{s,t}$ and $\varrho_{s,t}$. As we did for Table III, we finally regress in a third-stage each of these series on $Agg.Disp._{t-1}$, the contemporaneous four-factor alphas ($R_{m,t}$, HML_t, SMB_t, and UMD_t) and a set of additional forecasting variables measured in month $t-1$, D/P_{t-1}, $Inflation_{t-1}$, and $TedSpread_{t-1}$.

Standard errors are Newey-West adjusted, and allow for 11 lags of serial correlation.

Figure 7 summarizes our findings in a simple graphical analysis. In this figure, we compute, for the 20 β-sorted portfolios constructed from speculative stocks as defined above (bottom panel) and the 20 β-sorted portfolios constructed from non-speculative stocks, the average excess 12-month return for high (red dots) and low (blue dots) disagreement months (defined as top vs. bottom quartile of aggregate disagreement). For non-speculative stocks, we see that the Security Market line is not related in a clear way with aggregate disagreement. For speculative stocks, however, Figure 7 suggests that when aggregate disagreement is high, the SML exhibit an inverted-U shape while there is no such kink in months with low aggregate disagreement. This first pass at the data is thus consistent with Prediction 2, namely that aggregate disagreement makes the SML flatter only for speculative stocks.

[Insert Figure 7 here]

Table V reports the result from the actual regression analysis. Panel A of this table shows the estimation results of the third-stage regression when using portfolios constructed from speculative stocks. Panel B presents the results from portfolios constructed from non-speculative stocks. The first four columns (1)-(4) exhibit the estimation results for our main coefficient of interest $\chi_{s,t}$, which measures the slope of the conditional Security Market Line, conditional on the idiosyncratic variance of portfolios. Consistent with prediction 2, an increase in aggregate disagreement in month $t-1$ is associated with a significantly flatter Security Market Line – holding portfolio variance constant – only in Panel A, that is, only for stocks with a β/σ^2 ratio above the NYSE median ratio. In Panel B, where the portfolios are formed from stocks with a β/σ^2 ratio below the NYSE median ratio, aggregate disagreement is not significantly related to the slope of the SML. Across our four specifications, which include additional controls, including the contemporaneous 4-factor returns, the results are similar: a higher aggregate disagreement in month $t-1$ leads to a significantly flatter slope of the SML (with t-stats rangin from 2.1 to 2.9) when β-sorted portfolios are formed using speculative stocks; there is no significant relationship between aggregate disagreement and the slope of the SML for these portfolios that are constructed using non-speculative stocks. These results are consistent with Prediction 2.

The next four columns (5)-(8) of Table V show the regression estimates when $\varrho_{s,t}$ is the dependent variable. $\varrho_{s,t}$ represents the effect of idiosyncratic variance (the log of) on the returns of these β-sorted portfolios. Our model predicts that for speculative stocks, stocks with high idiosyncratic variance should have higher expected returns, especially when aggregate disagreement is high. In Panel A, column (5)-(8), we see that both constant and the coefficients in front of $Agg.Disp._{t-1}$ are positive, consistent with our model, but that they are not statistically significant. In some of the specifications below (most notably the specification using equal-weighted portfolios), we find that these coefficients are not only positive but also statistically significant. However, Table V shows that our model does not fully capture how idiosyncratic variance is priced in the cross-section of stock returns.

In Panel B, we find that (1) idiosyncratic variance has no significant effect on the returns of β-sorted portfolios constructed from non-speculative stocks (the constant is insignificant and small in magnitude) (2) for these non-speculative portfolios, an increase in disagreement is associated with a lower effect of idiosyncratic variance on the returns of these β-sorted portfolios (this effect is insignificant in all but column (6) where the t-stat is 1.8). This negative sign is inconsistent with our model since in the model, non-speculative stocks should have returns that are independent of aggregate disagreement. However, in differential terms, these results could be reconciled with the model, to the extent that they show that the effect of aggregate disagreement on the price of idiosyncratic variance is significantly larger for speculative stocks than for non-speculative stocks.

[Insert Table V here]

We confirm the robustness of this analysis by performing a battery of additional tests. In Table VI, we use equal-weighted portfolios instead of value-weighted portfolios. We obtain even more supporting evidence in that the coefficients of interests are both economically larger and statistically more significant. As mentionned above, we even find some support with these equal-weighted portfolios for the prediction relating aggregate disagreement to the price of idiosyncratic variance.

[Insert Table VI here]

In Table AVII, the pre-ranking βs are estimated by regressing monthly stock returns over the past 3 years on the contemporaneous market returns. Results are essentially similar. In Table AVIII, we reproduce the analysis of Table V using portfolio returns over an horizon of 1, 3, 6 and 18 months. The results are not statistically significant using a 1-month horizon. However, starting at the 3-month horizon and above, the results are consistent with the baseline 12-month horizon result shown in Table V:

overall, the prediction that aggregate disagreement leads to a flatter SML for speculative stocks is strongly supported in the data, while the prediction relating aggregate disagreement to the price of idiosyncratic risk finds only mixed support.[27]

Our analysis so far has used an arbitrary cutoff to define speculative stocks, namely the median NYSE β/σ^2 ratio. In Figure 8, we reproduce a similar analysis to that performed in Table V but where we use different cut-offs to define speculative versus non-speculative stocks. For each of these cutoffs, we plot the coefficient estimate of the regression of aggregate disagreement on the slope of the SML $\chi_{s,t}$, obtained from the specification in Column (2), which includes only the realized 4-factor returns as controls. We select this specification as it typically yields the smallest point estimates. The left (resp. right) Panel shows the results obtained for portfolios formed from speculative (resp. non-speculative) stocks. The cut-offs we use range from the 30th percentile of the NYSE distribution of the β_i/σ_i^2 ratio to its 70th percentile. Accross all these different specifications, we obtain consistent results in that aggregate disagreement leads to a flatter SML only for speculative stocks. The effect of disagreement on the slope of the SML for speculative stocks become larger (in absolute value) as the threshold to define speculative stocks become more conservative.

[Insert Figure 8 here]

IV. Conclusion

We show that incorporating the speculative motive for trade into asset pricing models yields strikingly different results from the risk-sharing or liquidity motives. High beta assets are more speculative because they are more sensitive to disagreement about common cash-flows. Hence, they experience greater divergence of opinion and in the presence of short-sales constraint for some investors, they end up being over-priced relative to low beta assets. When aggregate disagreement is low, the risk-return relationship is upward sloping. As aggregate disagreement rises, the slope of the Security Market Line is piecewise constant, higher in the low beta range, and potentially negative for the high beta range. Empirical tests using measures of disagreement based on security analyst forecasts are consistent with these predictions. We believe our simple and tractable model provides a plausible explanation for part of the high-risk, low-return puzzle. The broader thrust of our analysis has been to point out that one can construct a behavioral macro-finance model in which aggregate sentiment can influence the cross-section of asset prices in non-trivial ways.

References

Almazan, Andres, Keith C. Brown, Murray Carlson, and David A. Chapman, 2004, Why constrain your mutual fund manager?, *Journal of Financial Economics* 73, 289–321.

Ang, Andrew, Robert J. Hodrick, Yuhang Xing, and Xiaoyan Zhang, 2006, The cross-section of volatility and expected returns, *Journal of Finance* 61, 259–299.

Baker, Malcolm, Brendan Bradley, and Jeffrey Wurgler, 2011, Benchmarks as limits to arbitrage: Understanding the low-volatility anomaly, *Financial Analysts Journal* 67, 1–15.

Barber, Brad M., and Terrance Odean, 2000, Trading is hazardous to your wealth: The common stock investment performance of individual investors, *The Journal of Finance* 55, 773–806.

Barber, Brad M., and Terrance Odean, 2001, Boys will be boys: Gender, overconfidence, and common stock investment, *The Quarterly Journal of Economics* 116, 261–292.

Barber, Brad M., and Terrance Odean, 2008, All that glitters: The effect of attention and news on the buying behavior of individual and institutional investors, *Review of Financial Studies* 21, 785–818.

Barberis, Nicholas, and Ming Huang, 2001, Mental accounting, loss aversion, and individual stock returns, *Journal of Finance* 56, 1247–1292.

Berk, Jonathan, 1995, A critique of size-related anomalies, *Review of Financial Studies* 8, 275–286.

Black, Fischer, 1972, Capital market equilibrium with restricted borrowing, *Journal of Business* 4, 444–455.

Black, Fischer, Michael C. Jensen, and Myron Scholes, 1972, The capital asset pricing model: Some empirical tests, in Michael C. Jensen, ed., *Studies in the Theory of Capital Markets* (New York: Praeger).

Blitz, David C., and Pim Van Vliet, 2007, The volatility effect: Lower risk without lower return, *Journal of Portfolio Management* 34, 102–113.

Brennan, Michael J., 1993, Agency and asset pricing, University of california at los angeles, anderson graduate school of management, Anderson Graduate School of Management, UCLA.

Campbell, John Y, Sanford J Grossman, and Jiang Wang, 1993, Trading volume and serial correlation in stock returns, *The Quarterly Journal of Economics* 108, 905–39.

Campbell, John Y, and Robert J Shiller, 1988, The dividend-price ratio and expectations of future dividends and discount factors, *Review of financial studies* 1, 195–228.

Chen, Joseph, Harrison Hong, and Jeremy C. Stein, 2002, Breadth of ownership and stock returns, *Journal of Financial Economics* 66, 171–205.

Cohen, Randolph B. Cohen Christopher Polk Randolph B., Christopher Polk, and Tuomo Vuolteenaho, 2005, Money illusion in the stock market: The modigliani-cohn hypothesis, *The Quarterly Journal of Economics* 120, 639–668.

Cukierman, Alex, and Paul Wachtel, 1979, Differential inflationary expectations and the variability of the rate of inflation: Theory and evidence, *American Economic Review* 69, 595–609.

Daniel, Kent D., David Hirshleifer, and Avanidhar Subrahmanyam, 2001, Overconfidence, arbitrage, and equilibrium asset pricing, *Journal of Finance* 56, 921–965.

Delong, J Bradford, Andrei Shleifer, Lawrence Summers, and Mark Waldmann, 1990, Positive feedback investment strategies and destabilizing rational speculation, *Journal of Finance* 45, 379–95.

Diether, Karl B., Christopher J. Malloy, and Anna Scherbina, 2002, Differences of opinion and the cross section of stock returns, *Journal of Finance* 57, 2113–2141.

Dimson, Elroy, 1979, Risk measurement when shares are subject to infrequent trading, *Journal of Financial Economics* 7, 197–226.

Fama, Eugene F, and Kenneth R French, 1992, The cross-section of expected stock returns, *Journal of Finance* 47, 427–65.

Frazzini, Andrea, and Lasse Pedersen, 2010, Betting against beta, Working paper, New York University.

Griffin, John M., and Jin Xu, 2009, How smart are the smart guys? a unique view from hedge fund stock holdings, *Review of Financial Studies* 22, 2531–2570.

Harrison, J Michael, and David M Kreps, 1978, Speculative investor behavior in a stock market with heterogeneous expectations, *The Quarterly Journal of Economics* 92, 323–36.

Hong, Harrison, Jose Scheinkman, and Wei Xiong, 2006, Asset float and speculative bubbles, *Journal of Finance* 61, 1073–1117.

Hong, Harrison, and Jeremy C. Stein, 2007, Disagreement and the stock market, *Journal of Economic Perspectives* 21, 109–128.

Huberman, Gur, 2001, Familiarity breeds investment, *Review of financial Studies* 14, 659–680.

Jarrow, Robert A, 1980, Heterogeneous expectations, restrictions on short sales, and equilibrium asset prices, *Journal of Finance* 35, 1105–13.

Jegadeesh, Narasimhan, and Sheridan Titman, 1993, Returns to buying winners and selling losers: Implications for stock market efficiency, *Journal of Finance* 48, 65–91.

Kandel, Eugene, and Neil D Pearson, 1995, Differential interpretation of public signals and trade in speculative markets, *Journal of Political Economy* 103, 831–72.

Koski, Jennifer Lynch, and Jeffrey Pontiff, 1999, How are derivatives used? evidence from the mutual fund industry, *Journal of Finance* 54, 791–816.

Kumar, Alok, and Charles M.C. Lee, 2006, Retail investor sentiment and return comovements, *The Journal of Finance* 61, 2451–2486.

Lakonishok, Josef, Andrei Shleifer, and Robert W Vishny, 1994, Contrarian investment, extrapolation, and risk, *Journal of Finance* 49, 1541–78.

Lamont, Owen, 2004, Go down fighting: Short sellers vs. firms, Nber working papers, National Bureau of Economic Research, Inc.

Lamont, Owen A., 2002, Macroeconomic forecasts and microeconomic forecasters, *Journal of Economic Behavior & Organization* 48, 265–280.

Li, Dan, and Geng Li, 2014, Are household investors noise traders? belief dispersion and stock trading volume, *Federal Reserve Board Working Paper* .

Li, Wekai, 2014, Macro disagreement and the cross-section of stock rteurns, *Hong Kong University Working Paper* .

Mankiw, N. Gregory, Ricardo Reis, and Justin Wolfers, 2004, Disagreement about inflation expectations, in *NBER Macroeconomics Annual 2003, Volume 18*, NBER Chapters, 209–270 (National Bureau of Economic Research, Inc).

Merton, Robert C, 1987, A simple model of capital market equilibrium with incomplete information, *Journal of Finance* 42, 483–510.

Miller, Edward M, 1977, Risk, uncertainty, and divergence of opinion, *Journal of Finance* 32, 1151–68.

Morris, Stephen, 1996, Speculative investor behavior and learning, *The Quarterly Journal of Economics* 111, 1111–33.

Noxy-Marx, Robert, 2014, Understanding defensive equity, *University of Rochester Paper* .

Odean, Terrance, 1999, Do investors trade too much?, *American Economic Review* 89, 1279–1298.

Pontiff, Jeffrey, 1996, Costly arbitrage: Evidence from closed-end funds, *The Quarterly Journal of Economics* 111, 1135–51.

Scheinkman, Jose A., and Wei Xiong, 2003, Overconfidence and speculative bubbles, *Journal of Political Economy* 111, 1183–1219.

Sharpe, William, 1964, Capital asset prices: A theory of market equilibrium under conditions of risk, *Journal of Finance* XIX, 425–442.

Shleifer, Andrei, and Robert W Vishny, 1997, The limits of arbitrage, *Journal of Finance* 52, 35–55.

Summers, Lawrence H, 1986, Does the stock market rationally reflect fundamental values?, *The Journal of Finance* 41, 591–601.

Wurgler, Jeffrey, and Ekaterina Zhuravskaya, 2002, Does arbitrage flatten demand curves for stocks?, *The Journal of Business* 75, 583–608.

Yu, Jialin, 2010, Disagreement and portfolio return predictability, Working papers, forthcoming *Journal of Financial Economics.*

Figures and Tables

Figure 1. Time-series of Aggregate Disagreement

Note: Sample Period: 12/1981-12/2014. Sample: CRSP stock file excluding penny stocks (price < \$5) and microcaps (stocks in bottom 2 deciles of the monthly size distribution using NYSE breakpoints). Each month, we calculate for each stock the standard deviation of analyst forecasts on the stock' long run growth of EPS, which is our measure of stock-level disagreement. We also estimate for each stock i $\hat{\beta}_{i,t-1}$, the stock market beta of stock i at the end of the previous month. These betas are estimated with a market model using daily returns over the past calendar year and 5 lags of the market returns. Aggregate Disagreement is the monthly $\hat{\beta}_{i,t-1}$-weighted average of stock-level disagreement.

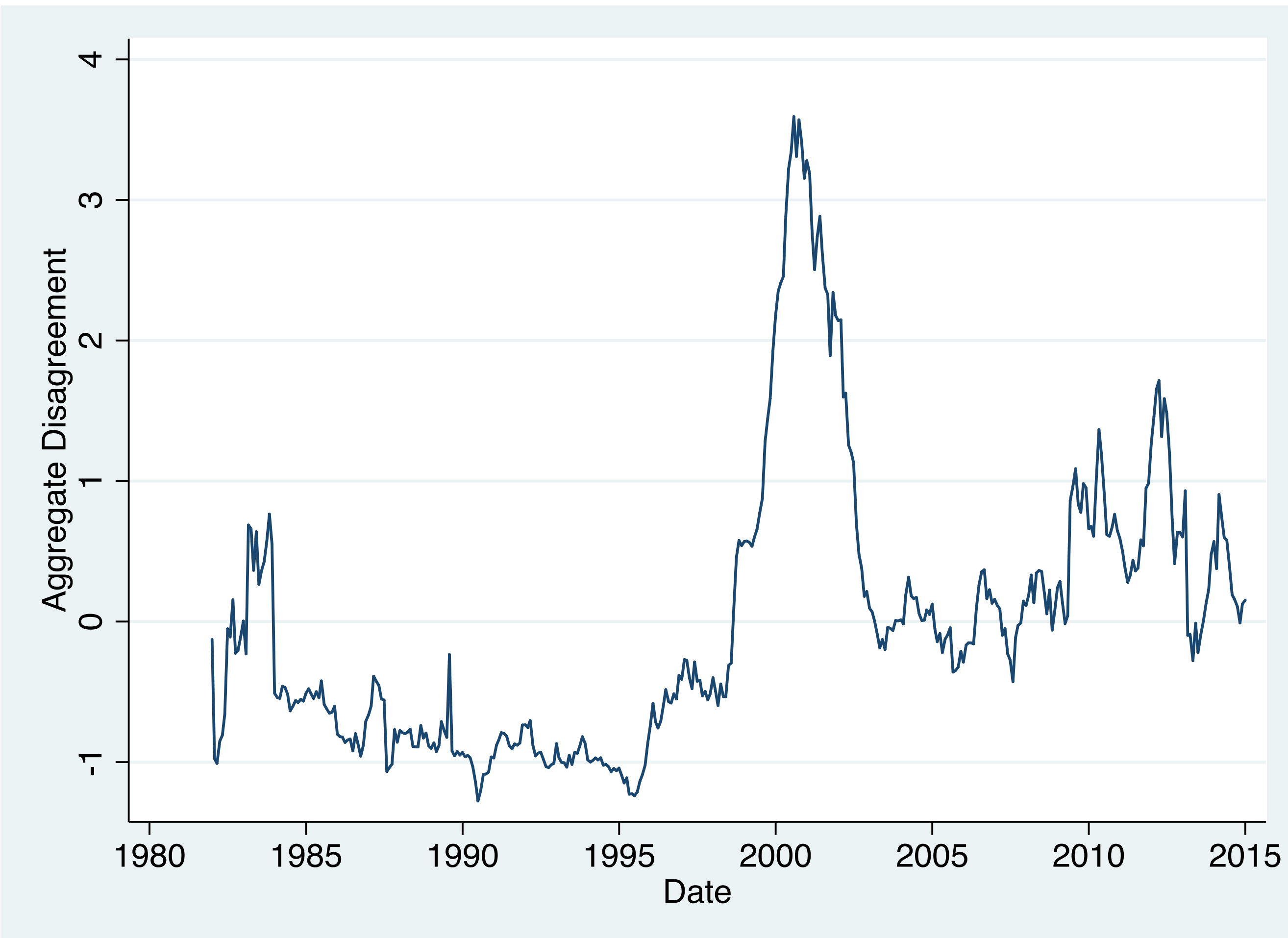
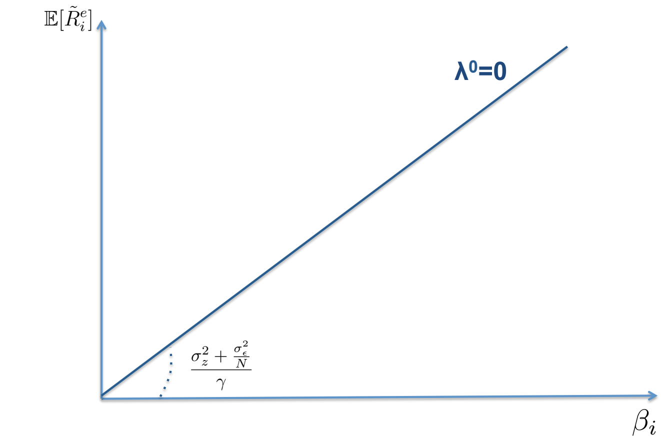
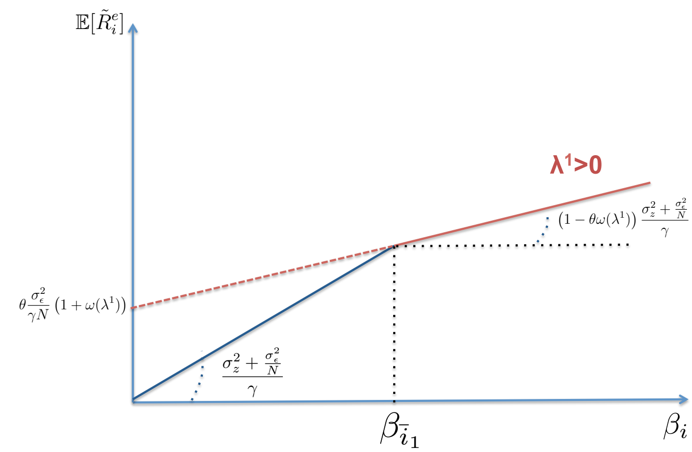


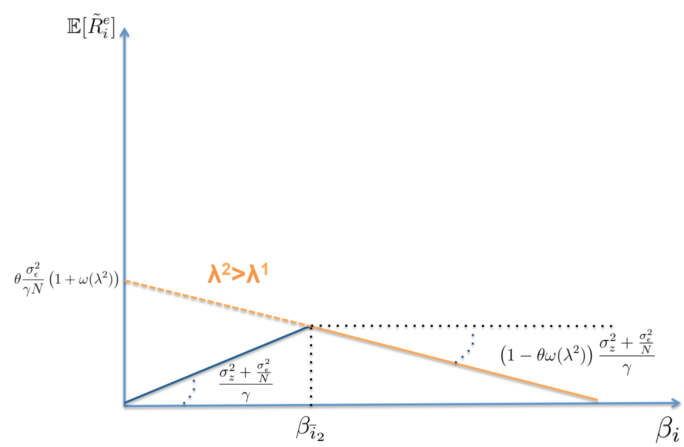
Figure 2. Security Market Line for Different Levels of Aggregate Disagreement



(a) No Aggregate Disagreement



(b) Medium Aggregate Disagreement



(c) High Aggregate Disagreement

Figure 3. Timeline of the dynamic model of Section I.G

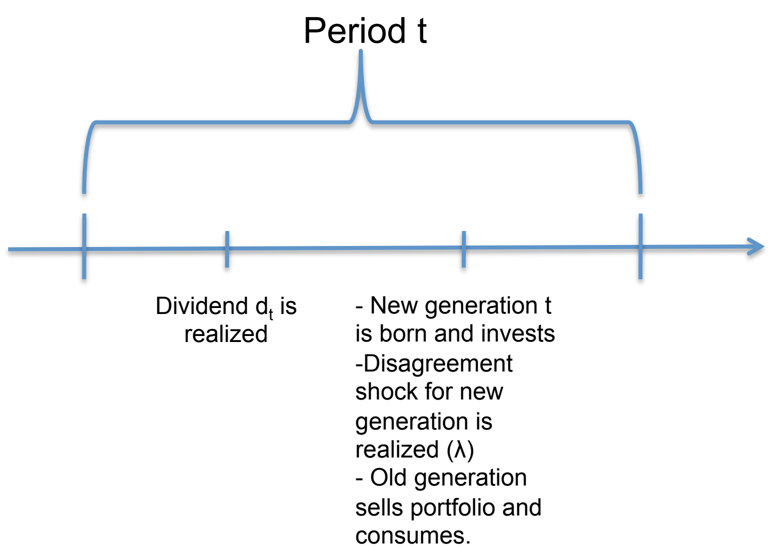
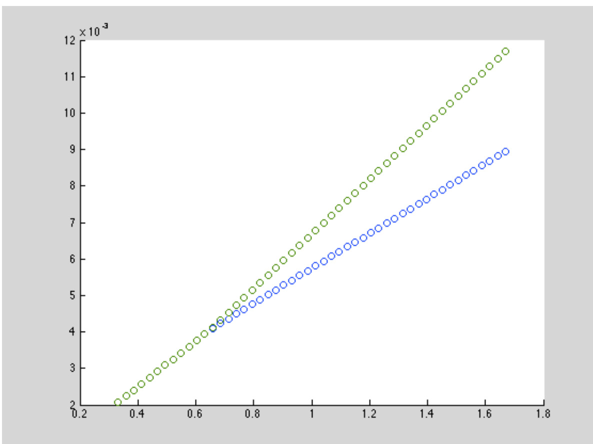
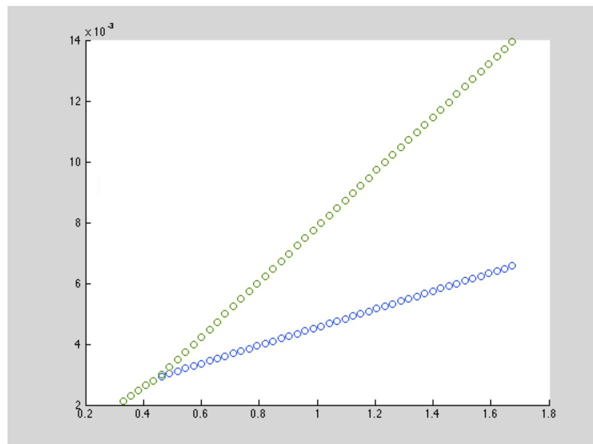


Figure 4. Calibration of the dynamic model

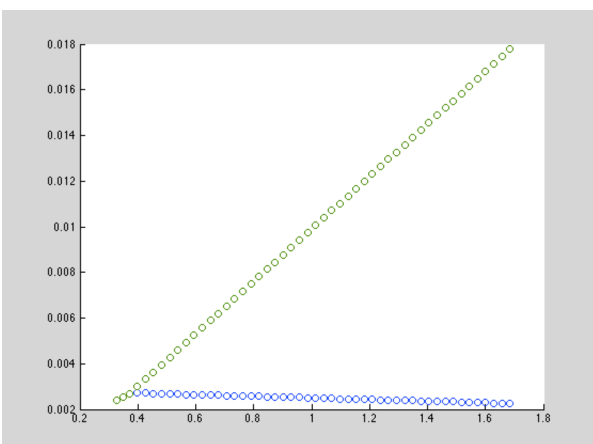
Note: This figure plots the Security Market Line in the high aggregate disagreement state (blue dots) and in the low aggregate disagreement state (green dots) obtained from the simulation of the dynamic model. Across simulations, we use the following parameters: $\theta = .5$, $N = 50$ and $\rho = .95$. Each of the four panels set a value of λ (.008 in panel (a), .013 in panel (b), .022 in panel (c) and .05 in panel (d)) and then find the values for σ_z^2, σ_ϵ^2 and γ that match the empirical average idiosyncratic variance of monthly stock returns, the empirical variance of the monthly market return and the empirical average return on the market portfolio over the sample period.



