Activism, Strategic Trading, and Market Liquidity

Kerry.E.Back
Rice University

Pierre Collin-Dufresne
EPFL, SFI and CEPR

Vyacheslav (Slava) Fos
Boston College

Tao Li
City University Hong Kong

Alexander Ljungqvist
NYU and NBER

Share-holder Activism

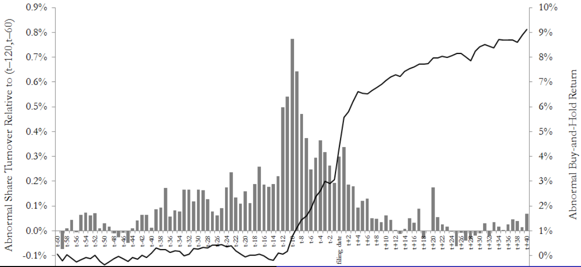
- Activists play central role in modern corporate governance and are often successful in increasing the value of targeted companies (Icahn, Buffett, Ackman, Peltz, Loeb).
- Recent issue of The Economist called them: "Capitalism's unlikely heroes."
- Assets under management more than doubled since 2008 to close to $120 billion of capital in 2014, where it attracted a fifth of all flows into hedge funds.
- According to the Economist: "Last year Activists launched 344 campaigns against public companies, large and small. In the past five years one company in two in the S&P 500 index of Americas most valuable listed firms has had a big activist fund on its share register, and one in seven has been on the receiving end of an activist attack."

Activism: Schedule 13D Disclosure Rules

- Activists typically accumulate shares by trading anonymously in secondary markets.
- When their stake hits 5%, SEC requires they disclose within 10 days:

(I) their holdings and intentions
(e.g., *Corporate governance action, Management shake-up, M&A transaction, Capital structure change, Cost reduction measures, Dividend payouts, Share buybacks, . . .*)

(II) all their trades during prior 60 days (CD and Fos (2015)):



Activism and Shareholder Value

- Schedule 13D activists:
  - own 7.2% stake on average on the filing date
  - increase share-holder value significantly (+6% excess returns in 30days pre-filing) and persistently
  - target more liquid stocks (and trade when liquidity is high).
- Recently senators Baldwin of Wisconsin and Merkley of Oregon propose new legislature (the "Brokaw Act") to shorten the disclosure window to 2 days to *"remove the opportunity for risk-less gains that activists achieve."*
- Big law firms such as Wachtell, Lipton, Rosen and Katz lobby the SEC to review the 13D disclosure rules to make it more difficult for activists to acquire shares *"in the interest of transparency and fairness for small shareholders."*

⇒ Raises questions about economic efficiency (and market liquidity).

Background

- Link between market liquidity (price efficiency), corporate governance (activism), and firm value (economic efficiency):
  - Suppose activist can create (or destroy) value at some cost (e.g., governance).
  - Profitability depends on ability to buy (or sell) shares before market reflects full value (Maug (1998)).
  - Conversely, if market reflects value of activism, market liquidity may allow activist to sell out of her stake and hurt share-holders (Coffee (1991), Bhide (1993)).
- Kyle (1985) proposes seminal model of strategic trading by informed investor:
  - Risk-neutral trader knows exogenous firm value V
  - Market marker sets price equal to expected value given she observes only total order flow (equal to informed trading plus noise).
  - $\Rightarrow$ (a) Optimal trading strategy, (b) Equilibrium price dynamics, (c) Market liquidity.
- We endogenize the liquidation value $V(X_T)$ by modeling the effort choice of the activist as a function of the accumulated stake.

Related Literature

- The microstructure literature
  Kyle, 1985; Glosten and Milgrom, 1985; Easley and O'Hare, 1987; Back, 1992
- Take-over literature
  Grossman and Hart (1980), Shleifer and Vishny (1986) , Kyle and Vila (1991)
- Corporate governance literature
  Coffee (1991), Bhide (1993), Admati, Pfleiderer, and Zechner (1994), Maug (1998)
- Dynamic model of governance
  DeMarzo and Urosevic (2006), Back, Li, Ljungqvist (2014), CD and Fos (2014)
- Market efficiency and disclosure rules:
  Grossman and Stiglitz (1980), Fishman and Haggerty (1995)
- Insider trading:
  Glosten (1989), Fishman and Haggerty (1992)

Model Setup

- Given a price function $P(t, Y_t)$, the activist seeks to maximize

$$\max_{v,\theta} \mathrm{E}\left[v X_T - C(v) - \int_0^T P(t, Y_t)\theta_t \, \mathrm{d}t \mid X_0 \right]. \quad (1)$$

  where
  - $C(v)$ is arbitrary (convex) effort cost paid by activist to achieve v.
  - $X_t = X_0 + \int_0^t \theta_s ds$ is aggregate stock position of activist.