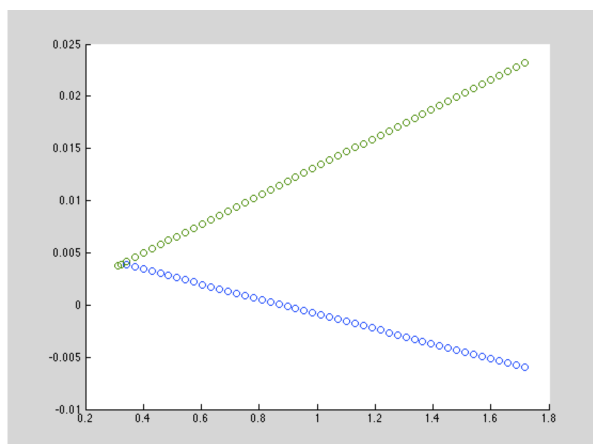
(a) $\lambda = .008$, $\sigma_z^2 = .0014$, $\sigma_\epsilon^2 = .0305$, $\gamma = .32$



(b) $\lambda = .013$, $\sigma_z^2 = .0013$, $\sigma_\epsilon^2 = .0305$, $\gamma = .31$



(c) $\lambda = .022$, $\sigma_z^2 = .0011$, $\sigma_\epsilon^2 = .0305$, $\gamma = .27$



(d) $\lambda = .05$, $\sigma_z^2 = .0305$, $\sigma_\epsilon^2 = .0005$, $\gamma = .16$

Figure 5. Stock Level Disagreement and β

Note: Sample Period: 12/1981-12/2014. Sample: CRSP stock file excluding penny stocks (price $<$ \$5) and microcaps (stocks in bottom 2 deciles of the monthly size distribution using NYSE breakpoints). At the beginning of each calendar month, stocks are ranked in ascending order on the basis of their estimated beta at the end of the previous month. Pre-formation betas are estimated with a market model using daily returns over the past calendar year and 5 lags of the market returns. The ranked stocks are assigned to one of twenty portfolios based on NYSE breakpoints. The graph plots the value-weighted average stock-level disagreement of stocks in these portfolios of the 20 β-sorted portfolios for months in the bottom quartile of aggregate disagreement (in blue) and months in the top quartile of aggregate disagreement (in red). Stock-level disagreement is the standard deviation of analyst forecasts on stocks' long run growth of EPS. Aggregate disagreement is the monthly β-weighted average of this stock level disagreement measure.

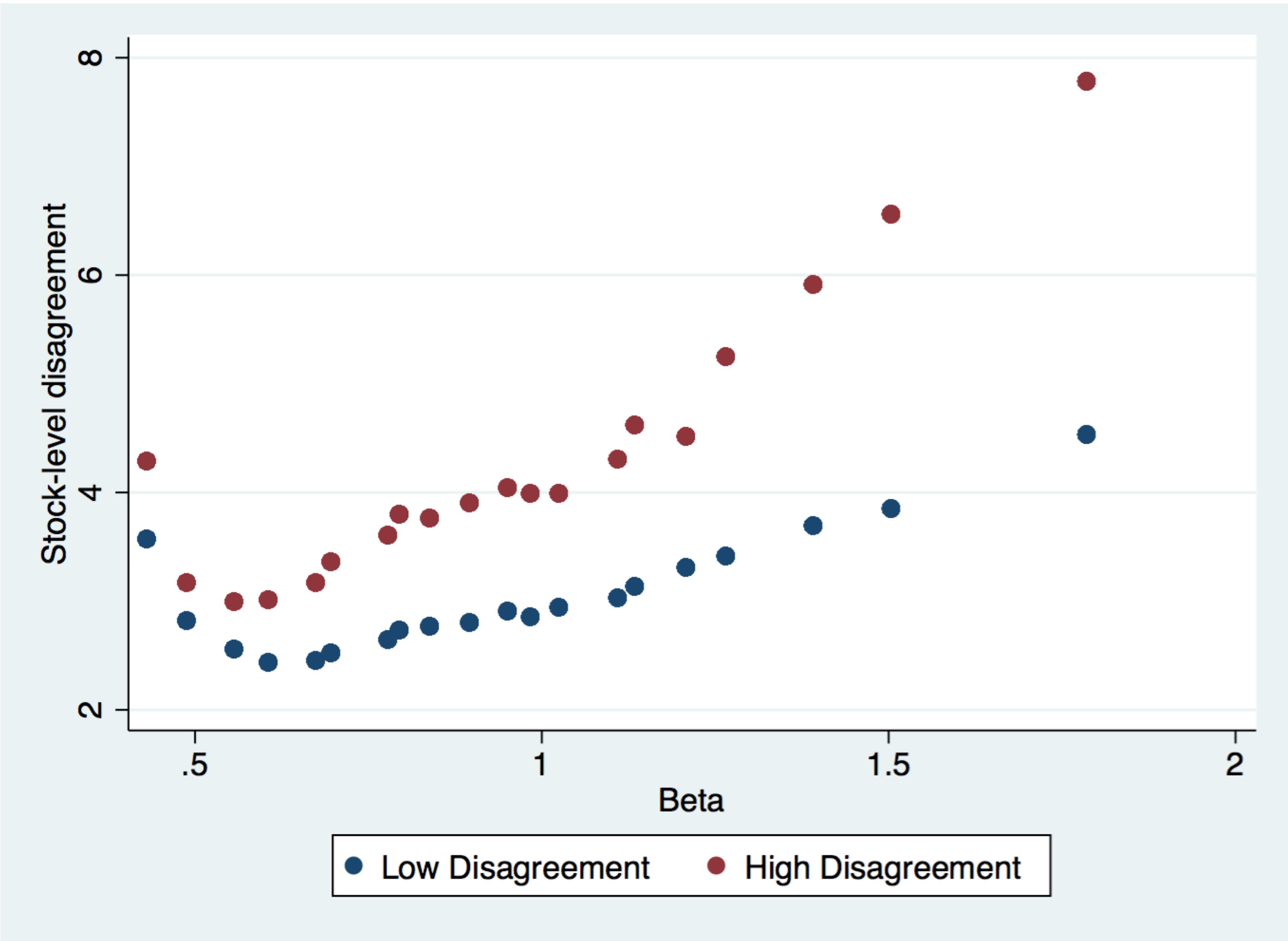
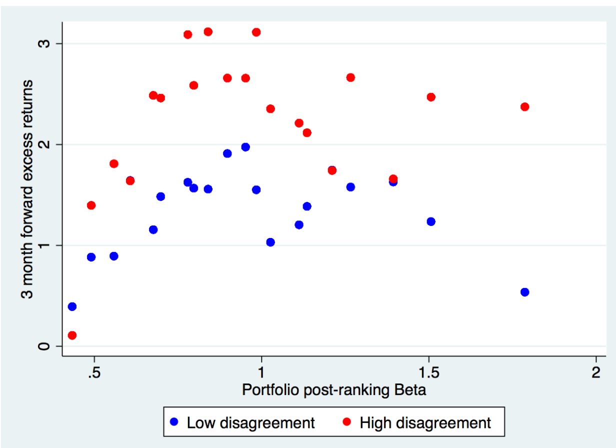
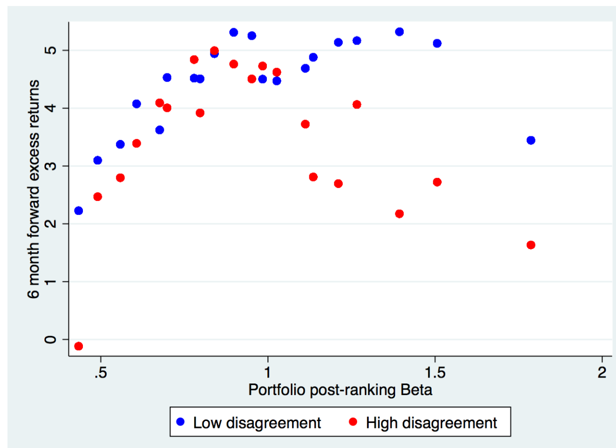


Figure 6. Excess Returns, β and Aggregate Disagreement

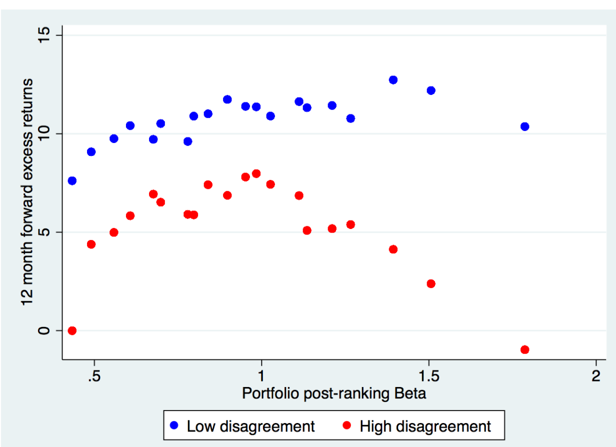
Note: Sample Period: 12/1981-12/2014. Sample: CRSP stock file excluding penny stocks (price < \$5) and microcaps (stocks in bottom 2 deciles of the monthly size distribution using NYSE breakpoints). At the beginning of each calendar month, stocks are ranked in ascending order on the basis of their estimated beta at the end of the previous month. Pre-formation betas are estimated with a market model using daily returns over the past calendar year and 5 lags of the market returns. The ranked stocks are assigned to one of twenty value-weighted portfolios based on NYSE breakpoints. The graph plots the average excess returns over the next 3 months (panel (a)) 6 months (panel (b)), 12 months (panel (c)) and 18 months (panel (d)) of the 20 β-sorted portfolios for months in the bottom quartile of aggregate disagreement (in blue) and months in the top quartile of aggregate disagreement (in red). Aggregate disagreement. is the monthly β-weighted average of stock level disagreement measured as the standard deviation of analyst forecasts on stocks' long run growth of EPS.



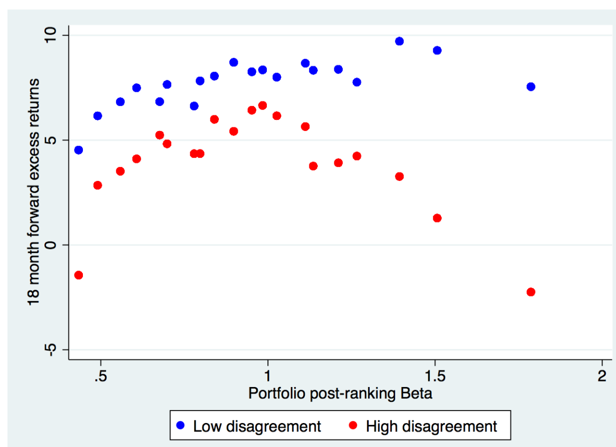
(a) 3 month value-weighted return



(b) 6 months value-weighted return



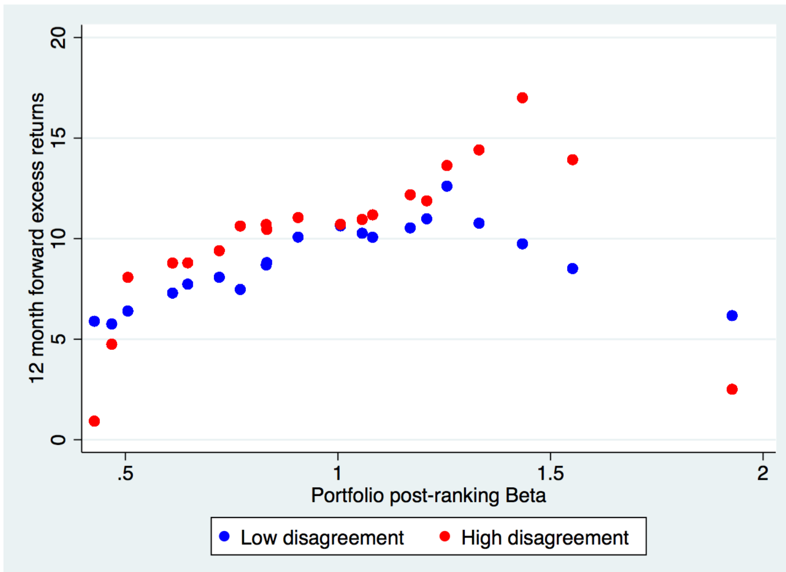
(c) 12 months value-weighted return



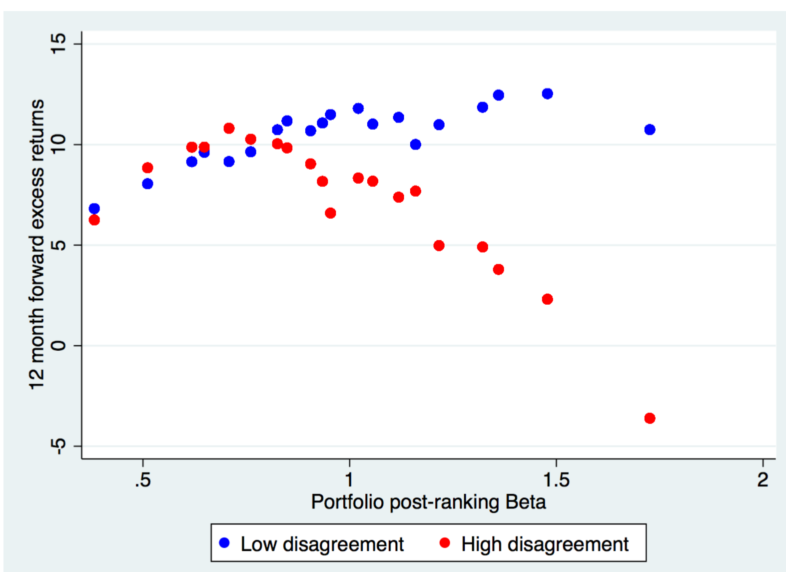
(d) 18 months value-weighted return

Figure 7. Excess Returns, β and Aggregate Disagreement: Speculative vs. non-speculative stocks

Note: Sample Period: 12/1981-12/2014. Sample: CRSP stock file excluding penny stocks (price < \$5) and microcaps (stocks in bottom 2 deciles of the monthly size distribution using NYSE breakpoints). At the beginning of each calendar month, stocks are ranked in ascending order on the basis of the estimated ratio of beta to idiosyncratic variance ($\frac{\beta}{\sigma^2}$) at the end of the previous month. Pre-formation betas and idiosyncratic variance are estimated with a market model using daily returns over the past calendar year and 5 lags of the market returns. The ranked stocks are assigned to two groups: speculative ($\frac{\hat{\beta}_i}{\hat{\sigma}_i^2} >$ NYSE median $\frac{\hat{\beta}}{\hat{\sigma}^2}$ in month t) and non-speculative stocks. Within each of these two groups, stocks are then ranked in ascending order of their estimated beta at the end of the previous month and are assigned to one of twenty value-weighted beta-sorted portfolios based on NYSE breakpoints. The graph plots the average excess returns over the next 12 months for the 20 β-sorted portfolios for months in the bottom quartile of aggregate disagreement (in blue) and months in the top quartile of aggregate disagreement (in red). Aggregate disagreement. is the monthly β-weighted average of stock level disagreement measured as the standard deviation of analyst forecasts on stocks' long run growth of EPS. Panel (a) plots these excess returns for non-speculative stocks, panel (b) for the speculative stocks.



(a) 12 months value-weighted return for **non-speculative stocks**



(b) 12 months value-weighted return for **speculative stocks**

Figure 8. Disagreement and Slope of the Security Market Line: Speculative vs. non speculative stocks

Note: Sample Period: 12/1981-12/2014. Sample: CRSP stock file excluding penny stocks (price < \$5) and microcaps (stocks in bottom 2 deciles of the monthly size distribution using NYSE breakpoints). At the beginning of each calendar month, stocks are ranked in ascending order on the basis of the estimated ratio of beta to idiosyncratic variance ($\frac{\beta}{\sigma^2}$) at the end of the previous month. Pre-formation betas and idiosyncratic variance are estimated with a market model using daily returns over the past calendar year and 5 lags of the market returns. The ranked stocks are assigned to two groups: speculative ($\frac{\hat{\beta}_i}{\hat{\sigma}_i^2} >$ NYSE q$^{\text{th}}$ percentile of $\frac{\hat{\beta}}{\hat{\sigma}^2}$ in month t) and non-speculative stocks. Within each of these two groups, stocks are ranked in ascending order of their estimated beta at the end of the previous month and assigned to one of 20 value-weighted beta-sorted portfolios using NYSE breakpoints. We compute the full sample beta of these 40 portfolios (20 beta-sorted portfolios for speculative stocks and for non-speculative stocks) using the same market model. $\beta_{P,s}$ is the resulting full sample beta, where $P = 1, \ldots, 20$ and s $\in$ {speculative, non speculative}. We then estimate the following cross-sectional regressions, where P is one of the 20 β-sorted portfolios, $s \in$ {speculative, non speculative} and t is a month :

$$r^{(12)}_{P,s,t} = \iota_{s,t} + \chi_{s,t} \times \beta_{P,s} + \psi_{s,t} \times \ln\left(\sigma_{P,s,t-1}\right) + \epsilon_{P,s,t},$$

where $\sigma_{P,s,t-1}$ is the median idiosyncratic volatility of stocks in portfolio (P,s) estimated in month $t-1$ and $r^{(12)}_{P,s,t}$ is the 12-months excess return of portfolio (P,s). We estimate the following time-series regression with Newey-West adjusted standard errors allowing for 11 lags:

$$\chi_{s,t} = c_s + \xi_s \cdot \text{Agg. Disp.}_{t-1} + \delta^m_s \cdot R^{(12)}_{m,t} + \delta^{HML}_s \cdot HML^{(12)}_t + \delta^{SMB}_s \cdot SMB^{(12)}_t + \delta^{UMD}_s \cdot UMD^{(12)}_t + \zeta_{t,s}$$

Agg. Disp.$_{t-1}$ is the monthly β-weighted average of stock level disagreement measured as the standard deviation of analyst forecasts on stocks' long run growth of EPS, $R^{(12)}_{m,t}$ (resp. $HML^{(12)}_t$, $SMB^{(12)}_t$ and $UMD^{(12)}_t$) are the 12-months excess return from t to $t+11$ on the market (resp. HML, SMB and UMD). The thresholds used to define speculative stocks are q=30, 35, 40, 45, 50, 55, 60, 65 and 70th percentile of the distribution of $\frac{\beta}{\sigma^2}$. The figure plots the estimated $\xi_{\text{spec.}}$ (left panel) and $\xi_{\text{non spec.}}$ (right panel) for each of these thresholds and their 95% confidence interval.

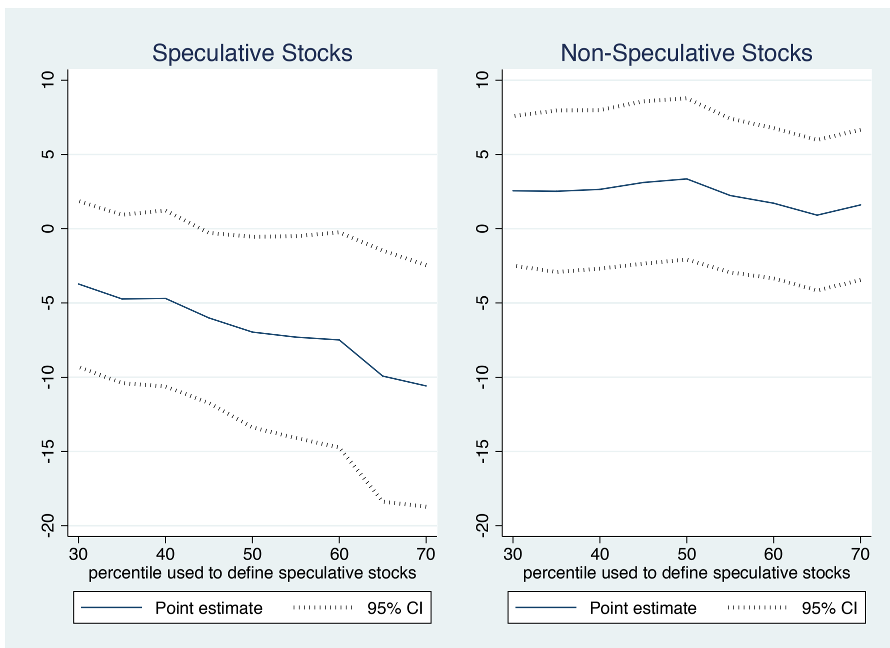


Table I. Summary Statistics for 20 β-sorted porftolios

Note: Sample Period: 12/1981-12/2014. Sample: CRSP stock file excluding penny stocks (price < \$5) and microcaps (stocks in bottom 2 deciles of the monthly size distribution using NYSE breakpoints). At the beginning of each calendar month, stocks are ranked in ascending order on the basis of their estimated beta at the end of the previous month. Pre-formation betas are estimated with a market model using daily returns over the past calendar year and 5 lags of the market returns. The ranked stocks are assigned to one of 20 value-weighted portfolios based on NYSE breakpoints. The table reports the full sample β of each of these 20 portfolios, computed using a similar risk model. Median Vol. is the median pre-ranking volatility of stocks in the portfolio. $R_t^{(1)}$ is the return of the portfolio from t to $t+1$, and $R_t^{(12)}$ from t to t+11. Stock Disp. is the average stock-level disagreement for stocks in each portfolio, where stock-level disagreement is defined as the standard-deviation of analysts' forecasts for the long run growth of the stock's EPS. % Mkt. Cap. is the average ratio of market capitalization of stocks in the portfolio divided by the total market capitalization of stocks in the sample. N stocks is the number of stocks on average in each portfolio.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
β	.43	.49	.55	.6	.67	.69	.77	.79	.83	.89	.95	.98	1.02	1.11	1.13	1.21	1.26	1.39	1.5	1.78
Median Vol.	1.56	1.14	1.1	1.11	1.13	1.14	1.15	1.17	1.22	1.23	1.28	1.33	1.34	1.43	1.5	1.58	1.7	1.88	2.26	3.16
$R_{i,t}^{(1)}$	.12	.39	.63	.61	.61	.73	.87	.61	.79	.6	.67	.64	.53	.57	.73	.54	.58	.44	-.17	-.72
$R_{i,t}^{(12)}$	3.21	5.58	7.98	7.74	8.19	7.94	8.1	8.15	8.92	6.88	8.28	8.51	7.46	7.82	8.08	7.86	8.23	6.62	-.55	-11.69
Stock Disp.	2.97	2.76	2.81	2.78	2.89	3.02	3.24	3.35	3.44	3.38	3.54	3.56	3.6	3.72	3.94	3.93	4.29	4.71	5.03	6.91
% Mkt. Cap.	2.77	3.58	4.15	4.73	4.86	5	5.32	5.44	5.49	5.84	5.6	5.58	5.62	5.63	5.36	5.45	5.31	5.09	5.21	7.48
N stocks	171	143	145	147	149	154	157	160	161	169	164	166	167	167	167	168	175	186	210	365

Table II. Summary Statistics for Time-Series Variables

Note: To construct Agg. Disp, we start from the CRSP stock file excluding penny stocks (price < \$5) and microcaps (stocks in bottom 2 deciles of the monthly size distribution using NYSE breakpoints). Each month, we calculate for each stock the standard deviation of analyst forecasts on the stock' long run growth of EPS, which is our measure of stock-level disagreement. We also estimate for each stock i $\hat{\beta}_{i,t-1}$, the stock market beta of stock i at the end of the previous month. These betas are estimated with a market model using daily returns over the past calendar year and 5 lags of the market returns. Agg. Disp. is the monthly $\hat{\beta}_{i,t-1}$-weighted average of stock-level disagreement. Agg. Disp. (compressed) uses $.5\hat{\beta}_i + .5$ as weight for stock i instead of $\hat{\beta}_i$. Agg. Disp ($\beta\times$ Value weight) uses $\hat{\beta}_i \times$ (Market Value)$_i$ as weight for stock i instead of $\hat{\beta}_i$. Top-down Disp. is the monthly standard deviation of analyst forecasts of annual S&P 500 earnings, scaled by the average forecast on S&P 500 earnings. SPF Disp. is the first principal component of the standard deviation of forecasts on GDP, IP, Corporate Profit and Unemployment rate in the Survey of Professional Forecasters (SPF) and is taken from Li and Li (2014). These measures of aggregate disagreement are standardized to have an in-sample mean of 0 and a standard deviation of 1. D/P is the aggregate dividend-to-price ratio from Robert Shiller's website. $R_{m,t}^{(12)}$, $\text{SMB}_t^{(12)}$, $\text{HML}_t^{(12)}$, $\text{UMD}_t^{(12)}$ are the 12 months monthly returns on the market, SMB, HML and UMD portfolios from Ken French's website and are expressed in %. TED is the TED spread and Inflation is the yearly inflation rate. The sample period goes from 12/1981 to 12/2014, and the summary statistics are displayed for months where both Agg. Disp and $R_{m,t}^{(12)}$ are non-missing.

	Mean	Std. Dev.	p10	p25	Median	p75	p90	Obs.
Agg. Disp.	-0.00	1.00	-0.96	-0.79	-0.21	0.42	1.28	385
Agg. Disp. (compressed)	0.00	1.00	-1.04	-0.79	-0.16	0.57	1.46	385
Agg. Disp. ($\beta\times$ Value weight)	0.00	1.00	-1.00	-0.84	-0.33	0.67	1.47	385
Top-down Disp.	0.00	1.00	-0.43	-0.37	-0.27	-0.13	0.64	353
SPF Disp.	0.00	1.00	-1.00	-0.78	-0.14	0.44	1.06	361
$\text{R}_{m,t}^{(12)}$	8.82	16.95	-15.47	0.41	10.53	19.73	27.93	385
$\text{SMB}_t^{(12)}$	1.23	9.91	-9.46	-5.32	0.18	6.93	14.11	385
$\text{HML}_t^{(12)}$	3.96	13.32	-10.45	-4.52	3.11	10.77	17.50	385
$\text{UMD}_t^{(12)}$	7.15	16.56	-9.29	-0.91	7.58	16.92	25.53	385
D/P	2.58	1.09	1.41	1.76	2.17	3.24	4.21	385
Inflation	0.03	0.01	0.01	0.02	0.03	0.03	0.04	385
TED spread	0.72	0.57	0.20	0.32	0.56	0.91	1.34	385

Table III. Disagreement and Concavity of the Security Market Line

Note: Sample Period: 12/1981-12/2014. Sample: CRSP stock file excluding penny stocks (price < \$5) and microcaps (stocks in bottom 2 deciles of the monthly size distribution using NYSE breakpoints). At the beginning of each calendar month, stocks are ranked in ascending order on the basis of their estimated beta at the end of the previous month. Pre-formation betas are estimated with a market model using daily returns over the past calendar year and 5 lags of the market returns. The ranked stocks are assigned to one of 20 value-weighted (panel A) or equal-weighted (panel B) portfolios based on NYSE breakpoints. We compute the full sample beta of these 20-beta sorted portfolios using the same market model. We then estimate every month the cross-sectional regression:

$$r_{P,t}^{(12)} = \kappa_t + \pi_t \times \beta_P + \phi_t \times (\beta_P)^2 + \epsilon_{P,t}, \quad \text{where} \quad P = 1, ..., 20$$

and $r_{P,t}^{(12)}$ is the 12-months excess return of the P^{th} beta-sorted portfolio and β_P is the full sample post-ranking beta of the P^{th} beta-sorted portfolio. We then estimate second-stage regressions in the time-series using OLS and Newey-West adjusted standard errors allowing for 11 lags:

$$\begin{cases} \phi_t = & c_1 + \psi_1 \cdot \text{Agg. Disp.}_{t-1} + \delta_1^m \cdot R_{m,t}^{(12)} + \delta_1^{HML} \cdot HML_t^{(12)} + \delta_1^{SMB} \cdot SMB_t^{(12)} + \delta_1^{UMD} \cdot UMD_t^{(12)} + \sum_{x \in X} \delta_1^x \cdot x_{t-1} + \zeta_t \\ \pi_t = & c_2 + \psi_2 \cdot \text{Agg. Disp.}_{t-1} + \delta_2^m \cdot R_{m,t}^{(12)} + \delta_2^{HML} \cdot HML_t^{(12)} + \delta_2^{SMB} \cdot SMB_t^{(12)} + \delta_2^{UMD} \cdot UMD_t^{(12)} + \sum_{x \in X} \delta_2^x \cdot x_{t-1} + \omega_t \\ \kappa_t = & c_3 + \psi_3 \cdot \text{Agg. Disp.}_{t-1} + \delta_3^m \cdot R_{m,t}^{(12)} + \delta_3^{HML} \cdot HML_t^{(12)} + \delta_3^{SMB} \cdot SMB_t^{(12)} + \delta_3^{UMD} \cdot UMD_t^{(12)} + \sum_{x \in X} \delta_3^x x_{t-1} + \nu_t \end{cases}$$

Column (1) and (5) controls for Agg. Disp.$_{t-1}$, the monthly β-weighted average of stock-level disagreement, which is measured as the standard deviation of analyst forecasts on stocks' long run growth of EPS. Column (2) and (6) add controls for the 12-months excess return from t to $t+11$ of the market ($R_{m,t}^{(12)}$), HML ($HML_t^{(12)}$), SMB ($SMB_t^{(12)}$), and UMD ($UMD_t^{(12)}$). Column (3) and (7) add controls for the aggregate Dividend/Price ratio in $t-1$ and the past-12 months inflation rate in t_1. Column (4) and (8) additionally control for the TED spread in month $t-1$. T-statistics are in parenthesis. *, **, and *** means statistically different from zero at 10, 5 and 1% level of significance.

Table III (Continued):

Dep. Var:	ϕ_t				π_t				κ_t			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Panel A: Value-Weighted Portfolios												
Agg. Disp.$_{t-1}$	-6.4**	-5.5*	-10***	-10***	7.4*	11**	16***	15***	-4.5**	-3.5*	-3.8	-3.2
	(-2.1)	(-1.9)	(-3.1)	(-3.1)	(1.7)	(2.1)	(2.7)	(2.6)	(-2.3)	(-1.7)	(-1.5)	(-1.3)
$R_{m,t}^{(12)}$		-.15	-.22*	-.22*		.92***	1.1***	1.1***		.26**	.21	.2
		(-1.2)	(-1.9)	(-1.8)		(3.6)	(4.1)	(4)		(2)	(1.6)	(1.5)
$HML_t^{(12)}$		-.54**	-.4**	-.4**		.53	.37	.41		.24	.24	.21
		(-2.4)	(-2.1)	(-2.1)		(1.3)	(.97)	(1.1)		(1.3)	(1.3)	(1.1)
$SMB_t^{(12)}$		.28	.48**	.48**		-.11	-.38	-.39		-.17	-.13	-.12
		(1)	(2)	(2)		(-.21)	(-.73)	(-.75)		(-.66)	(-.46)	(-.46)
$UMD_t^{(12)}$		-.0039	.057	.058		.0045	-.089	-.071		-.0071	.017	.0033
		(-.033)	(.56)	(.57)		(.02)	(-.39)	(-.32)		(-.061)	(.13)	(.028)
D/P_{t-1}			-2.3	-2.4			.86	-1			1.7	3.1
			(-.97)	(-.85)			(.17)	(-.19)			(.6)	(1)
Inflation$_{t-1}$			-5.9***	-6***			9.3**	8.2*			-2.4	-1.7
			(-3.6)	(-3.7)			(2.2)	(1.8)			(-1.1)	(-.63)
Ted Spread$_{t-1}$				.15				3.3				-2.4
				(.085)				(.9)				(-1.2)
Constant	-6.3**	-3.1	-3.7	-3.7	14***	4	4.4	4	2.4	-.58	-.36	-.073
	(-2.6)	(-1.2)	(-1.5)	(-1.5)	(3)	(.75)	(.84)	(.72)	(1)	(-.22)	(-.13)	(-.024)
Panel B: Equal-Weighted Portfolios												
Agg. Disp.$_{t-1}$	-6.8**	-4.9***	-6.5***	-6.8***	9.8**	10**	10**	9.6**	-3.4*	-3.1	-.67	.86
	(-2.6)	(-2.6)	(-3.1)	(-3.2)	(2.3)	(2.5)	(2.3)	(2.1)	(-2)	(-1.5)	(-.32)	(.39)
$R_{m,t}^{(12)}$		-.22**	-.3***	-.32***		1.1***	1.2***	1.3***		.16	.11	.13
		(-2.5)	(-3.5)	(-3.3)		(6.4)	(6.5)	(5.6)		(1.6)	(1.1)	(1.1)
$HML_t^{(12)}$		-.69***	-.65***	-.67***		.87***	.89***	.97***		.22	.13	.064
		(-4.4)	(-5)	(-5.3)		(2.7)	(3.1)	(3.2)		(1.4)	(.88)	(.4)
$SMB_t^{(12)}$		.0073	.12	.16		.69**	.58*	.47		-.11	-.16	-.11
		(.05)	(.86)	(1.1)		(2.5)	(1.8)	(1.4)		(-.69)	(-.88)	(-.62)
$UMD_t^{(12)}$		-.077	-.03	-.048		.059	-.01	.08		.028	.035	-.036
		(-1.1)	(-.55)	(-.75)		(.39)	(-.068)	(.53)		(.28)	(.34)	(-.37)
D/P_{t-1}			1.3	1.7			-6.1	-8.8*			4.8**	7***
			(.61)	(.65)			(-1.4)	(-1.7)			(2.4)	(3.1)
Inflation$_{t-1}$			-4.7***	-5***			7.1**	7.3**			-.9	-.63
			(-3.6)	(-3.8)			(2.1)	(2)			(-.48)	(-.3)
Ted Spread$_{t-1}$				-.93				5.1				-4**
				(-.57)				(1.3)				(-2)
Constant	-9.7***	-4.5**	-4.4**	-4.3**	22***	7.3	6.4	5.6	-.92	-3.3	-2.5	-1.6
	(-4.8)	(-2.3)	(-2.3)	(-2)	(5)	(1.6)	(1.4)	(1.1)	(-.46)	(-1.4)	(-1)	(-.6)
N	385	385	385	358	385	385	385	358	385	385	385	358

Table IV. Summary Statistics for 20 β-sorted porftolios: speculative and non-speculative stocks

Note: Sample Period: 12/1981-12/2014. Sample: CRSP stock file excluding penny stocks (price < \$5) and microcaps (stocks in bottom 2 deciles of the monthly size distribution using NYSE breakpoints). At the beginning of each calendar month, stocks are ranked in ascending order on the basis of the estimated ratio of beta to idiosyncratic variance ($\frac{\beta}{\sigma^2}$) at the end of the previous month. Pre-formation betas and idiosyncratic variance are estimated with a market model using daily returns over the past calendar year and 5 lags of the market returns. The ranked stocks are assigned to two groups: speculative ($\frac{\hat{\beta}_i}{\hat{\sigma}_i^2}$ > NYSE median $\frac{\hat{\beta}}{\hat{\sigma}^2}$ in month t) and non-speculative stocks. Within each of these two groups, stocks are ranked in ascending order of their estimated beta at the end of the previous month and are assigned to one of twenty value-weighted beta-sorted portfolios based on NYSE breakpoints. The table reports statistics for these portfolios: Panel A for non-speculative stocks; panel B for speculative stocks. β is the post-ranking full sample β, and is computed using a similar risk model to the one used to compute the pre-ranking β. Median Vol. is the median pre-ranking volatility of stocks in the portfolio. $\frac{\beta}{\sigma^2}$ is the ratio of β to the square of Median Vol.$R_t^{(1)}$ is the return of the portfolio from t to $t+1$, and $R_t^{(12)}$ from t to t+11. Stock Disp. is the average stock-level disagreement for stocks in each portfolio, where stock-level disagreement is defined as the standard-deviation of analysts' forecasts for the long run growth of the stock's EPS. % Mkt. Cap. is the average ratio of market capitalization of stocks in the portfolio divided by the total market capitalization of stocks in the sample. "N stocks" is the number of stocks on average in each portfolio.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
Panel A: Non-speculative stocks																				
β	.42	.46	.5	.61	.64	.72	.77	.83	.83	.9	1	1.05	1.08	1.17	1.2	1.25	1.33	1.43	1.55	1.92
Median Vol.	1.85	1.35	1.29	1.4	1.38	1.46	1.54	1.62	1.71	1.8	1.86	1.93	2.01	2.15	2.26	2.33	2.43	2.7	2.92	4.09
$\frac{\beta}{\sigma^2}$	.12	.25	.3	.31	.33	.33	.32	.31	.28	.27	.29	.28	.26	.25	.23	.22	.22	.19	.18	.11
$R_{i,t}^{(1)}$	.05	.21	.34	.35	.57	.46	.79	.59	.66	.82	.82	.61	.61	.93	.43	.9	.75	.51	.76	-.02
$R_{i,t}^{(12)}$	1.89	3.01	3.93	5.7	5.88	7.57	6.77	9.75	9.07	9.03	8.83	8.34	7.9	10.35	10.15	12.73	10.8	11.21	11.58	-2.78
Stock Disp.	3.12	2.85	3.06	3.56	3.34	3.36	3.54	3.8	3.87	4.16	4.74	4.73	4.83	4.95	5.17	5.66	5.96	6.06	6.24	7.99
% Mkt. Cap.	4.31	3.95	4.76	5.15	5.56	5.62	5.9	5.65	5.47	5.57	5.19	5	5.06	4.74	4.81	4.61	4.58	4.55	5.14	8.7
N stocks	96	73	71	75	81	84	87	89	93	96	97	98	100	101	108	111	118	126	151	339
Panel B: Speculative stocks																				
β	.38	.51	.61	.64	.7	.76	.82	.84	.9	.93	.95	1.02	1.05	1.11	1.15	1.21	1.32	1.36	1.47	1.72
Median Vol.	.78	.86	.93	.98	1.01	1.02	1.05	1.09	1.13	1.17	1.17	1.24	1.27	1.34	1.39	1.47	1.56	1.75	1.94	2.6
$\frac{\beta}{\sigma^2}$	.62	.68	.7	.66	.68	.71	.73	.7	.69	.68	.68	.65	.65	.62	.59	.56	.54	.44	.39	.25
$R_{i,t}^{(1)}$	.62	.81	.54	.78	.76	.66	.47	.61	.63	.55	.83	.48	.46	.73	.43	.65	.49	-.24	0	-.91
$R_{i,t}^{(12)}$	8.51	9.47	8.71	8.24	7.38	7.1	6.49	7.43	7.96	7.58	7.41	7.9	7.31	7.06	7.55	6.54	4.57	-1.52	-2.38	-11.62
Stock Disp.	1.95	2.17	2.33	2.66	2.76	2.9	2.98	3.07	3.16	3.24	3.23	3.17	3.31	3.49	3.5	3.75	4.21	4.55	5.03	6.57
% Mkt. Cap.	4.16	4.48	4.98	5.05	5.19	5.37	5.88	5.84	5.73	5.5	5.35	5.24	5.3	5.17	4.98	4.68	5.01	5.15	4.81	6.64
N stocks	57	58	58	60	63	64	65	67	68	67	68	69	67	66	65	66	65	66	68	90

Table V. Disagreement and Slope of the Security Market Line: speculative vs. non speculative stocks

Note: Sample Period: 12/1981-12/2014. Sample: CRSP stock file excluding penny stocks (price < \$5) and microcaps (stocks in bottom 2 deciles of the monthly size distribution using NYSE breakpoints). At the beginning of each calendar month, stocks are ranked in ascending order on the basis of the ratio of their estimated beta at the end of the previous month and their estimated idiosyncratic variance ($\frac{\beta}{\sigma^2}$). Pre-formation betas and idiosyncratic variance are estimated with a market model using daily returns over the past calendar year and 5 lags of the market returns. The ranked stocks are assigned to two groups: speculative ($\frac{\hat{\beta}_i}{\hat{\sigma}_i^2}$ > NYSE median $\frac{\hat{\beta}}{\hat{\sigma}^2}$ in month t) and non-speculative stocks. Within each of these two groups, stocks are ranked in ascending order of their estimated beta at the end of the previous month and are assigned to one of 20 beta-sorted portfolios using NYSE breakpoints. We compute the full sample beta of these 2×20 value-weighted portfolios (20 beta-sorted portfolios for speculative stocks; 20 beta-sorted portfolios for non-speculative stocks) using the same market model. $\beta_{P,s}$ is the resulting full sample beta, where $P = 1, \ldots, 20$ and s $\in$ {speculative, non speculative}. We estimate every month the following cross-sectional regressions, where P is one of the 20 β-sorted portfolios, $s \in$ {speculative, non speculative} and t is a month :

$$r^{(12)}_{P,s,t} = \iota_{s,t} + \chi_{s,t} \times \beta_{P,s} + \varrho_{s,t} \times \ln(\sigma_{P,s,t-1}) + \epsilon_{P,s,t},$$

where $\sigma_{P,s,t-1}$ is the median idiosyncratic volatility of stocks in portfolio (P, s) estimated at the end of month $t-1$ and $r^{(12)}_{P,s,t}$ is the value-weighted 12-months excess return of portfolio (P, s). We then estimate second-stage regressions in the time-series using OLS and Newey-West adjusted standard errors allowing for 11 lags:

$$\begin{cases} \chi_{s,t} = & c_{1,s} + \psi_{1,s} \cdot \text{Agg. Disp.}_{t-1} + \delta^m_{1,s} \cdot R^{(12)}_{m,t} + \delta^{HML}_{1,s} \cdot HML^{(12)}_t + \delta^{SMB}_{1,s} \cdot SMB^{(12)}_t + \delta^{UMD}_{1,s} \cdot UMD^{(12)}_t + \sum_{x \in X} \delta^x_{1,s} \cdot x_{t-1} + \zeta_{t,s} \\ \varrho_{t,s} = & c_{2,s} + \psi_{2,s} \cdot \text{Agg. Disp.}_{t-1} + \delta^m_{2,s} \cdot R^{(12)}_{m,t} + \delta^{HML}_{2,s} \cdot HML^{(12)}_t + \delta^{SMB}_{2,s} \cdot SMB^{(12)}_t + \delta^{UMD}_{2,s} \cdot UMD^{(12)}_t + \sum_{x \in X} \delta^x_{2,s} \cdot x_{t-1} + \omega_{t,s} \\ \iota_{t,s} = & c_{3,s} + \psi_{3,s} \cdot \text{Agg. Disp.}_{t-1} + \delta^m_{3,s} \cdot R^{(12)}_{m,t} + \delta^{HML}_{3,s} \cdot HML^{(12)}_t + \delta^{SMB}_{3,s} \cdot SMB^{(12)}_t + \delta^{UMD}_{3,s} \cdot UMD^{(12)}_t + \sum_{x \in X} \delta^x_{3,s} x_{t-1} + \nu_{t,s} \end{cases}$$

Column (1) and (5) controls for Agg. Disp.$_{t-1}$, the monthly β-weighted average of stock level disagreement measured as the standard deviation of analyst forecasts on stocks' long run growth of EPS. Column (2) and (6) add controls for the 12-months excess return from t to $t+11$ of the market ($R^{(12)}_{m,t}$), HML ($HML^{(12)}_t$), SMB ($SMB^{(12)}_t$), and UMD ($UMD^{(12)}_t$). Column (3) and (7) add controls for the aggregate Dividend/Price ratio in $t-1$ and the past-12 months inflation rate in t_1. Column (4) and (8) additionally control for the TED spread in month $t-1$. T-statistics are in parenthesis. *, **, and *** means statistically different from zero at 10, 5 and 1% level of significance.

Table V (Continued):

Dep. Var:	$\chi_{s,t}$				$\varrho_{s,t}$				$\iota_{s,t}$			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Panel A: Speculative stocks ($\frac{\hat{\beta}_i}{\hat{\sigma}_i^2}$ > NYSE median $\frac{\hat{\beta}}{\hat{\sigma}^2}$)												
Agg. Disp.$_{t-1}$	-13**	-7**	-11***	-11***	2.8	2.5	1.6	.89	5.4*	5.9**	10***	11***
	(-2.5)	(-2.1)	(-2.7)	(-2.9)	(1)	(1)	(.57)	(.31)	(1.7)	(2.1)	(3.2)	(3.4)
$R_{m,t}^{(12)}$		.67***	.63***	.64***		-.056	-.069	-.058		.36***	.41***	.4***
		(4.8)	(4.3)	(4.3)		(-.45)	(-.48)	(-.38)		(2.9)	(3.4)	(3.3)
$HML_t^{(12)}$		-.72***	-.6***	-.57**		.17	.19	.24		.7***	.56***	.51***
		(-2.9)	(-2.7)	(-2.5)		(1)	(1.2)	(1.4)		(3.3)	(3.1)	(2.9)
$SMB_t^{(12)}$		.61**	.76***	.76***		-.35	-.31	-.32		-.5**	-.69***	-.68***
		(2.4)	(2.9)	(2.9)		(-1.4)	(-1.3)	(-1.4)		(-2.2)	(-3.1)	(-3.2)
$UMD_t^{(12)}$		.042	.085	.1		-.2*	-.19**	-.17*		.031	-.017	-.038
		(.27)	(.59)	(.75)		(-1.9)	(-2)	(-1.7)		(.24)	(-.15)	(-.38)
D/P$_{t-1}$			-2.7	-4.3			-.57	-2.5			3.4	5.6
			(-.65)	(-.94)			(-.2)	(-.8)			(1)	(1.4)
Inflation$_{t-1}$			-4*	-5**			-1.1	-2.2			4.6***	5.8***
			(-1.8)	(-2.6)			(-.49)	(-1.2)			(3)	(4)
Ted Spread$_{t-1}$				2.8				3.4				-3.8*
				(.95)				(1.1)				(-1.9)
Constant	-1.3	-5.5***	-6***	-6.4***	.028	1.7	1.6	1.2	9.9***	4.4**	5.1***	5.5***
	(-.35)	(-2.6)	(-2.6)	(-2.8)	(.011)	(.87)	(.81)	(.63)	(3.5)	(2.4)	(2.6)	(2.9)
Panel B: Non speculative stocks ($\frac{\hat{\beta}_i}{\hat{\sigma}_i^2} \leq$ NYSE median $\frac{\hat{\beta}}{\hat{\sigma}^2}$)												
Agg. Disp.$_{t-1}$	.074	3.4	-3.4	-3.8	-4.7	-4*	-1.9	-2.2	2.7	2.8	7.3***	7.7***
	(.043)	(1.2)	(-1.4)	(-1.6)	(-1.3)	(-1.8)	(-.71)	(-.84)	(.88)	(1.2)	(3.2)	(3.4)
$R_{m,t}^{(12)}$		.3*	.34***	.34***		.3**	.23*	.23*		.5***	.53***	.52***
		(1.8)	(3.1)	(3)		(2.5)	(1.9)	(1.9)		(3.6)	(4.8)	(4.5)
$HML_t^{(12)}$		-.43	-.21	-.19		-.41***	-.49***	-.47***		1***	.87***	.85***
		(-1.6)	(-.91)	(-.8)		(-3)	(-4.2)	(-3.8)		(4.5)	(4.7)	(4.9)
$SMB_t^{(12)}$		.25	.46**	.46**		.97***	.95***	.95***		-.64***	-.81***	-.81***
		(.82)	(2.3)	(2.3)		(4.9)	(5.7)	(5.8)		(-2.6)	(-4.5)	(-4.5)
$UMD_t^{(12)}$		.16	.18	.19		-.15*	-.13*	-.12		-.008	-.047	-.057
		(1)	(1.4)	(1.6)		(-1.8)	(-1.7)	(-1.5)		(-.059)	(-.42)	(-.51)
D/P$_{t-1}$			-9.5***	-11***			5.4	4.5			4.2*	5.2*
			(-3.3)	(-4)			(1.5)	(1.5)			(1.7)	(1.7)
Inflation$_{t-1}$			-2	-2.6			-2.3	-2.8			3.7*	4.2*
			(-.6)	(-.66)			(-.87)	(-.93)			(1.7)	(1.8)
Ted Spread$_{t-1}$				1.8				1.5				-1.8
				(.55)				(.66)				(-.75)
Constant	.87	-1.5	-3.1	-3.4	.12	-1.1	-.21	-.39	8.7***	1.1	1.9	2.1
	(.29)	(-.54)	(-1.3)	(-1.3)	(.044)	(-.5)	(-.097)	(-.18)	(2.8)	(.47)	(.87)	(.92)
N	385	385	385	385	385	385	385	385	385	385	385	385

Table VI. Disagreement and Slope of the Security Market Line: speculative vs. non speculative stocks; equal-weighted portfolios