- Market Maker has prior $X_0 \sim N(\mu_X, \sigma_X^2)$ and observes total order flow Y_t:

$$dY_t = \theta_t dt + \sigma dZ_t$$

  where Z_t is standard Brownian motion.

- An equilibrium is a pair (P, θ) s.t. trading strategy θ maximizes (18) given P and

$$P(t, Y_t) = \mathrm{E}\left[V(X_T) \mid \mathcal{F}_t^Y \right] \quad (2)$$

  for each t, given θ and where $V(x) = \mathrm{argmax}_v\{vx - C(v)\}$

Some Examples of Cost function

- Symmetric quadratic (continuous) cost: $C(v) = (v - v_0)^2/(2\psi)$:

  Linear $V(x) = v_0 + \psi x$

- Asymmetric Quadratic cost: $C(v) = \begin{cases} (v - v_0)^2/(2\psi) & \text{if } v \geq v_0\,, \\ \infty & \text{otherwise}\,. \end{cases}$

  Piece-wise linear and convex $V(x) = v_0 + \psi x^+$

- Exponential case $C(v) = \frac{1}{\psi} v \ln(\frac{v}{v_0}) - \frac{1}{\psi} v$

  Strictly convex $V(x) = v_0 e^{\psi x}$

- Binary (all or nothing): It costs $c > 0$ to increase stock value from v_0 to $v_0 + \Delta$.

  Digital $V(x) = v_0 + \Delta \mathbf{1}_{[c/\Delta,\infty)}(x)$

Equilibrium

Theorem

The pricing rule $P(t, Y_t) = \mathrm{E}\left[h(Y_T)\,|\mathcal{F}_t^Y\right]$ *with* $h(y) = V\left(\mu_x + \Lambda\, y\right)$ *and the trading strategy:*

$$\theta_t = \frac{1}{T-t}\frac{(X_t - \mu_x - \Lambda Y_t)}{\Lambda - 2}\,, \qquad (3)$$

where $\Lambda = 1 + \sqrt{1 + \frac{\sigma_x^2}{\sigma^2 T}}$ *only depends on the signal to noise ratio. constitute an equilibrium such that:*

- $dP(t, Y_t) = \lambda(t, Y_t) dY_t$ *with* $\lambda(t, y) = \frac{\partial P(t,y)}{\partial y}$.
- *Price impact* $\lambda(t, Y_t)$ *is a martingale.*
- $P(T, Y_T) = V(X_T)$ *almost surely.*
- $\mathrm{E}[\theta_t \,|\, \mathcal{F}_t^Y] = 0$.
- $X_T \sim \textbf{Normal}\left[\mu_x, (\sigma\sqrt{T} + \sqrt{\sigma^2 T + \sigma_x^2})^2\right]$.

- A crucial result is that $dY_t = \theta_t dt + \sigma dZ_t$ is a Brownian Motion with volatility σ on its own (i.e., market maker's) filtration such that $X_T = \mu_x + \Lambda Y_T$ a.s.

- This implies that the optimal trading strategy is inconspicuous.

- Remarkably, the optimal trading strategy is independent of the effort cost $(C(v), V(x))$ when expressed as a function of Y_t, X_t.

- Instead, the cost function $C(v)$ determines $V(x)$ and thus affects the price function $P(t, Y)$ and the amount of effort expended.

- Different from Kyle, the optimal trading strategy depends positively on the number of accumulated shares (X_t)

$\rightarrow$ Amplification effect: The informed more than offsets the cumulative noise trading demand because the value of activism increases with his ownership.

$\rightarrow$ The endogenous value of the firm depends on the amount of realized liquidity trading.

A useful lemma

Lemma

Let ε be a standard normal random variable that is independent of Z. Let b be a nonnegative constant, and set $a = \sigma\sqrt{(2b+1)T}$. Then, the solution Y of the SDE

$$\mathrm{d}Y_t = \frac{a\varepsilon - bZ_t - (b+1)Y_t}{T-t}\,\mathrm{d}t + \mathrm{d}Z_t \tag{4}$$

on the time interval $[0,T)$ has the following properties: $Y_T \stackrel{def}{=} \lim_{t\to T} Y_t$ exists a.s., Y is a Brownian motion with zero drift and standard deviation σ on its own filtration on $[0,T]$, and, with probability 1,

$$Y_T = \frac{a\varepsilon - bZ_T}{b+1}\,. \tag{5}$$

Sketch of proof: Consider (4) as an observation equation for $\zeta_t = \frac{a\varepsilon - bZ_t}{b+1}$. Since $dY_t = \frac{(b+1)}{T-t}(\zeta_t - Y_t)dt + dZ_t$ is linear, conditionally Gaussian filtering gives the dynamics of $\hat{\zeta}_t = \mathrm{E}[\zeta_t \mid \mathcal{F}_t^Y]$ and its conditional variance. Under the condition of the lemma the latter goes to zero (L^2 convergence).