Note: Sample Period: 12/1981-12/2014. Sample: CRSP stock file excluding penny stocks (price < \$5) and microcaps (stocks in bottom 2 deciles of the monthly size distribution using NYSE breakpoints). At the beginning of each calendar month, stocks are ranked in ascending order on the basis of the ratio of their estimated beta at the end of the previous month and their estimated idiosyncratic variance ($\frac{\beta}{\sigma^2}$). Pre-formation betas and idiosyncratic variance are estimated with a market model using daily returns over the past calendar year and 5 lags of the market returns. The ranked stocks are assigned to two groups: speculative ($\frac{\hat{\beta}_i}{\hat{\sigma}_i^2}$ > NYSE median $\frac{\hat{\beta}}{\hat{\sigma}^2}$ in month t) and non-speculative stocks. Within each of these two groups, stocks are ranked in ascending order of their estimated beta at the end of the previous month and are assigned to one of 20 beta-sorted portfolios using NYSE breakpoints. We compute the full sample beta of these 2×20 equal-weighted portfolios (20 beta-sorted portfolios for speculative stocks; 20 beta-sorted portfolios for non-speculative stocks) using the same market model. $\beta_{P,s}$ is the resulting full sample beta, where $P = 1, \ldots, 20$ and s $\in$ {speculative, non speculative}. We estimate every month the following cross-sectional regressions, where P is one of the 20 β-sorted portfolios, $s \in$ {speculative, non speculative} and t is a month :

$$r^{(12)}_{P,s,t} = \iota_{s,t} + \chi_{s,t} \times \beta_{P,s} + \varrho_{s,t} \times \ln\left(\sigma_{P,s,t-1}\right) + \epsilon_{P,s,t},$$

where $\sigma_{P,s,t-1}$ is the median idiosyncratic volatility of stocks in portfolio (P, s) estimated at the end of month $t-1$ and $r^{(12)}_{P,s,t}$ is the equal-weighted 12-months excess return of portfolio (P, s). We then estimate second-stage regressions in the time-series using OLS and Newey-West adjusted standard errors allowing for 11 lags:

$$\begin{cases} \chi_{s,t} = & c_{1,s} + \psi_{1,s} \cdot \text{Agg. Disp.}_{t-1} + \delta^{m}_{1,s} \cdot R^{(12)}_{m,t} + \delta^{HML}_{1,s} \cdot HML^{(12)}_t + \delta^{SMB}_{1,s} \cdot SMB^{(12)}_t + \delta^{UMD}_{1,s} \cdot UMD^{(12)}_t + \sum_{x \in X} \delta^{x}_{1,s} \cdot x_{t-1} + \zeta_{t,s} \\ \varrho_{t,s} = & c_{2,s} + \psi_{2,s} \cdot \text{Agg. Disp.}_{t-1} + \delta^{m}_{2,s} \cdot R^{(12)}_{m,t} + \delta^{HML}_{2,s} \cdot HML^{(12)}_t + \delta^{SMB}_{2,s} \cdot SMB^{(12)}_t + \delta^{UMD}_{2,s} \cdot UMD^{(12)}_t + \sum_{x \in X} \delta^{x}_{2,s} \cdot x_{t-1} + \omega_{t,s} \\ \iota_{t,s} = & c_{3,s} + \psi_{3,s} \cdot \text{Agg. Disp.}_{t-1} + \delta^{m}_{3,s} \cdot R^{(12)}_{m,t} + \delta^{HML}_{3,s} \cdot HML^{(12)}_t + \delta^{SMB}_{3,s} \cdot SMB^{(12)}_t + \delta^{UMD}_{3,s} \cdot UMD^{(12)}_t + \sum_{x \in X} \delta^{x}_{3,s} x_{t-1} + \nu_{t,s} \end{cases}$$

Column (1) and (5) controls for Agg. Disp.$_{t-1}$, the monthly β-weighted average of stock level disagreement measured as the standard deviation of analyst forecasts on stocks' long run growth of EPS. Column (2) and (6) add controls for the 12-months excess return from t to $t+11$ of the market ($R^{(12)}_{m,t}$), HML ($HML^{(12)}_t$), SMB ($SMB^{(12)}_t$), and UMD ($UMD^{(12)}_t$). Column (3) and (7) add controls for the aggregate Dividend/Price ratio in $t-1$ and the past-12 months inflation rate in t_1. Column (4) and (8) additionally control for the TED spread in month $t-1$. T-statistics are in parenthesis. *, **, and *** means statistically different from zero at 10, 5 and 1% level of significance.

Table VI (Continued):

Dep. Var:		$\chi_{s,t}$					$\varrho_{s,t}$				$\iota_{s,t}$	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Panel A: Speculative stocks ($\frac{\hat{\beta}_i}{\hat{\sigma}_i^2}$ > NYSE median $\frac{\hat{\beta}}{\hat{\sigma}^2}$)												
Agg. Disp.$_{t-1}$	-19***	-11***	-15***	-16***	8.5***	6.2**	6.7	6.7*	9.2***	7***	11***	12***
	(-3.1)	(-2.9)	(-3.2)	(-3.4)	(2.7)	(2)	(1.6)	(1.7)	(2.7)	(2.6)	(3.7)	(3.8)
$R_{m,t}^{(12)}$		.55***	.49***	.5***		.23	.24	.24		.45***	.52***	.5***
		(3.1)	(2.9)	(2.9)		(1.6)	(1.5)	(1.5)		(3)	(4)	(3.9)
$HML_t^{(12)}$		-1.2***	-1.1***	-1***		.74***	.73***	.73***		1.1***	.95***	.9***
		(-4.8)	(-4.6)	(-4.4)		(4)	(3.7)	(3.9)		(5.9)	(6.1)	(5.8)
$SMB_t^{(12)}$		.45*	.62**	.61**		.016	-.012	-.012		-.088	-.28	-.28
		(1.8)	(2.5)	(2.4)		(.067)	(-.047)	(-.048)		(-.44)	(-1.6)	(-1.6)
$UMD_t^{(12)}$		-.13	-.08	-.063		-.037	-.047	-.046		.059	.0012	-.021
		(-.67)	(-.44)	(-.38)		(-.23)	(-.3)	(-.29)		(.41)	(.0099)	(-.2)
D/P$_{t-1}$			-1.8	-3.6			-.032	-.15			2.5	4.7
			(-.66)	(-1.1)			(-.01)	(-.046)			(.91)	(1.4)
Inflation$_{t-1}$			-5**	-6**			1	.94			5.6***	6.9***
			(-2)	(-2.3)			(.38)	(.36)			(3.3)	(3.9)
Ted Spread$_{t-1}$				3				.2				-3.9**
				(1.1)				(.066)				(-2)
Constant	-8*	-7.8**	-8.2**	-8.6***	7**	2.3	2.4	2.3	14***	5.2**	5.8**	6.3***
	(-1.9)	(-2.5)	(-2.5)	(-2.6)	(2.4)	(.69)	(.67)	(.65)	(4.3)	(2.3)	(2.4)	(2.7)
Panel B: Non speculative stocks ($\frac{\hat{\beta}_i}{\hat{\sigma}_i^2} \leq$ NYSE median $\frac{\hat{\beta}}{\hat{\sigma}^2}$)												
Agg. Disp.$_{t-1}$	.61	3.6	-1.5	-1.8	-5.3	-3.5	-.74	-1.8	4.7*	2.4	5.5**	6.6***
	(.27)	(.98)	(-.38)	(-.47)	(-1.4)	(-1.3)	(-.2)	(-.55)	(1.8)	(1.2)	(2.5)	(3.1)
$R_{m,t}^{(12)}$		-.0069	.051	.056		.83***	.67***	.69***		.31*	.46***	.44***
		(-.035)	(.3)	(.31)		(4.2)	(3.7)	(4.1)		(1.7)	(4.2)	(4)
$HML_t^{(12)}$		-.78**	-.61**	-.59*		.1	-.0089	.056		1***	.96***	.9***
		(-2.3)	(-2)	(-2)		(.5)	(-.047)	(.29)		(4.5)	(4.5)	(4.5)
$SMB_t^{(12)}$		.22	.35	.35		.95***	.95***	.95***		-.26	-.46**	-.45**
		(.53)	(1.1)	(1)		(2.6)	(3.3)	(3.5)		(-1.1)	(-2.1)	(-2.1)
$UMD_t^{(12)}$		-.079	-.072	-.065		.013	.063	.091		.15	.063	.036
		(-.42)	(-.41)	(-.37)		(.1)	(.54)	(.72)		(.87)	(.51)	(.33)
D/P$_{t-1}$			-8.2**	-8.9**			9.2*	6.2			-2.1	.82
			(-2)	(-2.2)			(1.9)	(1.5)			(-.74)	(.27)
Inflation$_{t-1}$			-.32	-.72			-5.4*	-7**			8.4**	10***
			(-.1)	(-.2)			(-1.7)	(-2.5)			(2.6)	(2.9)
Ted Spread$_{t-1}$				1.2				5.1				-5**
				(.29)				(1.4)				(-2)
Constant	-4.5	-1	-2.4	-2.6	5.9	-3	-1.6	-2.2	9.3***	1.8	1.6	2.2
	(-1.3)	(-.33)	(-.82)	(-.87)	(1.6)	(-1.1)	(-.6)	(-.9)	(2.9)	(.79)	(.77)	(1.1)
N	385	385	385	385	385	385	385	385	385	385	385	385

Notes

[1]A non-exhaustive list of studies include Blitz and Vliet (2007), Cohen et al. (2005), and Frazzini and Pedersen (2010).

[2]The value-growth effect (Fama and French (1992), Lakonishok et al. (1994)), buying stocks with low price-to-fundamental ratios and shorting those with high ones, generates a reward-to-risk or Sharpe (1964) ratio that is two-thirds of a zero-beta adjusted strategy of buying low beta stocks and shorting high beta stocks. The corresponding figure for the momentum effect (Jegadeesh and Titman (1993)), buying past year winning stocks and shorting past year losing ones, is roughly three-fourths of the long low beta, short high beta strategy.

[3]Indeed, most behavioral models would also not deliver such a pattern. In Barberis and Huang (2001), mental accounting by investors still leads to a positive relationship between risk and return. The exception is the model of overconfident investors and the cross-section of stock returns in Daniel et al. (2001) that might yield a negative relationship as well but not the new patterns with beta we document below.

[4]See Hong and Stein (2007) for a discussion of the various rationales. A large literature starting with Odean (1999) and Barber and Odean (2001) argues that retail investors engage in excessive trading due to overconfidence.

[5]See Lamont (2004) for a discussion of the many rationales for the bias against shorting in financial markets, including historical events such as the Great Depression in which short-sellers were blamed for the Crash of 1929.

[6]The consideration of a general disagreement structure about both means and covariances of asset returns with short-sales restrictions in a CAPM setting is developed in Jarrow (1980). He shows that short-sales restrictions in one asset might increase the prices of others. It turns out that a focus on a simpler one-factor disagreement structure about common cash-flows yields closed form solutions and a host of testable implications for the cross-section of asset prices that would otherwise not be possible.

[7]High beta stocks might also be more difficult to arbitrage because of incentives for benchmarking and other agency issues (Brennan (1993), Baker et al. (2011)).

[8]When aggregate disagreement is so large that pessimists are sidelined on all assets, the relationship between risk and return is entirely downward sloping as the entire market becomes overpriced. We assume that all assets in our model have a strictly positive loading on the aggregate factor. Thus, it is always possible that pessimists want to be short an asset, provided aggregate disagreement is large enough.

[9]The "square" portfolio, which corresponds to the monthly coefficient estimate of a regression of portfolio returns on the portfolio's β^2, is a portfolio that goes long the top 3 and bottom 6 β-sorted portfolios and short the remaining portfolios. It thus captures intuitively the inverted-U shape of the Security Market Line in our theoretical analysis.

[10]This normalization of supply to $1/N$ is without loss of generality. If asset i is in supply s_i, then what matters is the ranking of assets along the $\frac{b_i}{s_i}$ dimension. The rest of the analysis is then left unchanged.

[11]The condition defining the marginal asset $\bar{i}$, $\bar{i} = \min\{k \in [0, N+1] \mid \lambda > u_k\}$ corresponds to an N-asset generalization of the condition defining the equilibrium with short-sales constraint in the one-asset model of Chen et al. (2002). An intuition for this condition is that disagreement has to be larger than the risk-premium for bearing the risk of an asset for short-sales constraints to bind. Otherwise, even pessimists would like to be long the risky asset. The sequence (u_i), which plays a key

role in this condition, corresponds to the equilibrium holding in asset i of pessimist investors. Naturally, these u_i's depend on the risk-tolerance γ, the supply of risky assets $1/N$ and the covariance of asset i with other assets.

[12]The derivation of this formula can be found in Appendix D.

[13]Most of the assumptions made in this model are discussed in Section I.A in the context of our static model.

[14]$\lambda_t^a = 0$, $\lambda_t^A = \lambda$ and $\lambda_t^B = -\lambda$ when $\tilde{\lambda}_t = \lambda$ or $\lambda_t^a = \lambda_t^A = \lambda_t^B = 0$ when $\tilde{\lambda}_t = 0$ and $\lambda_t^a = 0$

[15]In the low disagreement state, $\tilde{\lambda} = 0$ so there is no disagreement among investors and hence there cannot be any binding short-sales constraint.

[16]In a simple 3 asset version of this model with trading costs, we can show that mispricing is in fact increasing with λ.

[17]We performed calibrations using $N = 25$ and $N = 75$ and found similar qualitative results.

[18]The volatility and average excess return on the market are directly computed from the monthly market return series obtained from Ken French's website. To compute the average idiosyncratic variance of stock returns, we first estimate a CAPM equation for each stock in our sample using monthly excess returns, we then compute the variance of the residuals from this equation by stock and finally define the average idiosyncratic variance as the average of these variances across all stocks in our sample.

[19]The pre-ranking β are constructed as detailed in Section II.B.

[20]In Table I.A AIII, we show that our main results are robust to different weighting-schemes: (1) weighting using compressed betas (2) weighting using the product of beta and size. These measures also use pre-ranking betas.

[21]We have also checked hermite polynomials in this specification but the quadratic functional fits the best.

[22]Noxy-Marx (2014) also shows that a significant part of the returns on defensive equity strategies is driven by exposure to a profitability factor. In unreported regressions, available from the authors upon request, we show that the inclusion of this additional factor does not affect our results.

[23]In addition, Li (2014) also tests our model using dispersion of macro-forecasts for each of these macro-variables separately. But rather than using 20-β portfolios, he forms optimal tracking portfolios for each of these macro-variables and calculates each stock's macro-beta with respect to these macro-tracking portfolios and finds that when aggregate disagreement is high, higher macro-beta stocks under-perform lower macro-beta stocks.

[24]We use the logarithm of the idiosyncratic volatility as a control variable in the first-stage regression to account for the skewness in this variable

[25]This result is consistent with the result in Noxy-Marx (2014) – the novelty here is that our model proposes an explanation for this correlation.

[26]This is true for all but the top β portfolio created from speculative stocks, which has a β/σ^2 ratio of .25 only. Excluding this portfolio does not change our analysis qualitatively.

[27] This is true except at the 3-month horizon where both the prediction relating aggregate disagreement to the price of β and the price of idiosyncratic variance are verified.

Internet Appendix
For Online Publication Only

A. Proofs of the Model

A. Proof of Theorem 1

Proof. We solve the model here allowing for heteroskedastic dividend σ_i^2. Theorem 1 can then be proved as a special case $\sigma_\epsilon^2 = \sigma_i^2$. We assume that assets are ranked in ascending order of β/σ^2.

We first posit an equilibrium structure and check ex-post that it is indeed an equilibrium and then that it is the unique equilibrium. Let $\bar{i} \in [2, N]$ and let μ_i^m be the share holdings of asset k by group m where $m \in \{a, A, B\}$. Consider an equilibrium where group B investors are long on assets $i < \bar{i}$ and hold no position (i.e., $\mu_i^B = 0$) for assets $i \geq \bar{i}$ and group A investors are long all assets $i \in [1, N]$. Since group A investors are long, their holdings satisfy the following first order conditions:

$$\forall i \in [1, N]: \quad d + \lambda b_i - P_i(1 + r) = \frac{1}{\gamma}\left(\left(\sum_{k=1}^{N} b_k \mu_k^A\right) b_i \sigma_z^2 + \mu_i^A \sigma_i^2\right)$$

Since group B investors are long only on assets $i < \bar{i}$, their holdings for these assets must also satisfy the following first order condition:

$$\forall i \in [1, \bar{i} - 1], \quad d - \lambda b_i - P_i(1 + r) = \frac{1}{\gamma}\left(\left(\sum_{k=1}^{\bar{i}-1} b_k \mu_k^B\right) b_i \sigma_z^2 + \mu_i^B \sigma_i^2\right)$$

For assets $i \geq \bar{i}$, group B investors have 0 holdings and so $\mu_i^B = 0$. For these assets, it must be the case that the group B investors' marginal utility of holding the asset, taken at the equilibrium holdings, is strictly negative (otherwise, group B investors would have an incentive to increase their holdings). This is equivalent to:

$$\forall i \geq \bar{i}, \quad d - \lambda b_i - P_i(1 + r) - \frac{1}{\gamma}\left(\left(\sum_{k=1}^{\bar{i}-1} b_k \mu_k^B\right) b_i \sigma_z^2\right) < 0$$

Finally, since arbitrageurs are not short-sales constrained, their holdings always satisfy the following first-order condition:

$$\forall i \in [1, N]: \quad d - P_i(1 + r) = \frac{1}{\gamma}\left(\left(\sum_{k=1}^{N} b_k \mu_k^a\right) b_i \sigma_z^2 + \mu_i^a \sigma_i^2\right)$$

The market clearing condition for asset i is simply: $\alpha \frac{\mu_i^A + \mu_i^B}{2} + (1 - \alpha)\mu_i^a = \frac{1}{N}$. We sum the first-order conditions of investors a, A and B for assets $i < \bar{i}$, and of investors a and A only for assets $i \geq \bar{i}$, weighting the sum by the size of each investors group (i.e., $\frac{\alpha}{2}$ for group A and B and $1-\alpha$ for group a). This results in the following equations:

$$\begin{cases} d - P_i(1+r) = \dfrac{1}{\gamma}\left(b_i\sigma_z^2 + \dfrac{\sigma_i^2}{N}\right) \quad \text{for } i < \bar{i} \\ \left(1-\dfrac{\alpha}{2}\right)(d - P_i(1+r)) + \dfrac{\alpha}{2}\lambda b_i = \dfrac{1}{\gamma}\left(b_i\sigma_z^2 + \dfrac{\sigma_i^2}{N} - \dfrac{\alpha}{2}\sigma_z^2 b_i \displaystyle\sum_{k=1}^{\bar{i}-1} b_k\mu_k^B\right) \quad \text{for } i \geq \bar{i} \end{cases} \tag{5}$$

Call $S^B = \sum_{k=1}^{\bar{i}-1} b_k\mu_k^B$. S^B represents the exposure of group B investors to the aggregate factor $\tilde{z}$. We look for an expression for S^B. We start by using the first order conditions of group B investors on assets $k < \bar{i}$ and plug in the equilibrium price of assets $k < \bar{i}$ found in the first equation of system (5):

$$\forall k < \bar{i}, \qquad -\lambda\gamma b_k + b_k\sigma_z^2 + \frac{\sigma_k^2}{N} = S^B b_k\sigma_z^2 + \mu_k^B\sigma_k^2$$

We can now simply multiply the previous equation by b_k and divided it by σ_k^2 for all $k < \bar{i}$ and sum up the resulting equations for $k < \bar{i}$, which results in:

$$S^B = -\lambda\gamma\left(\sum_{k<\bar{i}}\frac{b_k^2}{\sigma_k^2}\right) - S\sigma_z^2\left(\sum_{k<\bar{i}}\frac{b_k^2}{\sigma_k^2}\right) + \sigma_z^2\left(\sum_{k<\bar{i}}\frac{b_k^2}{\sigma_k^2}\right) + \sum_{k<\bar{i}}\frac{b_k}{N} \tag{6}$$

From the previous expression, we can now derive S^B:

$$S^B = 1 - \frac{\left(\sum_{k\geq\bar{i}}\frac{b_k}{N}\right) + \lambda\gamma\left(\sum_{k<\bar{i}}\frac{b_k^2}{\sigma_k^2}\right)}{1+\sigma_z^2\left(\sum_{k<\bar{i}}\frac{b_k^2}{\sigma_k^2}\right)}$$

Now that we have a closed-form expression for S^B, we simply plug it into the second equation of system 5. Define $\theta = \frac{\frac{\alpha}{2}}{1-\frac{\alpha}{2}}$. The price of assets $i \geq \bar{i}$ is then given by:

$$P_i(1+r) = d - \frac{1}{\gamma}\left(b_i\sigma_z^2 + \frac{\sigma_i^2}{N}\right) + \underbrace{\frac{\theta}{\gamma}\left(b_i\sigma_z^2\underbrace{\frac{\lambda\gamma - \sigma_z^2\sum_{k\geq\bar{i}}\frac{b_k}{N}}{\sigma_z^2\left(1+\sigma_z^2\left(\sum_{k<\bar{i}}\frac{b_k^2}{\sigma_i^2}\right)\right)}}_{=\omega(\lambda)} - \frac{\sigma_i^2}{N}\right)}_{\pi^i=\text{speculative premium}} \tag{7}$$

The first equation of system 5 provides us with a simple expression for the price of assets $i < \bar{i}$:

$$P_i(1+r) = d - \frac{1}{\gamma}\left(b_i\sigma_z^2 + \frac{\sigma_i^2}{N}\right) \tag{8}$$

In order to derive the conditions under which the proposed equilibrium is indeed an equilibrium (i.e., $\bar{i}$ is indeed the marginal asset), we need to derive the equilibrium holdings of group B investors:

$$\mu_i^{B,\star} = \begin{cases} \dfrac{1}{N} + \dfrac{b_i}{\sigma_i^2}\left(\dfrac{\sigma_z^2\left(\sum_{i\geq\bar{i}}\frac{b_i}{N}\right)-\lambda\gamma}{1+\sigma_z^2\left(\sum_{i<\bar{i}}\frac{b_i^2}{\sigma_i^2}\right)}\right) & \text{for } i<\bar{i} \\ 0 \quad \text{for } i\geq\bar{i} \end{cases}$$

We are now ready to derive the conditions under which the proposed equilibrium is indeed an equilibrium. The marginal asset is asset $\bar{i}$ if and only if $\frac{\partial U^B}{\partial\mu_{\bar{i}}^B}(\mu^{B,\star})<0$ and $\mu_{\bar{i}-1}^B\geq 0$, where $\mu^{B,\star}$ is group B investors' holdings derived above. The condition that the marginal utility of investing in asset $\bar{i}$ for pessimist agents is equivalent to $\pi_{\bar{i}}>0$ so that $\bar{i}$ is the marginal asset if and only if:

$$\frac{\sigma_z^2}{\gamma N}\sum_{k\geq\bar{i}}b_k+\frac{1}{\gamma N\frac{b_{\bar{i}-1}}{\sigma_{\bar{i}-1}^2}}\left(1+\sigma_z^2\sum_{k<\bar{i}}\frac{b_k^2}{\sigma_k^2}\right)\geq\lambda>\frac{\sigma_z^2}{\gamma N}\sum_{k\geq\bar{i}}b_k+\frac{1}{\gamma N\frac{b_{\bar{i}}}{\sigma_{\bar{i}}^2}}\left(1+\sigma_z^2\sum_{k<\bar{i}}\frac{b_k^2}{\sigma_k^2}\right)$$

Call $u_k=\frac{1}{\gamma N\frac{b_k}{\sigma_k}}\left(1+\sigma_z^2\left(\sum_{i<k}\frac{b_i^2}{\sigma_i^2}\right)\right)+\frac{\sigma_z^2}{\gamma}\left(\sum_{i\geq k}\frac{b_i}{N}\right)$. Clearly, u_k is a strictly decreasing sequence as:

$$\begin{aligned} u_{i-1}-u_i &= \frac{1}{\gamma N\frac{b_{i-1}}{\sigma_{i-1}^2}}\left(1+\sigma_z^2\left(\sum_{j<i-1}\frac{b_j^2}{\sigma_j^2}\right)\right)+\frac{\sigma_z^2}{\gamma}\left(\sum_{j\geq i-1}\frac{b_j}{N}\right)-\frac{1}{\gamma N\frac{b_i}{\sigma_i^2}}\left(1+\sigma_z^2\left(\sum_{j<i}\frac{b_j^2}{\sigma_j^2}\right)\right)-\frac{\sigma_z^2}{\gamma}\left(\sum_{j\geq i}\frac{b_j}{N}\right) \\ &= \frac{1}{\gamma N}\left(1+\sigma_z^2\left(\sum_{j<i-1}\frac{b_j^2}{\sigma_j^2}\right)\right)\left(\frac{\sigma_{i-1}^2}{b_{i-1}}-\frac{\sigma_i^2}{b_i}\right)-\frac{1}{\gamma N\frac{b_i}{\sigma_i^2}}\sigma_z^2\frac{b_{i-1}^2}{\sigma_{i-1}^2}+\frac{\sigma_z^2}{\gamma N}b_{i-1} \\ &= \frac{1}{\gamma N}\left(1+\sigma_z^2\left(\sum_{j<i-1}\frac{b_j^2}{\sigma_j^2}\right)\right)\left(\frac{\sigma_{i-1}^2}{b_{i-1}}-\frac{\sigma_i^2}{b_i}\right)+\frac{\sigma_z^2}{\gamma N}\frac{b_{i-1}^2}{\sigma_{i-1}^2}\left(\frac{\sigma_{i-1}^2}{b_{i-1}}-\frac{\sigma_i^2}{b_i}\right) \\ &= \frac{1}{\gamma N}\left(1+\sigma_z^2\left(\sum_{j<i}\frac{b_j^2}{\sigma_j^2}\right)\right)\left(\frac{\sigma_{i-1}^2}{b_{i-1}}-\frac{\sigma_i^2}{b_i}\right)>0 \end{aligned}$$

Define $u_0=+\infty$ and $u_{N+1}=0$. Then the sequence $(u_i)_{i\in[0,N+1]}$ spans $\mathbb{R}^+$ and the marginal asset is simply defined as: $\bar{i}=\min\{k|\lambda>u_k\}$. We know that $\bar{i}>0$ since $u_0=+\infty$. If $\bar{i}=N+1$, then group B investors are long all assets and all the previous formula apply except that there is no asset such that $i\geq\bar{i}$. If $\bar{i}\in[1,n]$, then the equilibrium has the proposed structure, that is, investors B are long only assets $i<\bar{i}$.

We have so far assumed that $\bar{i}>1$. The equilibrium is easily derived when $\bar{i}=1$, that is, when all assets are over-priced. In this case, $S^B=0$ and we directly have:

$$d-(1+r)P_i=\frac{1}{\gamma}(1+\theta)\left(b_i\sigma_z^2+\frac{\sigma_\epsilon^2}{N}\right)-\theta\lambda b_i.$$

This corresponds to the formula derived in Theorem 1 where we define $\sum_{i<1}b_i^2=0$. $\bar{i}=1$ is an equilibrium if and only if $\mu_1^{B,\star}<0$, which is equivalent to $\lambda<u_N$, as stated in the Theorem.

We now show that the equilibrium is unique. Let $J=j|\mu_j^B>0$ – J is the set of assets that pessimists are long. It is direct to show in this case that prices are given by:

$$P_i(1+r) = \begin{cases} d - \frac{1}{\gamma}\left(b_i\sigma_z^2 + \frac{\sigma_\epsilon^2}{N}\right) & \text{for } i \in J \\ d - \frac{1}{\gamma}\left(b_i\sigma_z^2 + \frac{\sigma_i^2}{N}\right) + \underbrace{\frac{\theta}{\gamma}\left(b_i\left(\frac{\lambda\gamma - \frac{\sigma_z^2}{N}\left(\sum_{i\notin J} b_i\right)}{1+\sigma_z^2\left(\sum_{i\in J}\frac{b_i^2}{\sigma_i^2}\right)}\right) - \frac{\sigma_i^2}{N}\right)}_{\pi^i=\text{speculative premium}} & \text{for } i \notin J \end{cases} \tag{9}$$

The holdings of the pessimists can then be written as:

$$\mu_i^{B,\star} = \begin{cases} \frac{1}{N} + \frac{b_i}{\sigma_i^2}\left(\frac{\sigma_z^2\left(\sum_{i\notin J}\frac{b_i}{N}\right) - \lambda\gamma}{1+\sigma_z^2\left(\sum_{i\in J}\frac{b_i^2}{\sigma_i^2}\right)}\right) & \text{for } i \in J \\ 0 \quad \text{for } i \notin J \end{cases}$$

In particular, this implies that for $j \in J$, we need to have $\frac{b_j}{\sigma_j^2}\left(\frac{\lambda\gamma - \sigma_z^2\left(\sum_{i\notin J}\frac{b_i}{N}\right)}{1+\sigma_z^2\left(\sum_{i\in J}\frac{b_i^2}{\sigma_i^2}\right)}\right) < \frac{1}{N}$ and for $i \notin J$ that: $b_i\left(\frac{\lambda\gamma - \frac{\sigma_z^2}{N}\left(\sum_{i\notin J} b_i\right)}{1+\sigma_z^2\left(\sum_{i\in J}\frac{b_i^2}{\sigma_i^2}\right)}\right) > \frac{\sigma_i^2}{N}$.

It thus follows that for all $j \in J$ and for all $i \notin J$:

$$\frac{b_i}{\sigma_i^2} > \frac{1}{N}\left(\frac{1+\sigma_z^2\left(\sum_{i\in J}\frac{b_i^2}{\sigma_i^2}\right)}{\lambda\gamma - \sigma_z^2\left(\sum_{i\notin J}\frac{b_i}{N}\right)}\right) > \frac{b_j}{\sigma_j^2}.$$

It follows that the equilibrium structure is necessarily in the form of a cutoff and hence our equilibrium is unique.

□

B. Proof of Corollary 1

Proof. Corollary 1 characterizes overpricing. Overpricing for assets $i \geq \bar{i}$ is defined as the difference between the equilibrium price and the price that would prevail in the absence of heterogenous beliefs and short sales constraints ($\alpha = 0$). Overpricing is just simply equal to the speculative premium:

$$\forall i \geq \bar{i}, \quad \text{Overpricing}^i = \pi^i = \frac{\theta}{\gamma}\left(b_i\sigma_z^2\omega(\lambda) - \frac{\sigma_i^2}{N}\right)$$

By definition of the equilibrium, $\lambda > u_{\bar{i}}$, which is equivalent to $\frac{b_{\bar{i}}}{\sigma_{\bar{i}}^2}\sigma_z^2\omega(\lambda) > \frac{1}{N}$. Since assets are ranked in ascending order of $\frac{b_i}{\sigma_i^2}$, this directly implies that for $i \geq \bar{i}$, $\pi^i > 0$ and assets $i \geq \bar{i}$ are in fact overpriced. That mispricing is increasing with the fraction of short-sales constrained investors α is direct as θ is a strictly increasing function of α. That mispricing increases with b_i and decreases with σ_i^2 is also directly seen from the definition of mispricing.[28].

$$\forall j > i \geq \bar{i}, \quad \text{Overpricing}^j - \text{Overpricing}^i = \frac{\theta}{\gamma}\sigma_z^2\omega(\lambda)(b_j - b_i)$$

□

C. Proof of Corollary 2

Proof. Corollary 2 characterizes the amount of shorting in the equilibrium. We first need to derive the equilibrium holdings of arbitrageurs. Group a holdings need to satisfy the following first-order conditions:

$$\forall i \in [1, N], \quad d - P_i(1 + r) = \frac{1}{\gamma}\left(b_i \sigma_z^2 \left(\sum_{k=1}^{N} \mu_k^a b_k\right) + \mu_i^a \sigma_i^2\right)$$

Define $S^a = \sum_{k=1}^N \mu_k^a b_k$. Using the equilibrium pricing equation in equation 7 and equation 8, this first-order condition can be rewritten as:

$$\forall k \in [1, N], \quad b_k \sigma_z^2 + \frac{\sigma_k^2}{N} - \gamma \pi^k 1_{k \geq \bar{i}} = b_k \sigma_z^2 S^a + \mu_k^a \sigma_k^2$$

We multiply each of these equations by b_k, divide them by σ_k^2 and sum up the resulting equations for all $i \in [1, N]$ to obtain:

$$S^a = 1 - \frac{\gamma \sum_{k \geq \bar{i}} b_k \frac{\pi^k}{\sigma_k^2}}{1 + \sigma_z^2 \left(\sum_{k=1}^N \frac{b_k^2}{\sigma_k^2}\right)}$$

We can now inject this expression for S^a in group a investors' first-order conditions derived above. This yields the following expression for group a investors' holdings of assets $i \in [1, N]$:

$$\forall i \in [1, N], \quad \mu_i^a \sigma_i^2 = \frac{\sigma_i^2}{N} - \gamma \pi^i 1_{\{i \geq \bar{i}\}} + b_i \sigma_z^2 \frac{\gamma \sum_{k \geq \bar{i}} b_k \frac{\pi^k}{\sigma_k^2}}{1 + \sigma_z^2 \left(\sum_{k=1}^N \frac{b_k^2}{\sigma_k^2}\right)}$$

First remark that if $i < \bar{i}$, $\mu_i^a > 0$, so that arbitrageurs are long assets $i < \bar{i}$. Now consider the case $i \geq \bar{i}$. Notice from the expression of the speculative premium that:

$$\forall k, i \geq \bar{i}, \quad \pi_k + \frac{\theta \sigma_k^2}{\gamma N} = \frac{b_k}{b_i}\left(\pi^i + \frac{\theta \sigma_i^2}{\gamma N}\right)$$

Thus, multiplying the previous expression by b_k, dividing by σ_k^2 and summing over all $k \geq \bar{i}$:

$$\sum_{k \geq \bar{i}} b_k \frac{\pi_k}{\sigma_k^2} + \frac{\theta}{\gamma N}\left(\sum_{k \geq \bar{i}} b_k\right) = \left(\sum_{k \geq \bar{i}} \frac{b_k^2}{\sigma_k^2}\right)\left(\frac{\pi^i + \frac{\theta \sigma_i^2}{\gamma N}}{b_i}\right)$$

Thus, for $i \geq \bar{i}$:

$$\begin{aligned}
\gamma\pi^i - b_i\sigma_z^2 \frac{\gamma \sum_{k\geq \bar{i}} b_k \frac{\pi^k}{\sigma_k^2}}{1+\sigma_z^2\left(\sum_{k=1}^N \frac{b_k^2}{\sigma_k^2}\right)} &= \gamma\pi^i - \frac{b_i\sigma_z^2}{1+\sigma_z^2\left(\sum_{k=1}^N \frac{b_k^2}{\sigma_k^2}\right)}\left(\left(\sum_{k\geq\bar{i}}\frac{b_k^2}{\sigma_k^2}\right)\left(\frac{\gamma\pi^i+\frac{\theta}{N}\sigma_i^2}{b_i}\right) - \frac{\theta}{N}\left(\sum_{k\geq\bar{i}} b_k\right)\right) \\
&= \gamma\pi^i \frac{1+\sigma_z^2\left(\sum_{k<\bar{i}}\frac{b_k^2}{\sigma_k^2}\right)}{1+\sigma_z^2\left(\sum_{k=1}^N \frac{b_k^2}{\sigma_k^2}\right)} - \frac{\sigma_z^2}{1+\sigma_z^2\left(\sum_{k=1}^N \frac{b_k^2}{\sigma_k^2}\right)}\frac{\theta}{N}\left(\sum_{k\geq\bar{i}}\sigma_i^2\frac{b_k^2}{\sigma_k^2} - b_i\sum_{k\geq\bar{i}} b_k\right) \\
&= \theta\left[b_i\left(\frac{\lambda\gamma - \frac{\sigma_z^2}{N}\left(\sum_{i\geq\bar{i}} b_i\right)}{1+\sigma_z^2\left(\sum_{i=1}^N \frac{b_i^2}{\sigma_i^2}\right)}\right) - \frac{\sigma_i^2}{N}\frac{1+\sigma_z^2\left(\sum_{k<\bar{i}}\frac{b_k^2}{\sigma_k^2}\right)}{1+\sigma_z^2\left(\sum_{k=1}^N \frac{b_k^2}{\sigma_k^2}\right)}\right. \\
&\qquad \left. -\frac{1}{N}\frac{\sigma_z^2}{1+\sigma_z^2\left(\sum_{k=1}^N \frac{b_k^2}{\sigma_k^2}\right)}\left(\sigma_i^2\sum_{k\geq\bar{i}}\frac{b_k^2}{\sigma_k^2} - b_i\sum_{k\geq\bar{i}} b_k\right)\right] \\
&= \theta\left[b_i\frac{\lambda\gamma}{1+\sigma_z^2\left(\sum_{k=1}^N \frac{b_k^2}{\sigma_k^2}\right)} - \frac{\sigma_i^2}{N}\right]
\end{aligned}$$

We can now derive the actual holding of arbitrageurs on assets $i \geq \bar{i}$:

$$\forall i \geq \bar{i}, \quad \mu_i^a = \frac{1+\theta}{N} - \theta\frac{b_i}{\sigma_i^2}\frac{\lambda\gamma}{1+\sigma_z^2\left(\sum_{k=1}^N \frac{b_k^2}{\sigma_k^2}\right)}$$

First, notice that arbitrageurs' holdings are decreasing with i since $\frac{b_i}{\sigma_i^2}$ increases strictly with i. There is at least one asset shorted by group a investors provided that $\mu_N^a < 0$, which is equivalent to $\lambda > \hat{\lambda} = \frac{1+\theta}{\theta}\frac{1+\sigma_z^2\left(\sum_{k=1}^N \frac{b_k^2}{\sigma_k^2}\right)}{N}\frac{\sigma_N^2}{\gamma b_N}$. Provided this is verified, there exists a unique $\tilde{i} \in [1, N]$ such that $\mu_i^a < 0 \Leftrightarrow i \geq \tilde{i}$. We know already that $\tilde{i} \geq \bar{i}$ since for $i < \bar{i}$, group a investors holdings are strictly positive. It is direct to see from the expression for group a investors holdings that provided that $i \geq \tilde{i}$, we have:

$$\frac{\partial|\mu_i^a|}{\partial\lambda} > 0, \quad \frac{\partial|\mu_i^a|}{\partial\frac{b_i}{\sigma_i^2}} > 0 \quad \text{and} \quad \frac{\partial^2|\mu_i^a|}{\partial\lambda\partial b_i} > 0$$

There is more shorting on assets with larger ratio of cash flow beta to idiosyncratic variance. There is more shorting the larger is aggregate disagreement. The effect of aggregate disagreement on shorting is larger for assets with a high ratio of cash-flow beta to idiosyncratic variance. □

D. Proof of formula 3 for expected excess returns

Proof. From Theorem 1, we know that:

$$P_i(1+r) = d - \frac{1}{\gamma}\left(b_i\sigma_z^2 + \frac{\sigma_I^2}{N}\right) + \mathbf{1}_{i\geq\bar{i}}\frac{\theta}{\gamma}\left(b_i\sigma_z^2\omega(\lambda) - \frac{\sigma_I^2}{N}\right)$$

Call $\tilde{R}_i^e$ the percentage excess return per share on asset i. $\tilde{R}_i^e = d + b_i\tilde{z} + \tilde{\epsilon}_i - (1+r)P_i$ and $\mathbb{E}[\tilde{r}_i^e] = d - (1+r)P_i$. Define the market portfolio as the portfolio of all assets in the market. Since all assets have a supply of 1/N, the excess return per share on the market portfolio is $\tilde{R}_m^e = \sum_{j=1}^N \frac{\tilde{R}_j^e}{N}$. Note $P_m = \sum_{j=1}^N \frac{P_j}{N}$ the price of the market portfolio. Stock i's beta is defined as $\beta_i = \frac{\text{Cov}\left(\tilde{R}_i^e, \tilde{R}_m^e\right)}{Var(\tilde{R}_m^e)}$ and can be written as:

$$\beta_i = \frac{b_i\sigma_z^2 + \frac{\sigma_i^2}{N}}{\sigma_z^2 + \sum_{k=1}^N \frac{\sigma_k^2}{N^2}} \quad \text{so that:} \quad b_i\sigma_z^2 = \beta_i\left(\sigma_z^2 + \sum_{k=1}^N \frac{\sigma_k^2}{N^2}\right) - \frac{\sigma_i^2}{N}.$$

We can thus substitute b_i by β_i in the price formula and derive an expression for expected excess returns per share as a function of β_i:

$$\mathbb{E}[\tilde{R}_i^e] = \beta_i \frac{\sigma_z^2 + \sum_{k=1}^N \frac{\sigma_k^2}{N^2}}{\gamma}\left(1 - \mathbf{1}_{i\geq\bar{i}}\theta\omega(\lambda)\right) + \theta\frac{\sigma_i^2}{\gamma N}\mathbf{1}_{i\geq\bar{i}}(1+\omega(\lambda))$$

□

E. Proof of Corollary 3

Proof. We make this proof in the context of homoskedastic dividends: $\sigma_i^2 = \sigma_\epsilon^2$. We can write the actual excess returns as:

$$\tilde{R}_i^e = \begin{cases} \beta_i \dfrac{\sigma_z^2 + \frac{\sigma_\epsilon^2}{N}}{\gamma} + \tilde{\eta}^i & \text{for } i < \bar{i} \\ \beta_i \dfrac{\sigma_z^2 + \frac{\sigma_\epsilon^2}{N}}{\gamma}\left(1 - \theta\omega(\lambda)\right) + \dfrac{\sigma_\epsilon^2}{\gamma N}\theta(1+\omega(\lambda)) + \tilde{\eta}^i & \text{for } i \geq \bar{i} \end{cases}$$

where $\tilde{\eta}_i = b_i\tilde{z} + \tilde{\epsilon}_i$.

Using the fact that by definition, $\sum_{i=1}^N b_i = \sum_{i=1}^N \beta_i = N$, a cross-sectional regression of realized excess returns per share $\left(\tilde{R}_i^e\right)_{i\in[1,N]}$ on $(\beta_i)_{i\in[1,N]}$ and a constant would deliver the following coefficient estimate:

$$\begin{aligned} \hat{\mu} &= \frac{\sum_{i=1}^N \beta_i\tilde{R}_i - \sum_{i=1}^N \tilde{R}_i}{\sum_{i=1}^N \beta_i^2 - N} \\ &= \frac{\sigma_z^2 + \frac{\sigma_\epsilon^2}{N}}{\gamma}\left(1 + \frac{\gamma}{\sigma_z^2}\tilde{z} - \left(\frac{\sum_{i\geq\bar{i}}\beta_i^2 - \sum_{i\geq\bar{i}}\beta_i}{\sum_{i=1}^N \beta_i^2 - N}\right)\theta\omega(\lambda)\right) + \frac{\sum_{i\geq\bar{i}}(\beta_i - 1)}{\sum_{i=1}^N \beta_i^2 - N}\frac{\sigma_\epsilon^2}{\gamma N}\theta\left(1+\omega(\lambda)\right) \end{aligned}$$

Let $\frac{u_{\bar{i}-1}}{\gamma} > \lambda_1 > \lambda_2 > \frac{u_{\bar{i}}}{\gamma}$. Call $\bar{i}_1$ ($\bar{i}_2$) the threshold associated with disagreement λ_1 (resp. λ_2). We have that $\bar{i}_1 = \bar{i_2} = \bar{i}$. Then:

$$\hat{\mu}(\lambda_1) - \hat{\mu}(\lambda_2) = -\frac{1}{\gamma}\frac{\theta\left(\omega(\lambda_1) - \omega(\lambda_2)\right)}{\sum_{i=1}^N \beta_i^2 - N}\left(\sigma_z^2\left(\sum_{i\geq\bar{i}}^N \beta_i^2 - \sum_{i\geq\bar{i}}^N \beta_i\right) + \frac{\sigma_\epsilon^2}{N}\sum_{i\geq\bar{i}}^N (\beta_i - 1)^2\right)$$

We show that $\sum_{i\geq\bar{i}}\beta_i^2 \geq \sum_{i\geq\bar{i}}\beta_i$. Since the average β is one, we can write β_i as: $\beta_i = 1 + y_i$ with y_i such that $\sum y_i = 0$. Using this decomposition, we have that:

$$\sum_{i=1}^N \beta_i^2 = N + 2\underbrace{\sum_{i=1}^N y_i}_{=0} + \sum_{i=1}^N y^2 > N = \sum_{i=1}^N \beta_i.$$

Thus, the relationship is true for $\bar{i} = 1$. Now assume it is true for $\bar{i} = k > 1$. We have: $\sum_{i \geq k+1} \beta_i^2 - \sum_{i \geq k+1} \beta_i = \sum_{i \geq k} \beta_i^2 - \sum_{i \geq k} \beta_i + \beta_k - \beta_k^2$. Either $\beta_k > 1$ in which case it is evident that $\sum_{i \geq k+1} \beta_i^2 - \sum_{i \geq k+1} \beta_i > 0$ as $\beta_k > 1$ implies that $\beta_i > 1$ for $i \geq k$. Or $\beta_k \leq 1$ in which case $\beta_k - \beta_k^2 > 0$ and using the recurrence assumption, $\sum_{i \geq k+1} \beta_i^2 - \sum_{i \geq k+1} \beta_i > 0$. Thus, this proves that: $\hat{\mu}(\lambda_1) - \hat{\mu}(\lambda_2) < 0$.

We show now that for all $i \in [1, N]$, $\hat{\mu}(\lambda)$ is continuous at u_i where u_i is the sequence defined in Theorem 1 and defined by $b_i \sigma_z^2 \omega(u_i) = \frac{\sigma_\epsilon^2}{N}$. When $\lambda = u_i^+$, we have $\bar{i} = i$. When $\lambda = u_i^-$, we have $\bar{i} = i + 1$. First, notice that $\omega(\lambda)$ is continuous at u_i and:

$$\omega(u_i^-) = \omega(u_i^+) = \omega(u_i) = \frac{\sigma_\epsilon^2}{\sigma_z^2} \frac{1}{N\gamma} \frac{1}{b_i}$$

Thus:

$$\begin{aligned}
\hat{\mu}\left(u_i^+\right) - \hat{\mu}\left(u_i^-\right) &= -\frac{\theta}{\gamma} \frac{\beta_i - 1}{\sum_{k=1}^N \beta_k^2 - N} \left(-\beta_k \omega(u_k) \left(\sigma_z^2 + \frac{\sigma_\epsilon^2}{N} \right) + \frac{\sigma_\epsilon^2}{N} \left(1 + \omega(u_k)\right) \right) \\
&= -\frac{\theta}{\gamma} \frac{\beta_i - 1}{\sum_{k=1}^N \beta_k^2 - N} \left(-b_i \sigma_z^2 \omega(u_i) + \frac{\sigma_\epsilon^2}{N} \right) = 0 \quad \text{by definition of } u_i.
\end{aligned}$$

Thus $\hat{\mu}$ is continuous and strictly decreasing for λ in $]u_{i+1}, u_i[$, and it is continuous at $\lambda = u_i$, so that it is overall strictly decreasing in aggregate disagreement λ. Since the derivative of the slope of the security market line w.r.t. λ is linear in θ, it is trivial that $\frac{\partial^2 \hat{\mu}}{\partial \lambda \partial \theta} < 0$, that is, the negative effect of λ on the slope of the security market line is stronger when there is a larger fraction of short-sales constrained agents, that is, when θ is larger.

We can show that the slope of the security market line, $\hat{\mu}$, is strictly decreasing with θ, the fraction of short-sales constrained investors in the model. Since the marginal asset $\bar{i}$ is independent of θ and since we have already shown that: $\sum_{i \geq \bar{i}} \beta_i^2 - \sum_{i \geq \bar{i}} \beta_i$, we directly have that:

$$\frac{\partial \hat{\mu}}{\partial \theta} = -\frac{\omega(\lambda)}{\gamma \left(\sum_{i=1}^N \beta_i^2 - N \right)} \left(\frac{\sigma_\epsilon^2}{N} \sum_{i \geq \bar{i}} (\beta_i - 1)^2 + \sigma_z^2 \left(\sum_{i \geq \bar{i}} \beta_i^2 - \sum_{i \geq \bar{i}} \beta_i \right) \right) < 0$$

□

F. Proof of Theorem 2

Proof. We first consider the case where $\tilde{\lambda}_t = 0$. There is no disagreement among investors so all investors are long all assets $i \in [1, N]$. There is thus a unique first order-condition for all investors' type – for all $j \in [1, N]$ and $k = a$, A or B:

$$d - (1+r) P_t^j(0) + \mathbb{E}_t[P_{t+1}^j(\tilde{\lambda}_{t+1}) | \tilde{\lambda}_t = 0] = \frac{1}{\gamma} \left(b_j \sigma_z^2 \sum_{i \leq N} \mu_i^k(0) b_i + \mu_j^k(0) \sigma_j^2 + \rho(1-\rho) \Delta P_{t+1}^j \left(\sum_{i \leq N} \mu_i^k(0) \left(\Delta P_{t+1}^i \right) \right) \right)$$

Summing up this equation across investors' types, using the market clearing condition and dropping the time subscript leads to:[29]

$$d - (1+r) P^j(0) + \mathbb{E}_t[P^j(\tilde{\lambda}_{+1}) | \tilde{\lambda}_t = 0] = \frac{1}{\gamma} \left(b_j \sigma_z^2 + \frac{\sigma_j^2}{N} + \rho(1-\rho) \Delta P^j \left(\sum_{i \leq N} \frac{(\Delta P^i)}{N} \right) \right) \tag{10}$$

Consider now the case where $\lambda_t = \lambda$. Importantly, note that investors disagree on the expected value of the aggregate factor $\tilde{z}_{t+1}$, but they agree on the expected value of asset i's resale price $\mathbb{E}_t[P^j(\tilde{\lambda}_{t+1})]$. This is because investors agree to disagree, so they recognize the existence in the next generation of investors with heterogeneous beliefs – and in particular with beliefs different from theirs. However, they nevertheless evaluate the $t+1$ expected dividend stream differently. We proceed as in the static model. We assume there is a marginal asset $\bar{i}$, such that there are no binding short-sales constraints for assets $j < \bar{i}$ and strictly binding short-sales constraints for assets $j \geq \bar{i}$. We check ex post the conditions under which this is indeed an equilibrium. Under the proposed equilibrium structure, the first-order condition of the three groups of investors born at date t for assets $j < \bar{i}$ is easily written since, in the proposed equilibrium structure, these assets do not experience binding short-sales constraints:

$$\begin{gathered} d + b_j \lambda_t^k - (1+r)P^j(\lambda) + \mathbb{E}_t[P^j(\tilde{\lambda}_{t+1})|\tilde{\lambda}_t = \lambda] = \\ \frac{1}{\gamma}\left(b_j\sigma_z^2\left(\sum_{i\leq N}\mu_i^k(\lambda)b_i\right) + \mu_j^k(\lambda)\sigma_j^2 + \rho(1-\rho)\Delta P^j\left(\sum_{i\leq N}\mu_i^k(\lambda)\left(\Delta P^i\right)\right)\right) \end{gathered}$$

Summing up across investors types (using the weight of each investors' group) and using the market clearing condition leads to:

$$\forall j < \bar{i}, \quad d - (1+r)P^j(\lambda) + \mathbb{E}_t[P^j(\tilde{\lambda}_{+1})|\tilde{\lambda}_t = \lambda] = \frac{1}{\gamma}\left(b_j\sigma_z^2 + \frac{\sigma_j^2}{N} + \rho(1-\rho)\Delta P^j\sum_{i\leq N}\frac{\Delta P^i}{N}\right) \tag{11}$$

Subtracting equation (10) –prices in the low disagreement state– from equation (11) leads to:

$$\forall j < \bar{i}, \quad -(1+r)\Delta P^j + \rho\Delta P^j - (1-\rho)\Delta P^j = 0 \Leftrightarrow P^j(\lambda) = P^j(0),$$

since $\rho < 1$.

Thus, for all $j < \bar{i}$, $\Delta P^j = 0$. The payoff of assets below $\bar{j}$ is not sufficiently exposed to aggregate disagreement to make pessimist investors willing to go short. Hence, even in the high disagreement state, these assets experience no mispricing and in particular, their price is independent of the realization of aggregate disagreement. Aggregate disagreement thus only creates resale price risk on these assets that experience binding short-sales constraint in the high aggregate disagreement states, that is, the high $\frac{b}{\sigma^2}$ assets with $i \geq \bar{i}$.

We now turn to the assets with binding short-sales constraints in the high disagreement states, that is, assets $j > \bar{i}$. For these assets, we know that under the proposed equilibrium $\mu_j^B(\lambda) = 0$ and we have the following first-order conditions for HF and optimist MFs respectively:

$$\begin{cases} d + b_j\lambda - (1+r)P^j(\lambda) + \mathbb{E}_t[P^j(\tilde{\lambda}_{t+1})|\tilde{\lambda} = \lambda] = \dfrac{1}{\gamma}\left(b_j\sigma_z^2\displaystyle\sum_{i\leq N}\mu_i^A(\lambda)b_i + \mu_j^A(\lambda)\sigma_j^2 + \rho(1-\rho)\Delta P_{t+1}^j\left(\sum_{i\leq N}\mu_i^A(\lambda)\Delta P_{t+1}^i\right)\right) \\ d - (1+r)P^j(\lambda) + \mathbb{E}_t[P^j(\tilde{\lambda}_{t+1})|\tilde{\lambda} = \lambda] = \dfrac{1}{\gamma}\left(b_j\sigma_z^2\displaystyle\sum_{i\leq N}\mu_i^a(\lambda)b_i + \mu_j^a(\lambda)\sigma_j^2 + \rho(1-\rho)\Delta P_{t+1}^j\left(\sum_{i\leq N}\mu_i^a(\lambda)\Delta P_{t+1}^i\right)\right) \end{cases}$$

Define $\Gamma = \sum_{i\geq\bar{i}}\frac{\Delta P^i}{N}$, the average price difference between high and low aggregate disagreement states across all assets. Summing up these equations across investors' types (using the weight of each investors' group) and using the market clearing conditions lead to:

$$\tfrac{\alpha}{2}b_j\lambda + (1-\tfrac{\alpha}{2})\left(d-(1+r)P^j(\lambda)+\mathbb{E}_t[P^j(\tilde{\lambda}_{t+1})|\tilde{\lambda}_t=\lambda]\right) =$$

$$\tfrac{1}{\gamma}\left(b_j\sigma_z^2+\tfrac{\sigma_j^2}{N}+\rho(1-\rho)\Delta P^j\Gamma-\tfrac{\alpha}{2}b_j\sigma_z^2\underbrace{\sum_{i<\bar{i}}\mu_i^B(\lambda)b_i}_{S^1}-\tfrac{\alpha}{2}\rho(1-\rho)\Delta P^j\underbrace{\sum_{i<\bar{i}}\mu_i^B(\lambda)\left(\Delta P^i\right)}_{S^2=0}\right)$$

In the previous equation, $S^2=0$ since for all $i<\bar{i}$, $\Delta P^i=0$. To recover S^1, we use B-investors' first-order condition on assets $j<\bar{i}$, the equilibrium prices derived above for assets $j<\bar{i}$ and the fact that for all $i<\bar{j}$, $\Delta P^j=0$. This leads to the following equation :

$$\forall j<\bar{i},\quad b_j\sigma_z^2\underbrace{\sum_{i\leq N}\mu_i^B b_i}_{S^1}+\mu_j^B\sigma_j^2=-\lambda\gamma b_j+b_j\sigma_z^2+\frac{\sigma_j^2}{N}$$

Multiplying the previous expression by b_j, dividing by σ_j^2, and summing up the equations over j gives the following formula for S^1:

$$S^1=1-\frac{\left(\sum_{i\geq\bar{i}}\frac{b_i}{N}\right)+\lambda\gamma\left(\sum_{i<\bar{i}}\frac{b_i^2}{\sigma_i^2}\right)}{1+\sigma_z^2\left(\sum_{i<\bar{i}}\frac{b_i^2}{\sigma_i^2}\right)}$$

This allows us to derive the excess return on assets $j\geq\bar{i}$: å

$$\underbrace{d-(1+r)P^j(\lambda)+\mathbb{E}_t[P^j(\tilde{\lambda}_{t+1})|\tilde{\lambda}_t=\lambda]}_{\text{Excess Return}}=$$

$$\underbrace{\frac{1}{\gamma}\left(b_j\sigma_z^2+\frac{\sigma_j^2}{N}+(1+\theta)\rho(1-\rho)\Delta P^j\Gamma\right)}_{\text{Risk Premium}}-\underbrace{\frac{\theta}{\gamma}\left(b_j\frac{\lambda\gamma-\frac{\sigma_z^2}{N}\sum_{k\geq\bar{i}}b_k}{1+\sigma_z^2\left(\sum_{i<\bar{i}}\frac{b_i^2}{\sigma_i^2}\right)}-\frac{\sigma_j^2}{N}\right)}_{\text{speculative premium}=\pi^j}$$

Note that the risk premium embeds a term that reflects the resale price risk of high b assets. Subtracting equation (10) from the previous equation yields, for all $j\geq\bar{i}$:

$$-(1+r)\Delta P^j+(2\rho-1)\Delta P^j=-\pi^j+\frac{\theta\rho(1-\rho)}{\gamma}\Gamma\Delta P^j\Rightarrow\left((1+r)-(2\rho-1)+\frac{\theta\rho(1-\rho)}{\gamma}\Gamma\right)\Delta P^j=\pi^j \tag{12}$$

Remember that $\Gamma=\sum_{i\geq\bar{i}}\frac{\Delta P^i}{N}$. We can thus obtain a formula for Γ by adding up the previous equations for all $j\geq\bar{i}$ and dividing by N:

$$\left((1+r)-(2\rho-1)+\frac{\theta\rho(1-\rho)}{\gamma}\Gamma\right)\Gamma=\frac{1}{N}\sum_{j\geq\bar{i}}\pi^j$$

There is a unique $\Gamma^+>0$ which satisfies the previous equation, call it Γ^+:

$$\Gamma^+=\frac{-(1+r)+(2\rho-1)+\sqrt{\left((1+r)-(2\rho-1)\right)^2+\frac{4}{N}\frac{\theta\rho(1-\rho)}{\gamma}\sum_{j\geq\bar{i}}\pi^j}}{2\frac{\theta\rho(1-\rho)}{\gamma}}$$

There is also a unique $\Gamma^- < 0$ which satisfies equation 12:

$$\Gamma^- = \frac{-(1+r)+(2\rho-1)-\sqrt{((1+r)-(2\rho-1))^2+\frac{4}{N}\frac{\theta\rho(1-\rho)}{\gamma}\sum_{j\geq\bar{i}}\pi^j}}{2\frac{\theta\rho(1-\rho)}{\gamma}}$$

Let $\Gamma^\star$ be the actual value of Γ, the average price difference between high and low aggregate disagreement states across all assets. $\Gamma^\star \in \{\Gamma^-, \Gamma^+\}$. For $j \geq \bar{i}$, the price difference is simply expressed as a function of the speculative premium π^j and $\Gamma^\star$:

$$\Delta P^j = \frac{\pi^j}{1+r-(2\rho-1)+\frac{\theta\rho(1-\rho)}{\gamma}\Gamma^\star}.$$

For the equilibrium to exist, it needs to be that for each asset $j \geq \bar{i}$, the pessimists do not want to hold asset j, that is, the marginal utility of holding assets $j \geq \bar{i}$ at the optimal holding is 0. This is equivalent to:

$$\forall j \geq \bar{i} \quad d - b_j\lambda - (1+r)P^j(\lambda) + \rho P^j(\lambda) + (1-\rho)P^j(0) - \frac{1}{\gamma}b_j\sigma_z^2 \underbrace{\sum_{j<\bar{i}}\mu_i^B b_i}_{=S^1} < 0$$

We have:

$$\begin{aligned}
& d - b_j\lambda - (1+r)P^j(\lambda) + \rho P^j(\lambda) + (1-\rho)P^j(0) - \frac{1}{\gamma}b_j\sigma_z^2 S^1 \\
= \quad & -b_j\lambda + \frac{1}{\gamma}\left(b_j\sigma_z^2 + \frac{\sigma_j^2}{N} + \rho(1-\rho)(1+\theta)\Delta P^j\Gamma^\star\right) - \pi^j - \frac{1}{\gamma}b_j\sigma_z^2 S^1 \\
= \quad & -\frac{\pi^j}{\theta} - \pi^j + (1+\theta)\frac{\rho(1-\rho)}{\gamma}\Gamma^\star\Delta P^j \\
= \quad & \frac{1+\theta}{\theta}\pi^j\left(\frac{\frac{\theta\rho(1-\rho)}{\gamma}\Gamma^\star}{(1+r)-(2\rho-1)+\frac{\theta\rho(1-\rho)}{\gamma}\Gamma^\star} - 1\right) \\
= \quad & -\frac{1+\theta}{\theta}\frac{(1+r)-(2\rho-1)}{(1+r)-(2\rho-1)+\frac{\theta\rho(1-\rho)}{\gamma}\Gamma^\star} \times \pi^j
\end{aligned}$$

Assume that $\Gamma^\star = \Gamma^- < 0$. We know that:

$$\theta\rho(1-\rho)\frac{\Gamma^-}{\gamma} + (1+r) - (2\rho-1) = \frac{(1+r)-(2\rho-1)-\sqrt{((1+r)-(2\rho-1))^2+\frac{4}{N}\frac{\theta\rho(1-\rho)}{\gamma}\sum_{j\geq\bar{i}}\pi^j}}{2\frac{\theta\rho(1-\rho)}{\gamma}} < 0$$

Thus, if $\Gamma^\star = \Gamma^-$, then $-\frac{1+\theta}{\theta}\frac{(1+r)-(2\rho-1)}{(1+r)-(2\rho-1)+\frac{\theta\rho(1-\rho)}{\gamma}\Gamma} > 0$ so that it has to be that for all $j \geq \bar{i}$, $\pi^j < 0$. Thus: $\sum_{j\geq\bar{i}}\pi^j < 0$, so that

$$\left((1+r)-(2\rho-1)+\frac{\theta\rho(1-\rho)}{\gamma}\Gamma^-\right)\Gamma^- < 0$$

However, the previous expression is strictly positive since $\Gamma^- < 0$ and $(1+r)-(2\rho-1)+\frac{\theta\rho(1-\rho)}{\gamma}\Gamma^- < 0$ as well. Thus, we can't have $\Gamma^\star = \Gamma^-$ and it has to be that $\Gamma^\star = \Gamma^+$.

Since $\Gamma^\star > 0$, we have from the previous equilibrium condition that necessarily, for all $j \geq \bar{i}$, $\pi^j > 0$. Similarly, it is direct to show that for pessimists to have strictly positive holdings of assets $\bar{j}-1$, a necessary and sufficient condition is that $\pi^{\bar{j}-1} < 0$. Overall, this leads to the following equilibrium condition:

$$\frac{\sigma_z^2}{N}\left(\sum_{k\geq\bar{i}} b_k\right)+\frac{1}{N}\frac{\sigma_{\bar{j}-1}^2}{b_{\bar{j}-1}}\left(1+\sigma_z^2\sum_{k<\bar{i}}\frac{b_k^2}{\sigma_k^2}\right)\geq\lambda\gamma\geq\frac{\sigma_z^2}{N}\left(\sum_{k\geq\bar{i}} b_k\right)+\frac{1}{N}\frac{\sigma_{\bar{j}}^2}{b_{\bar{j}}}\left(1+\sigma_z^2\sum_{k<\bar{i}}\frac{b_k^2}{\sigma_k^2}\right)$$

We can define a sequence v_i, analogous to the sequence u_i defined in Theorem 1 as:

$$\forall i\in[1,N],\quad v_i=\frac{\sigma_z^2}{N}\left(\sum_{k\geq i} b_k\right)+\frac{1}{N}\frac{\sigma_i^2}{b_i}\left(1+\sigma_z^2\sum_{k<i}\frac{b_k^2}{\sigma_k^2}\right),\quad v_{N+1}=0\ \text{ and }\ v_0=+\infty$$

It is easily shown that this sequence is strictly decreasing since, for all $i\in[2,N]$:

$$v_i-v_{i-1}=\frac{1}{N}\left(\frac{\sigma_i^2}{b_i}-\frac{\sigma_{i-1}^2}{b_{i-1}}\right)\left(1+\sigma_z^2\sum_{k<\bar{i}}\frac{b_k^2}{\sigma_k^2}\right),$$

and assets are ranked in ascending order of $\frac{b_i}{\sigma_i^2}$.

The equilibrium condition can thus simply be written as $v_{\bar{i}-1}\geq\lambda\gamma\geq v_{\bar{i}}$ and $\bar{i}$ is thus defined as the smallest $i\in[1,N]$ such that $\lambda\gamma\geq v_i$.

We now move on to the expression for expected excess returns. Since $\Delta P^j=0$ for $j<\bar{i}$, we have that for all $j<\bar{i}$:

$$\mathbb{E}[R^j(\lambda)]=\mathbb{E}[R^j(0)]=d-rP^j(\lambda)=d-rP^j(0)=\frac{1}{\gamma}\left(b_j\sigma_z^2+\frac{\sigma_j^2}{N}\right)$$

For $j\geq\bar{i}$, however:

$$\begin{aligned}\mathbb{E}[R^j(0)] &= d-(1+r)P^j(0)+\rho P^j(0)+(1-\rho)P^j(\lambda)\\ &= \frac{1}{\gamma}\left(b_j\sigma_z^2+\frac{\sigma_j^2}{N}+\rho(1-\rho)\frac{\Gamma^\star}{(1+r)-(2\rho-1)+\frac{\theta\rho(1-\rho)}{\gamma}\Gamma^\star}\pi^j\right)\end{aligned}$$

The extra-term is the risk-premium required by investors for holding stocks which are sensitive to disagreement and are thus exposed to changes in prices coming from changes in the aggregate disagreement state variable. Of course, in the data, since ρ is very close to 1, this risk premium is going to be quantitatively small. Nevertheless, the intuition here is that high $\frac{b}{\sigma^2}$ stocks have low prices in the low disagreement states for two reasons: (1) they are exposed to aggregate risk $\tilde{z}$ (2) they are exposed to changes in aggregate disagreement $\tilde{\lambda}$. And finally:

$$\begin{aligned}\mathbb{E}[R^j(\lambda)] &= d-(1+r)P^j(\lambda)+\rho P^j(\lambda)+(1-\rho)P^j(0)\\ &= \frac{1}{\gamma}\left(b_j\sigma_z^2+\frac{\sigma_j^2}{N}+\rho(1-\rho)\frac{\Gamma^\star}{(1+r)-(2\rho-1)+\frac{\theta\rho(1-\rho)}{\gamma}\Gamma^\star}\pi^j\right)-\pi^j\\ &\quad+\theta\frac{\rho(1-\rho)}{\gamma}\frac{\Gamma^\star}{(1+r)-(2\rho-1)+\frac{\theta\rho(1-\rho)}{\gamma}\Gamma^\star}\pi^j\\ &= \frac{1}{\gamma}\left(b_j\sigma_z^2+\frac{\sigma_j^2}{N}+\rho(1-\rho)\frac{\Gamma^\star}{(1+r)-(2\rho-1)+\frac{\theta\rho(1-\rho)}{\gamma}\Gamma^\star}\pi^j\right)-\frac{1+r-(2\rho-1)}{(1+r)-(2\rho-1)+\frac{\theta\rho(1-\rho)}{\gamma}\Gamma^\star}\pi^j\end{aligned}$$

Thus, for assets $j\geq\bar{i}$, the expected return is strictly lower in high disagreement states than in low disagreement states.

The proof for the unicity of the equilibrium is similar to the proof of Theorem 1 and is thus omitted. □

G. Proof of Corrolary 4

Proof. Part (i) is a direct consequence of the formula for expected excess returns in Theorem 2. For (ii), we do a Taylor expansion around $\rho = 1$ for $\Gamma^\star$: $\Gamma^\star \approx \frac{1}{r}\sum_{j\geq \bar{i}} \frac{\pi^j}{N} > 0$, so that in the vicinity of $\rho = 1$ and for $j \geq \bar{i}$,

$$\mathbb{E}[R^j(\lambda)] \approx \frac{1}{\gamma}\left(b_j\sigma_z^2 + \frac{\sigma_j^2}{N}\right) - \frac{1 + r - (2\rho - 1)}{(1 + r) - (2\rho - 1) + \frac{\theta\rho(1-\rho)}{\gamma}\Gamma^\star}\pi^j$$

The slope of the security market line for assets $i < \bar{i}$ (expressed as a function of b_i – it would be equivalent as a function of β_i) is thus strictly lower for $i < \bar{i}$ than for $i \geq \bar{i}$ in the vicinity of $\rho = 1$, which proves (ii). (iii) can also be seen directly from the previous Taylor expansion and making λ grows to infinity. (iv) is also a direct consequence of the formula for expected excess returns in Theorem 2. $\square$

B. Additional Tables

Table AI. Disagreement and Concavity of the Security Market Line: Monthly βs

Note: Sample Period: 12/1981-12/2014. Sample: CRSP stock file excluding penny stocks (price < \$5) and microcaps (stocks in bottom 2 deciles of the monthly size distribution using NYSE breakpoints). At the beginning of each calendar month, stocks are ranked in ascending order on the basis of their estimated beta at the end of the previous month. Pre-formation betas are estimated with a market model using monthly returns over the past 3 calendar years. The ranked stocks are assigned to one of 20 value-weighted (panel A) or equal-weighted (panel B) portfolios based on NYSE breakpoints. We compute the full sample beta of these 20-beta sorted portfolios using the same market model. We then estimate every month the cross-sectional regression:

$$r_{P,t}^{(12)} = \kappa_t + \pi_t \times \beta_P + \phi_t \times (\beta_P)^2 + \epsilon_{P,t}, \quad \text{where} \quad P = 1, ..., 20$$

and $r_{P,t}^{(12)}$ is the 12-months excess return of the P^{th} beta-sorted portfolio and β_P is the full sample post-ranking beta of the P^{th} beta-sorted portfolio. We then estimate second-stage regressions in the time-series using OLS and Newey-West adjusted standard errors allowing for 11 lags:

$$\begin{cases} \phi_t = & c_1 + \psi_1 \cdot \text{Agg. Disp.}_{t-1} + \delta_1^m \cdot R_{m,t}^{(12)} + \delta_1^{HML} \cdot HML_t^{(12)} + \delta_1^{SMB} \cdot SMB_t^{(12)} + \delta_1^{UMD} \cdot UMD_t^{(12)} + \sum_{x \in X} \delta_1^x \cdot x_{t-1} + \zeta_t \\ \pi_t = & c_2 + \psi_2 \cdot \text{Agg. Disp.}_{t-1} + \delta_2^m \cdot R_{m,t}^{(12)} + \delta_2^{HML} \cdot HML_t^{(12)} + \delta_2^{SMB} \cdot SMB_t^{(12)} + \delta_2^{UMD} \cdot UMD_t^{(12)} + \sum_{x \in X} \delta_2^x \cdot x_{t-1} + \omega_t \\ \kappa_t = & c_3 + \psi_3 \cdot \text{Agg. Disp.}_{t-1} + \delta_3^m \cdot R_{m,t}^{(12)} + \delta_3^{HML} \cdot HML_t^{(12)} + \delta_3^{SMB} \cdot SMB_t^{(12)} + \delta_3^{UMD} \cdot UMD_t^{(12)} + \sum_{x \in X} \delta_3^x x_{t-1} + \nu_t \end{cases}$$

Column (1) and (5) controls for Agg. Disp.$_{t-1}$, the monthly β-weighted average of stock-level disagreement, which is measured as the standard deviation of analyst forecasts on stocks' long run growth of EPS. Column (2) and (6) add controls for the 12-months excess return from t to $t+11$ of the market ($R_{m,t}^{(12)}$), HML ($HML_t^{(12)}$), SMB ($SMB_t^{(12)}$), and UMD ($UMD_t^{(12)}$). Column (3) and (7) add controls for the aggregate Dividend/Price ratio in $t-1$ and the past-12 months inflation rate in t_1. Column (4) and (8) additionally control for the TED spread in month $t-1$. T-statistics are in parenthesis. *, **, and *** means statistically different from zero at 10, 5 and 1% level of significance.

Table AI (Continued):

Dep. Var:		ϕ_t				π_t				κ_t		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Panel A: Value-Weighted Portfolios												
Agg. Disp.$_{t-1}$	-3.7	-7.1**	-12***	-12***	3.2	14**	18**	17**	-2.8	-5.5*	-4.4	-4.1
	(-1.3)	(-2.5)	(-3.6)	(-3.6)	(.66)	(2.3)	(2.4)	(2.4)	(-1.1)	(-1.7)	(-1.2)	(-1.1)
$R^{(12)}_{m,t}$		-.32**	-.34**	-.34**		1.2***	1.2***	1.2***		.12	.17	.15
		(-2.3)	(-2.4)	(-2.4)		(4.6)	(4.2)	(4.3)		(.8)	(1.2)	(1)
$HML^{(12)}_t$		-.34*	-.23	-.23		-.18	-.28	-.23		.68***	.66***	.63***
		(-1.8)	(-1.4)	(-1.4)		(-.5)	(-.77)	(-.61)		(3.2)	(3)	(2.7)
$SMB^{(12)}_t$		.98***	1.2***	1.2***		-1.4**	-1.6***	-1.6***		.43	.36	.38
		(3)	(4.6)	(4.6)		(-2.4)	(-2.7)	(-2.8)		(1.3)	(1)	(1.2)
$UMD^{(12)}_t$		.2	.23*	.23		-.64**	-.65**	-.62**		.4**	.38**	.36**
		(1.3)	(1.7)	(1.6)		(-2.2)	(-2.3)	(-2.2)		(2.5)	(2.4)	(2.4)
D/P_{t-1}			-4.5	-4.2			5.4	2.4			-.8	1.3
			(-1.2)	(-1.1)			(.71)	(.3)			(-.21)	(.3)
Inflation$_{t-1}$			-4.5*	-4.3*			2.1	.29			2.8	4.1
			(-1.9)	(-1.8)			(.43)	(.057)			(1.2)	(1.4)
Ted Spread$_{t-1}$				-.62				5.8				-4.1
				(-.24)				(1.1)				(-1.2)
Constant	-4.9	-3.4	-4.2*	-4.1	11*	6.9	7.8	7.2	3.9	-3.3	-3.4	-2.9
	(-1.6)	(-1.5)	(-1.7)	(-1.6)	(1.7)	(1.4)	(1.5)	(1.3)	(1.2)	(-1.1)	(-1.2)	(-.98)
Panel B: Equal-Weighted Portfolios												
Agg. Disp.$_{t-1}$	-5.8**	-6***	-9.5***	-9.4***	8.1**	10**	12**	12**	-2.4	-2.5	-.13	.25
	(-2.3)	(-2.8)	(-3.5)	(-3.5)	(2.1)	(2.5)	(2.3)	(2.3)	(-1.5)	(-1.3)	(-.055)	(.11)
$R^{(12)}_{m,t}$		-.27**	-.38***	-.39***		1.1***	1.3***	1.4***		.17	.12	.082
		(-2)	(-3)	(-3.1)		(5.1)	(5.5)	(5.7)		(1.6)	(1)	(.75)
$HML^{(12)}_t$		-.78***	-.71***	-.73***		.92***	.9***	.97***		.24	.17	.12
		(-4.3)	(-5.3)	(-5.6)		(2.8)	(3.2)	(3.7)		(1.6)	(1.3)	(1)
$SMB^{(12)}_t$		.47	.67**	.69**		-.067	-.26	-.32		.15	.085	.12
		(1.5)	(2.5)	(2.6)		(-.14)	(-.57)	(-.69)		(.97)	(.48)	(.75)
$UMD^{(12)}_t$		.0072	.065	.053		-.27	-.35	-.3		.26**	.27**	.24***
		(.051)	(.58)	(.45)		(-1.2)	(-1.6)	(-1.4)		(2.5)	(2.4)	(2.6)
D/P_{t-1}			.44	1.7			-5.3	-10*			4.8*	7.9***
			(.14)	(.54)			(-.9)	(-1.8)			(1.9)	(3.5)
Inflation$_{t-1}$			-6.9***	-6.1**			8.8*	6			-.35	1.5
			(-2.6)	(-2.3)			(1.8)	(1.3)			(-.18)	(.73)
Ted Spread$_{t-1}$				-2.4				9.1**				-6.1***
				(-1.2)				(2.3)				(-3.4)
Constant	-9.7***	-4.9**	-4.9**	-4.6*	21***	9**	8.2*	7.2	-.18	-4.7**	-3.9*	-3.2
	(-3.7)	(-2.4)	(-2.1)	(-1.9)	(4.4)	(2)	(1.7)	(1.5)	(-.083)	(-2.1)	(-1.7)	(-1.4)
N	385	385	385	385	385	385	385	385	385	385	385	385

Table AII. Disagreement and Concavity of the Security Market Line: Different horizons

Note: Sample Period: 12/1981-12/2014. Sample: CRSP stock file excluding penny stocks (price < \$5) and microcaps (stocks in bottom 2 deciles of the monthly size distribution using NYSE breakpoints). At the beginning of each calendar month, stocks are ranked in ascending order on the basis of their estimated beta at the end of the previous month. Pre-formation betas are estimated with a market model using daily returns over the past calendar year and 5 lags of the market returns. The ranked stocks are assigned to one of 20 value-weighted (panel A) or equal-weighted (panel B) portfolios based on NYSE breakpoints. We compute the full sample beta of these 20-beta sorted portfolios using the same market model. We then estimate every month the cross-sectional regression:

$$r_{P,t}^{(k)} = \kappa_t^{(k)} + \pi_t^{(k)} \times \beta_P + \phi_t^{(k)} \times (\beta_P)^2 + \epsilon_{P,t}^{(k)}, \quad \text{where} \quad P = 1, ..., 20$$

and $r_{P,t}^{(12)}$ is the 12-months excess return of the P^{th} beta-sorted portfolio and β_P is the full sample post-ranking beta of the P^{th} beta-sorted portfolio. We then estimate second-stage regressions in the time-series using OLS and Newey-West adjusted standard errors allowing for 11 lags:

$$\begin{cases} \phi_t^{(k)} = & c_1 + \psi_1 \cdot \text{Agg. Disp.}_{t-1} + \sum_{z \in Z} \delta_1^z \cdot z_t^{(k)} + \sum_{x \in X} \delta_1^x \cdot x_{t-1} + \zeta_t \\ \pi_t^{(k)} = & c_2 + \psi_2 \cdot \text{Agg. Disp.}_{t-1} + \sum_{z \in Z} \delta_1^z \cdot z_t^{(k)} + \sum_{x \in X} \delta_2^x \cdot x_{t-1} + \omega_t \\ \kappa_t^{(k)} = & c_3 + \psi_3 \cdot \text{Agg. Disp.}_{t-1} + \sum_{z \in Z} \delta_3^z \cdot z_t^{(k)} + \sum_{x \in X} \delta_3^x x_{t-1} + \nu_t \end{cases}$$

Panel A use k=1 months, Panel B uses k=3 months, Panel C uses k=6 months, Panel D uses k=18 months. Column (1) and (5) controls for Agg. Disp.$_{t-1}$, the monthly β-weighted average of stock level disagreement measured as the standard deviation of analyst forecasts on stocks' long run growth of EPS. Column (2) and (6) add the factor $z \in Z$, where Z contains the k-months excess market return from t to $t+k-1$ and the k-months return on HML, SMB, and UMD from t to $t+k-1$; Column (3) and (7) add controls for the aggregate Dividend/Price ratio in $t-1$ and the past-12 months inflation rate in t_1; Column (4) and (8) additionally control for the TED spread in month $t-1$. T-statistics are in parenthesis. *, **, and *** means statistically different from zero at 10, 5 and 1% level of significance.

Dep. Var:	$\phi_t^{(k)}$				$\pi_t^{(k)}$				$\kappa_t^{(k)}$			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Panel A: k=1 months												
Agg. Disp.$_{t-1}$	-.21	-.31	-.57	-.66*	-.0077	.53	.89	.96	.042	-.075	-.16	-.15
	(-.38)	(-.93)	(-1.6)	(-1.8)	(-.0082)	(.75)	(1.2)	(1.2)	(.11)	(-.21)	(-.42)	(-.38)
Panel A: k=3 months												
Agg. Disp.$_{t-1}$	-.98	-.99	-1.9**	-2.1***	.48	1.6	2.8*	3*	-.02	-.076	-.3	-.32
	(-.81)	(-1.3)	(-2.3)	(-2.6)	(.24)	(1.1)	(1.7)	(1.8)	(-.025)	(-.1)	(-.36)	(-.37)
Panel A: k=6 months												
Agg. Disp.$_{t-1}$	-2.8	-2.5*	-4.4***	-4.7***	2.5	4.5	6.9**	6.9**	-1.1	-.92	-1.2	-1
	(-1.3)	(-1.7)	(-2.9)	(-3.1)	(.76)	(1.6)	(2.3)	(2.3)	(-.85)	(-.74)	(-.87)	(-.74)
Panel A: k=18 months												
Agg. Disp.$_{t-1}$	-6.4**	-7.4**	-12***	-12***	7.7*	12**	18***	17***	-4*	-3.2	-4	-3.5
	(-2)	(-2.3)	(-3.5)	(-3.4)	(1.7)	(2.1)	(2.7)	(2.7)	(-1.9)	(-1.3)	(-1.5)	(-1.4)

Table AIII. Disagreement and Concavity of the Security Market Line: Alternative measures of disagreement

Note: Sample Period: 12/1981-12/2014. Sample: CRSP stock file excluding penny stocks (price < \$5) and microcaps (stocks in bottom 2 deciles of the monthly size distribution using NYSE breakpoints). At the beginning of each calendar month, stocks are ranked in ascending order on the basis of their estimated beta at the end of the previous month. Pre-formation betas are estimated with a market model using daily returns over the past calendar year and 5 lags of the market returns. The ranked stocks are assigned to one of 20 value-weighted (panel A) or equal-weighted (panel B) portfolios based on NYSE breakpoints. We compute the full sample beta of these 20-beta sorted portfolios using the same market model. We then estimate every month the cross-sectional regression:

$$r_{P,t}^{(12)} = \kappa_t + \pi_t \times \beta_P + \phi_t \times (\beta_P)^2 + \epsilon_{P,t}, \quad \text{where} \quad P = 1, ..., 20$$

and $r_{P,t}^{(12)}$ is the 12-months excess return of the P^{th} beta-sorted portfolio and β_P is the full sample post-ranking beta of the P^{th} beta-sorted portfolio. We then estimate second-stage regressions in the time-series using OLS and Newey-West adjusted standard errors allowing for 11 lags:

$$\begin{cases} \phi_t = & c_1 + \psi_1 \cdot \text{Agg. Disp.}_{t-1} + \delta_1^m \cdot R_{m,t}^{(12)} + \delta_1^{HML} \cdot HML_t^{(12)} + \delta_1^{SMB} \cdot SMB_t^{(12)} + \delta_1^{UMD} \cdot UMD_t^{(12)} + \sum_{x \in X} \delta_1^x \cdot x_{t-1} + \zeta_t \\ \pi_t = & c_2 + \psi_2 \cdot \text{Agg. Disp.}_{t-1} + \delta_2^m \cdot R_{m,t}^{(12)} + \delta_2^{HML} \cdot HML_t^{(12)} + \delta_2^{SMB} \cdot SMB_t^{(12)} + \delta_2^{UMD} \cdot UMD_t^{(12)} + \sum_{x \in X} \delta_2^x \cdot x_{t-1} + \omega_t \\ \kappa_t = & c_3 + \psi_3 \cdot \text{Agg. Disp.}_{t-1} + \delta_3^m \cdot R_{m,t}^{(12)} + \delta_3^{HML} \cdot HML_t^{(12)} + \delta_3^{SMB} \cdot SMB_t^{(12)} + \delta_3^{UMD} \cdot UMD_t^{(12)} + \sum_{x \in X} \delta_3^x x_{t-1} + \nu_t \end{cases}$$

In Panel A, Agg. Disp. is the monthly β-weighted average of stock level disagreement measured as the standard deviation of analyst forecasts on stocks' long run growth of EPS, where the pre-ranking β have been compressed to 1 ($\beta^i = .5\hat{\beta} + .5$). In Panel B, Agg. Disp. is the monthly β-and-value weighted average of stock level disagreement measured as the standard deviation of analyst forecasts on stocks' long run growth of EPS. In Panel C, Agg. Disp. is the principal component of the monthly standard deviation of forecasts on GDP, IP, Corporate Profit and Unemployment rate in the Survey of Professional Forecasters (SPF) and is taken from Li and Li (2014). In Panel D, Agg. Disp is the "top-down" measure of market disagreement used in Yu (2010) and is measured as the standard deviation of analyst forecasts of annual S&P 500 earnings, scaled by the average forecast on S&P 500 earnings. Column (1) and (5) control only for Agg. Disp.$_{t-1}$. Column (2) and (6) add controls for the 12-months excess return from t to $t+11$ of the market ($R_{m,t}^{(12)}$), HML ($HML_t^{(12)}$), SMB ($SMB_t^{(12)}$), and UMD ($UMD_t^{(12)}$). Column (3) and (7) add controls for the aggregate Dividend/Price ratio in $t-1$ and the past-12 months inflation rate in t_1. Column (4) and (8) additionally control for the TED spread in month $t-1$. T-statistics are in parenthesis. *, **, and *** means statistically different from zero at 10, 5 and 1% level of significance.

Table AIII (Continued):

Dep. Var:	ϕ_t				π_t				κ_t			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Panel A: Compressed betas												
Agg. Disp.$_{t-1}$	-5.5*	-4.5	-8.7**	-8.6**	6.3	8.7*	14**	13**	-4*	-2.8	-2.9	-2.5
	(-1.7)	(-1.6)	(-2.5)	(-2.5)	(1.4)	(1.7)	(2.1)	(2)	(-1.9)	(-1.3)	(-1.1)	(-1)
Panel B: Value and beta-weighted												
Agg. Disp.$_{t-1}$	-5.3	-4.5	-6.9**	-6.8**	5.5	7.8	10*	9.3*	-3.4	-1.9	-1.5	-1
	(-1.6)	(-1.6)	(-2.4)	(-2.4)	(1.1)	(1.5)	(1.8)	(1.7)	(-1.5)	(-.85)	(-.63)	(-.45)
Panel C: SPF disagreement												
Agg. Disp.$_{t-1}$	-2.7	-2.4	-5.1*	-4.9*	6.4	2.5	10*	9.4	.9	-.1	-4.3	-3.9
	(-1.1)	(-1.1)	(-1.9)	(-1.7)	(1.3)	(.57)	(1.9)	(1.6)	(.49)	(-.05)	(-1.4)	(-1.2)
Panel D: Top-down measure												
Agg. Disp.$_{t-1}$	.17	.1	-3.6**	-3.7**	2.4	.27	5.7*	6.2*	-.11	-.1	-1.5	-1.7
	(.15)	(.076)	(-2.5)	(-2.5)	(1.2)	(.12)	(1.7)	(1.9)	(-.11)	(-.11)	(-.77)	(-.92)

Table AIV. Disagreement and Concavity of the Security Market Line: Controlling for Stock-Level Disagreement

Note: Sample Period: 12/1981-12/2014. Sample: CRSP stock file excluding penny stocks (price < $5) and microcaps (stocks in bottom 2 deciles of the monthly size distribution using NYSE breakpoints). At the beginning of each calendar month, stocks are ranked in ascending order on the basis of their estimated beta at the end of the previous month. Pre-formation betas are estimated with a market model using daily returns over the past calendar year and 5 lags of the market returns. The ranked stocks are assigned to one of 20 value-weighted (panel A) or equal-weighted (panel B) portfolios based on NYSE breakpoints. We compute the full sample beta of these 20-beta sorted portfolios using the same market model. We then estimate every month the cross-sectional regression:

$$r_{P,t}^{(12)} = \kappa_t + \pi_t \times \beta_P + \phi_t \times (\beta_P)^2 + \Omega_t \times \ln(\text{Disp}_{P,t}) + \epsilon_{P,t}, \quad \text{where} \quad P = 1, ..., 20$$

and $r_{P,t}^{(12)}$ is the 12-months excess return of the P^{th} beta-sorted portfolio, $\text{Disp}_{P,t}$ is the value-weighted average of the stock-level dispersion in analysts forecasts for stocks in portfolio P in month t and β_P is the full sample post-ranking beta of the P^{th} beta-sorted portfolio. We then estimate second-stage regressions in the time-series using OLS and Newey-West adjusted standard errors allowing for 11 lags:

$$\begin{cases} \phi_t = & c_1 + \psi_1 \cdot \text{Agg. Disp.}_{\cdot t-1} + \delta_1^m \cdot R_{m,t}^{(12)} + \delta_1^{HML} \cdot HML_t^{(12)} + \delta_1^{SMB} \cdot SMB_t^{(12)} + \delta_1^{UMD} \cdot UMD_t^{(12)} + \sum_{x \in X} \delta_1^x \cdot x_{t-1} + \zeta_t \\ \pi_t = & c_2 + \psi_2 \cdot \text{Agg. Disp.}_{\cdot t-1} + \delta_2^m \cdot R_{m,t}^{(12)} + \delta_2^{HML} \cdot HML_t^{(12)} + \delta_2^{SMB} \cdot SMB_t^{(12)} + \delta_2^{UMD} \cdot UMD_t^{(12)} + \sum_{x \in X} \delta_2^x \cdot x_{t-1} + \omega_t \\ \kappa_t = & c_3 + \psi_3 \cdot \text{Agg. Disp.}_{\cdot t-1} + \delta_3^m \cdot R_{m,t}^{(12)} + \delta_3^{HML} \cdot HML_t^{(12)} + \delta_3^{SMB} \cdot SMB_t^{(12)} + \delta_3^{UMD} \cdot UMD_t^{(12)} + \sum_{x \in X} \delta_3^x x_{t-1} + \nu_t \end{cases}$$

Column (1) and (5) controls for Agg. $\text{Disp.}_{\cdot t-1}$, the monthly β-weighted average of stock-level disagreement, which is measured as the standard deviation of analyst forecasts on stocks' long run growth of EPS. Column (2) and (6) add controls for the 12-months excess return from t to $t+11$ of the market ($R_{m,t}^{(12)}$), HML ($HML_t^{(12)}$), SMB ($SMB_t^{(12)}$), and UMD ($UMD_t^{(12)}$). Column (3) and (7) add controls for the aggregate Dividend/Price ratio in $t-1$ and the past-12 months inflation rate in t_1. Column (4) and (8) additionally control for the TED spread in month $t-1$. T-statistics are in parenthesis. *, **, and *** means statistically different from zero at 10, 5 and 1% level of significance.

Table AIV (Continued):

Dep. Var:	ϕ_t				π_t				κ_t			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Panel A: Value-Weighted Portfolios												
Agg. Disp.$_{t-1}$	-5.9*	-5.5*	-9.6***	-9.7***	6.4	11**	17***	17***	-3.2	-2.3	-1.4	-.51
	(-1.9)	(-1.9)	(-2.9)	(-3)	(1.5)	(2.2)	(2.7)	(2.7)	(-1.5)	(-.85)	(-.45)	(-.17)
$R^{(12)}_{m,t}$		-.16	-.22*	-.22		.99***	1.1***	1.1***		.29*	.26*	.25
		(-1.3)	(-1.7)	(-1.6)		(3.7)	(3.8)	(3.7)		(1.9)	(1.7)	(1.5)
$HML^{(12)}_t$		-.5**	-.38**	-.37**		.47	.3	.32		.31*	.28	.22
		(-2.6)	(-2.2)	(-2.2)		(1.2)	(.81)	(.87)		(1.7)	(1.5)	(1.3)
$SMB^{(12)}_t$		.34	.52**	.52**		-.33	-.58	-.58		-.19	-.2	-.2
		(1.4)	(2.3)	(2.3)		(-.65)	(-1.1)	(-1.1)		(-.73)	(-.72)	(-.75)
$UMD^{(12)}_t$		.018	.069	.069		.017	-.055	-.045		.062	.066	.042
		(.16)	(.67)	(.68)		(.072)	(-.24)	(-.2)		(.55)	(.57)	(.38)
D/P_{t-1}			-2.7	-2.8			3.5	2.4			1.8	4.3
			(-1.2)	(-1.1)			(.71)	(.46)			(.6)	(1.3)
Inflation$_{t-1}$			-4.8***	-4.9***			7*	6.4			-.51	.9
			(-3)	(-3.1)			(1.7)	(1.5)			(-.25)	(.4)
Ted Spread$_{t-1}$				.11				1.8				-4.3*
				(.063)				(.52)				(-1.9)
Constant	-5.7**	-2.8	-3.4	-3.4	13***	2.7	3.5	3.2	3.3	-.69	-.4	.12
	(-2.4)	(-1.2)	(-1.5)	(-1.4)	(2.8)	(.5)	(.65)	(.59)	(1.3)	(-.24)	(-.13)	(.036)
Panel B: Equal-Weighted Portfolios												
Agg. Disp.$_{t-1}$	-6.1**	-4.5**	-6.4***	-6.3***	8.9**	9.9**	11**	10**	-1.8	-1.8	.86	2.2
	(-2.3)	(-2.4)	(-3.1)	(-3.1)	(2.1)	(2.4)	(2.5)	(2.5)	(-.93)	(-.7)	(.31)	(.89)
$R^{(12)}_{m,t}$		-.22**	-.3***	-.3***		1.1***	1.2***	1.2***		.14	.14	.12
		(-2.5)	(-3.5)	(-3.4)		(6.5)	(6.7)	(6.4)		(1.2)	(1.3)	(.99)
$HML^{(12)}_t$		-.61***	-.56***	-.57***		.77***	.76***	.8***		.41**	.32*	.24
		(-4.2)	(-4.8)	(-5.1)		(2.6)	(3)	(3.1)		(2.2)	(1.8)	(1.5)
$SMB^{(12)}_t$		-.03	.086	.087		.62**	.48	.48		-.32*	-.41**	-.41**
		(-.2)	(.64)	(.64)		(2.3)	(1.6)	(1.6)		(-1.8)	(-2.1)	(-2.1)
$UMD^{(12)}_t$		-.091	-.045	-.048		.073	-.0019	.015		.03	.014	-.021
		(-1.3)	(-.82)	(-.89)		(.51)	(-.013)	(.11)		(.24)	(.11)	(-.22)
D/P_{t-1}			.89	1.2			-5.2	-7			3.2	6.7***
			(.47)	(.52)			(-1.3)	(-1.5)			(1.3)	(2.6)
Inflation$_{t-1}$			-4.6***	-4.4***			7.7**	6.7*			1.5	3.5
			(-3.4)	(-3)			(2.3)	(1.8)			(.69)	(1.5)
Ted Spread$_{t-1}$				-.61				3				-6.2***
				(-.36)				(.8)				(-2.7)
Constant	-9.1***	-4.1**	-4.1**	-4**	20***	6.3	5.7	5.3	.23	-2.5	-1.9	-1.1
	(-4.8)	(-2.2)	(-2.2)	(-2.1)	(5)	(1.5)	(1.3)	(1.1)	(.097)	(-.89)	(-.66)	(-.39)
N	385	385	385	385	385	385	385	385	385	385	385	385

Table AV. Disagreement and Concavity of the Security Market Line: Controlling for Idiosyncratic Volatility

Note: Sample Period: 12/1981-12/2014. Sample: CRSP stock file excluding penny stocks (price < \$5) and microcaps (stocks in bottom 2 deciles of the monthly size distribution using NYSE breakpoints). At the beginning of each calendar month, stocks are ranked in ascending order on the basis of their estimated beta at the end of the previous month. Pre-formation betas are estimated with a market model using daily returns over the past calendar year and 5 lags of the market returns. The ranked stocks are assigned to one of 20 value-weighted (panel A) or equal-weighted (panel B) portfolios based on NYSE breakpoints. We compute the full sample beta of these 20-beta sorted portfolios using the same market model. We then estimate every month the cross-sectional regression:

$$r_{P,t}^{(12)} = \kappa_t + \pi_t \times \beta_P + \phi_t \times (\beta_P)^2 + \Omega_t \times \ln(\sigma_{P,t}) + \epsilon_{P,t}, \quad \text{where} \quad P = 1, ..., 20$$

and $r_{P,t}^{(12)}$ is the 12-months excess return of the P^{th} beta-sorted portfolio, $\sigma_{P,t}$ is the value-weighted median of the idiosyncratic volatility of stocks in portfolio P in month t and β_P is the full sample post-ranking beta of the P^{th} beta-sorted portfolio. We then estimate second-stage regressions in the time-series using OLS and Newey-West adjusted standard errors allowing for 11 lags:

$$\begin{cases} \phi_t = & c_1 + \psi_1 \cdot \text{Agg. Disp.}_{t-1} + \delta_1^m \cdot R_{m,t}^{(12)} + \delta_1^{HML} \cdot HML_t^{(12)} + \delta_1^{SMB} \cdot SMB_t^{(12)} + \delta_1^{UMD} \cdot UMD_t^{(12)} + \sum_{x \in X} \delta_1^x \cdot x_{t-1} + \zeta_t \\ \pi_t = & c_2 + \psi_2 \cdot \text{Agg. Disp.}_{t-1} + \delta_2^m \cdot R_{m,t}^{(12)} + \delta_2^{HML} \cdot HML_t^{(12)} + \delta_2^{SMB} \cdot SMB_t^{(12)} + \delta_2^{UMD} \cdot UMD_t^{(12)} + \sum_{x \in X} \delta_2^x \cdot x_{t-1} + \omega_t \\ \kappa_t = & c_3 + \psi_3 \cdot \text{Agg. Disp.}_{t-1} + \delta_3^m \cdot R_{m,t}^{(12)} + \delta_3^{HML} \cdot HML_t^{(12)} + \delta_3^{SMB} \cdot SMB_t^{(12)} + \delta_3^{UMD} \cdot UMD_t^{(12)} + \sum_{x \in X} \delta_3^x x_{t-1} + \nu_t \end{cases}$$

Column (1) and (5) controls for Agg. Disp.$_{t-1}$, the monthly β-weighted average of stock-level disagreement, which is measured as the standard deviation of analyst forecasts on stocks' long run growth of EPS. Column (2) and (6) add controls for the 12-months excess return from t to $t+11$ of the market ($R_{m,t}^{(12)}$), HML ($HML_t^{(12)}$), SMB ($SMB_t^{(12)}$), and UMD ($UMD_t^{(12)}$). Column (3) and (7) add controls for the aggregate Dividend/Price ratio in $t-1$ and the past-12 months inflation rate in t_1. Column (4) and (8) additionally control for the TED spread in month $t-1$. T-statistics are in parenthesis. *, **, and *** means statistically different from zero at 10, 5 and 1% level of significance.

Table AV (Continued):

Dep. Var:	ϕ_t				π_t				κ_t			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Panel A: Value-Weighted Portfolios												
Agg. Disp.$_{t-1}$	-7*	-5.3	-8.7**	-8.6**	7.8	9.4	14*	13*	-6*	-3.4	-3.4	-2.6
	(-1.8)	(-1.6)	(-2.4)	(-2.3)	(1.4)	(1.5)	(1.9)	(1.8)	(-1.8)	(-1)	(-.92)	(-.73)
$R^{(12)}_{m,t}$		-.11	-.18	-.18		.75***	.91***	.93***		.36**	.29*	.28*
		(-.83)	(-1.3)	(-1.3)		(2.8)	(3)	(3)		(2.4)	(1.8)	(1.7)
$HML^{(12)}_t$		-.63***	-.53***	-.54***		.63	.52	.57		.078	.07	.022
		(-3)	(-2.8)	(-2.9)		(1.5)	(1.3)	(1.4)		(.37)	(.33)	(.11)
$SMB^{(12)}_t$		.2	.37	.37		.0085	-.24	-.24		-.25	-.2	-.2
		(.85)	(1.6)	(1.6)		(.017)	(-.44)	(-.46)		(-.87)	(-.68)	(-.69)
$UMD^{(12)}_t$		-.015	.038	.033		.14	.042	.065		.014	.043	.022
		(-.13)	(.35)	(.31)		(.55)	(.17)	(.27)		(.11)	(.33)	(.17)
D/P_{t-1}			-1.3	-.81			-1.4	-3.9			2.5	4.7
			(-.55)	(-.29)			(-.29)	(-.75)			(.91)	(1.6)
Inflation$_{t-1}$			-5.2***	-4.9***			9.6**	8.2*			-3	-1.8
			(-3)	(-3)			(2.2)	(1.8)			(-1.2)	(-.64)
Ted Spread$_{t-1}$				-.87				4.2				-3.8
				(-.45)				(1.1)				(-1.6)
Constant	-6.6***	-3.3	-3.7	-3.6	13***	3	3	2.5	2.3	-.98	-.63	-.18
	(-2.6)	(-1.4)	(-1.5)	(-1.4)	(2.7)	(.6)	(.59)	(.46)	(.9)	(-.34)	(-.21)	(-.057)
Panel B: Equal-Weighted Portfolios												
Agg. Disp.$_{t-1}$	-5.7**	-3.8*	-5.7***	-5.5***	6	5.4	8.2*	7.1	-1.3	-.99	.89	1.9
	(-2.1)	(-2)	(-2.6)	(-2.6)	(1.2)	(1.2)	(1.7)	(1.5)	(-.72)	(-.46)	(.37)	(.9)
$R^{(12)}_{m,t}$		-.22***	-.27***	-.28***		.77***	.86***	.88***		.2**	.23**	.21**
		(-2.6)	(-3)	(-3)		(4.5)	(4.5)	(4.2)		(2)	(2.6)	(2.1)
$HML^{(12)}_t$		-.67***	-.62***	-.63***		.9***	.82***	.88***		.27*	.21	.15
		(-4.7)	(-5.2)	(-5.7)		(3.1)	(3.2)	(3.6)		(1.8)	(1.4)	(1.1)
$SMB^{(12)}_t$		-.011	.087	.088		.37	.21	.21		-.11	-.19	-.18
		(-.067)	(.6)	(.61)		(1.2)	(.64)	(.61)		(-.62)	(-1)	(-.98)
$UMD^{(12)}_t$		-.083	-.049	-.053		.11	.052	.082		.029	.0071	-.019
		(-1.1)	(-.79)	(-.88)		(.52)	(.25)	(.45)		(.24)	(.063)	(-.21)
D/P_{t-1}			-.17	.32			-.23	-3.3			1.3	4.1*
			(-.086)	(.13)			(-.061)	(-.75)			(.59)	(1.8)
Inflation$_{t-1}$			-3.4**	-3.1**			5.8*	4			2.1	3.6
			(-2.5)	(-2.2)			(1.7)	(1.1)			(.93)	(1.5)
Ted Spread$_{t-1}$				-.84				5.4				-4.7**
				(-.5)				(1.3)				(-2.3)
Constant	-8.5***	-3.3*	-3.4*	-3.3*	16***	4.7	4.8	4.2	1.7	-1.2	-.89	-.33
	(-4.2)	(-1.8)	(-1.9)	(-1.8)	(4.3)	(1.2)	(1.2)	(1)	(.85)	(-.53)	(-.39)	(-.14)
N	385	385	385	385	385	385	385	385	385	385	385	385

Table AVI. Disagreement and the Slope of the Security Market Line

Note: Sample Period: 12/1981-12/2014. Sample: CRSP stock file excluding penny stocks (price < \$5) and microcaps (stocks in bottom 2 deciles of the monthly size distribution using NYSE breakpoints). At the beginning of each calendar month, stocks are ranked in ascending order on the basis of their estimated beta at the end of the previous month. Pre-formation betas are estimated with a market model using daily returns over the past calendar year and 5 lags of the market returns. The ranked stocks are assigned to one of 20 value-weighted (Panel A) or equal-weighted (Panel B) portfolios based on NYSE breakpoints. We compute the full sample beta of these 20-beta sorted portfolios using the same market model. We estimate every month the cross-sectional regression:

$$r_{P,t}^{(12)} = \kappa_t + \pi_t \times \beta_P + \epsilon_{P,t}, \quad \text{where} \quad P = 1, ..., 20$$

and $r_{P,t}^{(12)}$ is the 12-months excess return of the P^{th} beta-sorted portfolio and β_P is the full sample post-ranking beta of the P^{th} beta-sorted portfolio. We then estimate second-stage regressions in the time-series using OLS and Newey-West adjusted standard errors allowing for 11 lags:

$$\begin{cases} \pi_t = & c_1 + \psi_1 \cdot \text{Agg. Disp.}_{t-1} + \sum_{z \in Z} \delta_1^z \cdot z_t^{(k)} + \sum_{x \in X} \delta_1^x \cdot x_{t-1} + \omega_t \\ \kappa_t = & c_2 + \psi_2 \cdot \text{Agg. Disp.}_{t-1} + \sum_{z \in Z} \delta_2^z \cdot z_t^{(k)} + \sum_{x \in X} \delta_3^x x_{t-1} + \nu_t \end{cases}$$

Column (1) and (5) controls for Agg. Disp.$_{t-1}$, the monthly β-weighted average of stock level disagreement measured as the standard deviation of analyst forecasts on stocks' long run growth of Earnings per Share (EPS). Column (2) and (6) add the factor $z \in Z$, where Z contains the k-months excess market return from t to $t+k-1$ and the k-months return on HML, SMB, and UMD from t to $t+k-1$; Column (3) and (7) add controls for the aggregate Dividend/Price ratio in $t-1$ and the past-12 months inflation rate in t_1; Column (4) and (8) additionally control for the TED spread in month $t-1$. T-statistics are in parenthesis. *, **, and *** means statistically different from zero at 10, 5 and 1% level of significance.

Table AVI (Continued):

Dep. Var:	π_t				κ_t			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Panel A: Value-Weighted Portfolios								
Agg. Disp.$_{t-1}$	-6.1**	-.86	-4.9**	-5.7***	1.8	1.9	6.1***	6.6***
	(-2.1)	(-.39)	(-2.3)	(-2.7)	(.7)	(.89)	(2.8)	(3)
$R_{m,t}^{(12)}$		.6***	.58***	.59***		.4***	.43***	.42***
		(5.1)	(6.6)	(6.1)		(3.5)	(4.7)	(4.2)
$HML_t^{(12)}$		-.61***	-.48***	-.44***		.77***	.64***	.6***
		(-3.2)	(-3)	(-2.9)		(4)	(4.1)	(4)
$SMB_t^{(12)}$		.48**	.63***	.62***		-.44*	-.6***	-.59***
		(2.1)	(3.4)	(3.5)		(-1.9)	(-3.2)	(-3.3)
$UMD_t^{(12)}$		-.0036	.031	.051		-.0033	-.039	-.054
		(-.03)	(.31)	(.58)		(-.031)	(-.45)	(-.65)
D/P$_{t-1}$			-3.9*	-6**			3.9*	5.4*
			(-1.8)	(-2.3)			(1.7)	(1.8)
Inflation$_{t-1}$			-3.2*	-4.4**			3.4**	4.2**
			(-1.9)	(-2.1)			(2.2)	(2.3)
Ted Spread$_{t-1}$				3.6*				-2.5
				(1.8)				(-1.3)
Constant	.87	-2.6	-3.3	-3.8*	8.5***	2.5	3.3*	3.6*
	(.31)	(-1.2)	(-1.6)	(-1.8)	(3.2)	(1.2)	(1.7)	(1.7)
Panel B: Equal-Weighted Portfolios								
Agg. Disp.$_{t-1}$	-5.1*	-.46	-4.1**	-4.8***	3.8	2.1	6.3***	7***
	(-1.9)	(-.26)	(-2.5)	(-3)	(1.6)	(1.1)	(3.3)	(3.6)
$R_{m,t}^{(12)}$		.6***	.58***	.59***		.4***	.43***	.42***
		(5.2)	(6.6)	(6.1)		(3.3)	(4.5)	(4.1)
$HML_t^{(12)}$		-.66***	-.54***	-.5***		.96***	.83***	.79***
		(-4)	(-3.7)	(-3.5)		(5.4)	(5.6)	(5.9)
$SMB_t^{(12)}$		.7***	.84***	.84***		-.11	-.28*	-.28*
		(3.9)	(5.4)	(5.4)		(-.55)	(-1.7)	(-1.7)
$UMD_t^{(12)}$		-.11	-.077	-.057		.11	.068	.048
		(-.93)	(-.78)	(-.67)		(.95)	(.71)	(.57)
D/P$_{t-1}$			-3.3*	-5.3**			3.4	5.4**
			(-1.8)	(-2.6)			(1.6)	(2)
Inflation$_{t-1}$			-3.1*	-4.3**			4.1***	5.2***
			(-1.8)	(-2.2)			(2.6)	(3.1)
Ted Spread$_{t-1}$				3.6**				-3.5*
				(2.1)				(-1.9)
Constant	.2	-2.6	-3.2*	-3.7*	9.5***	1.6	2.2	2.7
	(.073)	(-1.4)	(-1.7)	(-1.9)	(3.5)	(.83)	(1.2)	(1.4)

Table AVII. Disagreement and Slope of the Security Market Line: speculative vs. non speculative stocks; monthly βs

Note: Sample Period: 12/1981-12/2014. Sample: CRSP stock file excluding penny stocks (price < \$5) and microcaps (stocks in bottom 2 deciles of the monthly size distribution using NYSE breakpoints). At the beginning of each calendar month, stocks are ranked in ascending order on the basis of the ratio of their estimated beta at the end of the previous month and their estimated idiosyncratic variance ($\frac{\beta}{\sigma^2}$). Pre-formation betas are estimated with a market model using monthly returns over the past 3 calendar years. The ranked stocks are assigned to two groups: speculative ($\frac{\hat{\beta}_i}{\hat{\sigma}_i^2} >$ NYSE median $\frac{\hat{\beta}}{\hat{\sigma}^2}$ in month t) and non-speculative stocks. Within each of these two groups, stocks are ranked in ascending order of their estimated beta at the end of the previous month and are assigned to one of 20 beta-sorted portfolios using NYSE breakpoints. We compute the full sample beta of these 2×20 equal-weighted portfolios (20 beta-sorted portfolios for speculative stocks; 20 beta-sorted portfolios for non-speculative stocks) using the same market model. $\beta_{P,s}$ is the resulting full sample beta, where $P = 1, \ldots, 20$ and s $\in$ {speculative, non speculative}. We estimate every month the following cross-sectional regressions, where P is one of the 20 β-sorted portfolios, $s \in$ {speculative, non speculative} and t is a month:

$$r^{(12)}_{P,s,t} = \iota_{s,t} + \chi_{s,t} \times \beta_{P,s} + \varrho_{s,t} \times \ln\left(\sigma_{P,s,t-1}\right) + \epsilon_{P,s,t},$$

where $\sigma_{P,s,t-1}$ is the median idiosyncratic volatility of stocks in portfolio (P,s) estimated at the end of month $t-1$ and $r^{(12)}_{P,s,t}$ is the equal-weighted 12-months excess return of portfolio (P,s). We then estimate second-stage regressions in the time-series using OLS and Newey-West adjusted standard errors allowing for 11 lags:

$$\begin{cases} \chi_{s,t} = & c_{1,s} + \psi_{1,s} \cdot \text{Agg. Disp.}_{t-1} + \delta^m_{1,s} \cdot R^{(12)}_{m,t} + \delta^{HML}_{1,s} \cdot HML^{(12)}_t + \delta^{SMB}_{1,s} \cdot SMB^{(12)}_t + \delta^{UMD}_{1,s} \cdot UMD^{(12)}_t + \sum_{x \in X} \delta^x_{1,s} \cdot x_{t-1} + \zeta_{t,s} \\ \varrho_{t,s} = & c_{2,s} + \psi_{2,s} \cdot \text{Agg. Disp.}_{t-1} + \delta^m_{2,s} \cdot R^{(12)}_{m,t} + \delta^{HML}_{2,s} \cdot HML^{(12)}_t + \delta^{SMB}_{2,s} \cdot SMB^{(12)}_t + \delta^{UMD}_{2,s} \cdot UMD^{(12)}_t + \sum_{x \in X} \delta^x_{2,s} \cdot x_{t-1} + \omega_{t,s} \\ \iota_{t,s} = & c_{3,s} + \psi_{3,s} \cdot \text{Agg. Disp.}_{t-1} + \delta^m_{3,s} \cdot R^{(12)}_{m,t} + \delta^{HML}_{3,s} \cdot HML^{(12)}_t + \delta^{SMB}_{3,s} \cdot SMB^{(12)}_t + \delta^{UMD}_{3,s} \cdot UMD^{(12)}_t + \sum_{x \in X} \delta^x_{3,s} x_{t-1} + \nu_{t,s} \end{cases}$$

Column (1) and (5) controls for Agg. Disp.$_{t-1}$, the monthly β-weighted average of stock level disagreement measured as the standard deviation of analyst forecasts on stocks' long run growth of EPS, where the β are the pre-ranking β computed above. Column (2) and (6) add controls for the 12-months excess return from t to $t+11$ of the market ($R^{(12)}_{m,t}$), HML ($HML^{(12)}_t$), SMB ($SMB^{(12)}_t$), and UMD ($UMD^{(12)}_t$). Column (3) and (7) add controls for the aggregate Dividend/Price ratio in $t-1$ and the past-12 months inflation rate in t_1. Column (4) and (8) additionally control for the TED spread in month $t-1$. T-statistics are in parenthesis. *, **, and *** means statistically different from zero at 10, 5 and 1% level of significance.

Table VI (Continued):

Dep. Var:	$\chi_{s,t}$				$\varrho_{s,t}$				$\iota_{s,t}$			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Panel A: Speculative stocks ($\frac{\hat{\beta}_i}{\hat{\sigma}_i^2}$ > NYSE median $\frac{\hat{\beta}}{\hat{\sigma}^2}$)												
Agg. Disp.$_{t-1}$	-6.5*	-3.3	-9.6***	-9.9***	.29	.33	1.2	1.1	2.7	4.5	13**	13**
	(-1.9)	(-1.6)	(-4.8)	(-5)	(.24)	(.27)	(.77)	(.74)	(.52)	(1)	(2.3)	(2.3)
$R^{(12)}_{m,t}$		.49***	.46***	.48***		.17**	.11	.11		1***	.87***	.86***
		(3.7)	(5.1)	(4.7)		(2.3)	(1.4)	(1.4)		(3.5)	(2.9)	(2.9)
$HML^{(12)}_t$		-.81***	-.66***	-.62***		.16	.13	.13		1.3**	1.1*	1.1*
		(-4.5)	(-4.1)	(-3.6)		(1.2)	(.84)	(.87)		(2.6)	(1.9)	(1.8)
$SMB^{(12)}_t$		.39	.66***	.64***		.12	.13	.12		-.15	-.4	-.39
		(1.2)	(2.8)	(2.9)		(.84)	(.86)	(.82)		(-.22)	(-.67)	(-.66)
$UMD^{(12)}_t$		-.38**	-.33**	-.32**		.14*	.16**	.16**		.73**	.74**	.73**
		(-2.1)	(-2.4)	(-2.5)		(1.9)	(2.2)	(2.2)		(2.2)	(2.4)	(2.3)
D/P_{t-1}			-5.8**	-7.8**			3.2*	2.9			15**	16*
			(-2.1)	(-2.4)			(1.9)	(1.5)			(2.1)	(1.9)
Inflation$_{t-1}$			-6***	-7.2***			-1.6	-1.8			.93	1.4
			(-3)	(-3.2)			(-1.2)	(-1.3)			(.18)	(.27)
Ted Spread$_{t-1}$				3.8*				.52				-1.6
				(1.6)				(.37)				(-.29)
Constant	-2.9	-1.8	-2.8	-3.2	2.3	-.94	-.44	-.5	19***	-.47	2	2.2
	(-.91)	(-.91)	(-1.3)	(-1.5)	(1.4)	(-.54)	(-.26)	(-.29)	(2.8)	(-.079)	(.31)	(.33)
Panel B: Non speculative stocks ($\frac{\hat{\beta}_i}{\hat{\sigma}_i^2} \leq$ NYSE median $\frac{\hat{\beta}}{\hat{\sigma}^2}$)												
Agg. Disp.$_{t-1}$	-1.7	.57	-5.2	-5.4	-1.4	-2	-1.9	-1.9	-3.6	-4.5	1.1	1.2
	(-.65)	(.18)	(-1.5)	(-1.6)	(-.66)	(-.99)	(-.82)	(-.84)	(-.5)	(-.62)	(.13)	(.14)
$R^{(12)}_{m,t}$		.27	.21	.23		.28***	.31**	.31**		1.4***	1.6***	1.6***
		(1.3)	(1.2)	(1.2)		(2.7)	(2.6)	(2.5)		(3.5)	(3.6)	(3.5)
$HML^{(12)}_t$		-1.1***	-.92***	-.89***		.021	.02	.028		1.3**	1.2**	1.2**
		(-3.2)	(-3.3)	(-3.3)		(.17)	(.18)	(.22)		(2.3)	(2.3)	(2.2)
$SMB^{(12)}_t$		.76	1**	1**		.55**	.53**	.52**		.99	.69	.7
		(1.6)	(2.4)	(2.4)		(2.4)	(2.2)	(2.1)		(.99)	(.69)	(.7)
$UMD^{(12)}_t$		-.052	.00023	.022		.016	.0065	.011		.17	.098	.091
		(-.23)	(.0011)	(.12)		(.14)	(.06)	(.095)		(.39)	(.23)	(.22)
D/P_{t-1}			-3.8	-6			-.74	-1.2			1	1.8
			(-1.1)	(-1.4)			(-.27)	(-.37)			(.11)	(.16)
Inflation$_{t-1}$			-6.9**	-8.2**			1	.75			9.2	9.7
			(-2)	(-2)			(.62)	(.43)			(1.3)	(1.3)
Ted Spread$_{t-1}$				4.2				.84				-1.4
				(1)				(.35)				(-.15)
Constant	-1.7	-.45	-1.2	-1.6	3.6	.23	.12	.023	20**	.018	.31	.47
	(-.41)	(-.12)	(-.31)	(-.45)	(1.6)	(.12)	(.061)	(.012)	(2.3)	(.0027)	(.041)	(.065)
N	385	385	385	385	385	385	385	385	385	385	385	385

Table AVIII. Disagreement and Slope of the Security Market Line: speculative vs. non speculative stocks. Other horizons

Note: Sample Period: 12/1981-12/2014. Sample: CRSP stock file excluding penny stocks (price < \$5) and microcaps (stocks in bottom 2 deciles of the monthly size distribution using NYSE breakpoints). At the beginning of each calendar month, stocks are ranked in ascending order on the basis of the ratio of their estimated beta at the end of the previous month and their estimated idiosyncratic variance ($\frac{\beta}{\sigma^2}$). Pre-formation betas are estimated with a market model using monthly returns over the past 3 calendar years. The ranked stocks are assigned to two groups: speculative ($\frac{\hat{\beta}_i}{\hat{\sigma}_i^2}$ > NYSE median $\frac{\hat{\beta}}{\hat{\sigma}^2}$ in month t) and non-speculative stocks. Within each of these two groups, stocks are ranked in ascending order of their estimated beta at the end of the previous month and are assigned to one of 20 beta-sorted portfolios using NYSE breakpoints. We compute the full sample beta of these 2×20 equal-weighted portfolios (20 beta-sorted portfolios for speculative stocks; 20 beta-sorted portfolios for non-speculative stocks) using the same market model. $\beta_{P,s}$ is the resulting full sample beta, where $P = 1, \ldots, 20$ and s $\in$ {speculative, non speculative}. We estimate every month the following cross-sectional regressions, where P is one of the 20 β-sorted portfolios, $s \in$ {speculative, non speculative} and t is a month:

$$r^{(k)}_{P,s,t} = \iota^{(k)}_{s,t} + \chi^{(k)}_{s,t} \times \beta_{P,s} + \varrho^{(k)}_{s,t} \times \ln\left(\sigma_{P,s,t-1}\right) + \epsilon^{(k)}_{P,s,t},$$

where $\sigma_{P,s,t-1}$ is the median idiosyncratic volatility of stocks in portfolio (P,s) estimated at the end of month $t-1$ and $r^{(k)}_{P,s,t}$ is the value-weighted k-months excess return of portfolio (P,s). We then estimate second-stage regressions in the time-series using OLS and Newey-West adjusted standard errors allowing for $k-1$ lags:

$$\begin{cases} \chi^{(k)}_{s,t} = & c_{1,s} + \psi_{1,s} \cdot \text{Agg. Disp.}_{t-1} + \delta^{m}_{1,s} \cdot R^{(k)}_{m,t} + \delta^{HML}_{1,s} \cdot HML^{(k)}_t + \delta^{SMB}_{1,s} \cdot SMB^{(k)}_t + \delta^{UMD}_{1,s} \cdot UMD^{(k)}_t + \sum_{x \in X} \delta^{x}_{1,s} \cdot x_{t-1} + \zeta_{t,s} \\ \pi^{(k)}_{t,s} = & c_{2,s} + \psi_{2,s} \cdot \text{Agg. Disp.}_{t-1} + \delta^{m}_{2,s} \cdot R^{(k)}_{m,t} + \delta^{HML}_{2,s} \cdot HML^{(k)}_t + \delta^{SMB}_{2,s} \cdot SMB^{(k)}_t + \delta^{UMD}_{2,s} \cdot UMD^{(k)}_t + \sum_{x \in X} \delta^{x}_{2,s} \cdot x_{t-1} + \omega_{t,s} \\ \kappa^{(k)}_{t,s} = & c_{3,s} + \psi_{3,s} \cdot \text{Agg. Disp.}_{t-1} + \delta^{m}_{3,s} \cdot R^{(k)}_{m,t} + \delta^{HML}_{3,s} \cdot HML^{(k)}_t + \delta^{SMB}_{3,s} \cdot SMB^{(k)}_t + \delta^{UMD}_{3,s} \cdot UMD^{(k)}_t + \sum_{x \in X} \delta^{x}_{3,s} x_{t-1} + \nu_{t,s} \end{cases}$$

Panel A use k=1 months, Panel B uses k=3 months, Panel C uses k=6 months, Panel D uses k=18 months. Column (1) and (5) controls for Agg. Disp.$_{t-1}$, the monthly β-weighted average of stock level disagreement measured as the standard deviation of analyst forecasts on stocks' long run growth of Earnings per Share (EPS). Column (2) and (6) control for the k-months excess market return from t to $t+k-1$, and the k-months return on HML, SMB, and UMD from t to $t+k-1$; Column (3) and (7) add controls for the aggregate Dividend/Price ratio in $t-1$ and the past-12 months inflation rate in t_1; Column (4) and (8) additionally control for the TED spread in month $t-1$. T-statistics are in parenthesis. *, **, and *** means statistically different from zero at 10, 5 and 1% level of significance.

Table AVIII (Continued):

Dep. Var:		$\chi_{s,t}^{(k)}$					$\varrho_{s,t}^{(k)}$				$\iota_t^{(k)}$	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Panel A: k=1 month												
Speculative stocks												
Agg. Disp.$_{t-1}$	-1.1	-.74	-.78	-.99*	.49	.55	.31	.42	.54	.57**	.75**	.89***
	(-1.4)	(-1.6)	(-1.5)	(-2)	(1.3)	(1.5)	(.73)	(.99)	(1.3)	(2.1)	(2.4)	(2.8)
Non-Speculative stocks												
Agg. Disp.$_{t-1}$	.15	.48*	-.0033	-.015	-.57	-.68**	-.34	-.41	.39	.34	.53*	.64**
	(.47)	(1.8)	(-.011)	(-.046)	(-1.2)	(-2.6)	(-1.1)	(-1.3)	(.8)	(1.2)	(1.7)	(2)
N	396	396	396	396	396	396	396	396	396	396	396	396
Panel B: k=3 month												
Speculative stocks												
Agg. Disp.$_{t-1}$	-3.6**	-2.3**	-3***	-3.4***	1.5*	1.4**	1.4*	1.2	1.8*	1.7**	2.5***	2.8***
	(-2.1)	(-2.5)	(-3)	(-3.4)	(1.8)	(2.1)	(1.8)	(1.5)	(1.7)	(2.5)	(3.4)	(3.8)
Non-Speculative stocks												
Agg. Disp.$_{t-1}$	-.26	.78	-.71	-.57	-.8	-.88	.045	-.2	1.2	.94	1.6**	1.7**
	(-.42)	(1.2)	(-.94)	(-.78)	(-.68)	(-1.1)	(.051)	(-.24)	(1.1)	(1.4)	(2.2)	(2.4)
N	394	394	394	394	394	394	394	394	394	394	394	394
Panel C: k=6 month												
Speculative stocks												
Agg. Disp.$_{t-1}$	-7.6**	-4.2**	-5.6***	-6.3***	2.5	1.7	1.3	1	3.5*	3.4***	5.2***	5.9***
	(-2.4)	(-2.5)	(-3)	(-3.5)	(1.6)	(1.4)	(.82)	(.67)	(1.9)	(2.6)	(3.7)	(4.4)
Non-Speculative stocks												
Agg. Disp.$_{t-1}$	-.071	2.2	-.96	-1.2	-2.9	-2.7**	-1.1	-1.3	2.4	1.8	3.6***	4***
	(-.062)	(1.6)	(-.64)	(-.85)	(-1.4)	(-2.3)	(-.82)	(-.99)	(1.2)	(1.5)	(2.8)	(3.3)
N	391	391	391	391	391	391	391	391	391	391	391	391
Panel D: k=18 month												
Speculative stocks												
s Agg. Disp.$_{t-1}$	-13**	-11***	-15***	-16***	2.9	4.4*	3.8	3.3	6*	8.8***	13***	13***
	(-2.3)	(-3.1)	(-3.3)	(-3.4)	(1.1)	(1.8)	(1.3)	(1.2)	(1.9)	(2.9)	(3.5)	(3.5)
Non-Speculative stocks												
Agg. Disp.$_{t-1}$	.042	.94	-5.6**	-6**	-4.2	-5.6*	-3.6	-4	3	5.4**	9.2***	9.5***
	(.022)	(.3)	(-2.1)	(-2.3)	(-1.1)	(-1.9)	(-1)	(-1.1)	(1.1)	(2)	(3.3)	(3.3)
N	379	379	379	379	379	379	379	379	379	379	379	379