Sketch of Proof

- Search for an equilibrium where $P(y,t)$ only depends on aggregate order flow. Then, risk-neutrality of the market maker implies $P(Y_t,t) = \mathrm{E}[h(Y_T) \mid \mathcal{F}_t^Y]$ for some function $h(Y)$.
- If the insider's trading strategy is unpredictable, then $P(y,t) = \mathrm{E}[h(y + Z_T - Z_t) \mid \mathcal{F}_t^Z]$. So the price function is pinned down by $h(\cdot)$ (given law of Z).
- If the insider leaves no-money on the table at maturity, then $h(Y_T) = V(X_T)$ *a.s.*.
- To find candidate $h(\cdot)$, we rely on previous lemma with $\varepsilon = \frac{X_0 - \mu_x}{\sigma_x}$. Indeed, substituting into (5) and using $Y_T = X_T - X_0 + Z_T$ we obtain that

  $$X_T = \mu_x + \Lambda Y_T$$

  for b, Λ as defined in the theorem (note $a = b\sigma_X$ required to cancel X_0). This implies $V(X_T) = V(\mu_x + \Lambda Y_T) \equiv h(Y_T)$ *a.s.* for this trading strategy and thus gives us the candidate $h(\cdot)$.
- It remains to verify the optimality for the insider of this strategy.

- Using Envelope theorem, Insider maximizes

$$\mathrm{E}\left[\int_0^T (V(X_t) - P(t, Y_t))\theta_t \,\mathrm{d}t \mid X_0\right]$$

- HJB equation for value function $J(Y_t, X_t, t)$ is linear in control θ. Obtain:

$$\begin{array}{lcl} (HJB) & 0 = & \frac{1}{2} J_{YY}\sigma^2 + J_t \\ (FOC) & 0 = & J_X + J_Y + V(X) - P(Y, t) \end{array}$$

- Using Feynman-Kac we seek (a 'no-trade') solution of the form $J(y, x, t) = \mathrm{E}_t[g(y + Z_T - Z_t, x)]$ for some function $g(\cdot, \cdot)$.
- To determine g we assume by 'continuity' that the second (FOC) equation holds at T. Integrating it we 'guess' $g(y, x) = \sup_{\overline{y}} \int_y^{\overline{y}} (V(x - y + z) - h(z))dz$
- Thus $\overline{y}^*(\cdot)$ satisfies $V(u + \overline{y}^*(u)) = h(\overline{y}^*(u))$, which gives $\overline{y}^*(u) = \frac{u - \mu_X}{\Lambda - 1}$, and the function

$$g(y, x) = \int_y^{\overline{y}^*(x-y)} (V(x - y + z) - h(z))dz$$

satisfies $g_x + g_y + V(x) - h(y) = 0$.

- It follows (with sufficient regularity) that for our candidate value function: $J_X(y,x) + J_Y(y,x,t) = \mathrm{E}_t[g_x(y + Z_T - Z_t, x) + g_y(y + Z_T - Z_t, x)] = \mathrm{E}_t[h(y + Z_T - Z_t) - V(x)] = P(y,t) - V(x)$
- Thus, the candidate J satisfies the HJB equation and FOC. It follows that for any θ_t:

$$J(Y_0, X_0,) = \mathrm{E}_0 \left[g(Y_T, X_T) + \int_0^T (V(X_t) - P(Y_t, t))\theta_t dt \right]$$

- Now note that $g(y,x) \geq 0$ and that for the particular strategy θ_t^* such that $\overline{y}^*(X_T^* - Y_T^*) = Y_T^*$ (which is equivalent to $h(Y_T^*) = V(X_T^*)$) we have $g(Y_T^*, X_T^*) = 0$. This completes the (sketch of) proof.

A few examples

Example

In the symmetric quadratic model, $V(x) = v_0 + \psi x$, so

$$h(y) = v_0 + \psi\mu_x + \psi\Lambda\, y\,.$$

The price function at any time $t \leq T$ is given by:

$$P(y,t) = v_0 + \psi\mu_x + \psi\Lambda y \tag{6}$$

The price impact function is given by:

$$\lambda(y,t) = \psi\Lambda \tag{7}$$

This case resembles the original Kyle model:

- Price impact is constant
- However, $\lim_{\sigma\to 0}\lambda = \psi > 0$ ('endogenous uncertainty'!).

Example

In the asymmetric quadratic model, $V(x) = v_0 + \psi x^+$, so

$$h(y) = v_0 + \psi\left(\mu_x + \Lambda y\right)^+$$
$$= \begin{cases} v_0 & \text{if } y < -\frac{\mu_x}{\Lambda} \\ v_0 + \psi\mu_x + \psi\Lambda\, y & \text{otherwise} \end{cases}, .$$

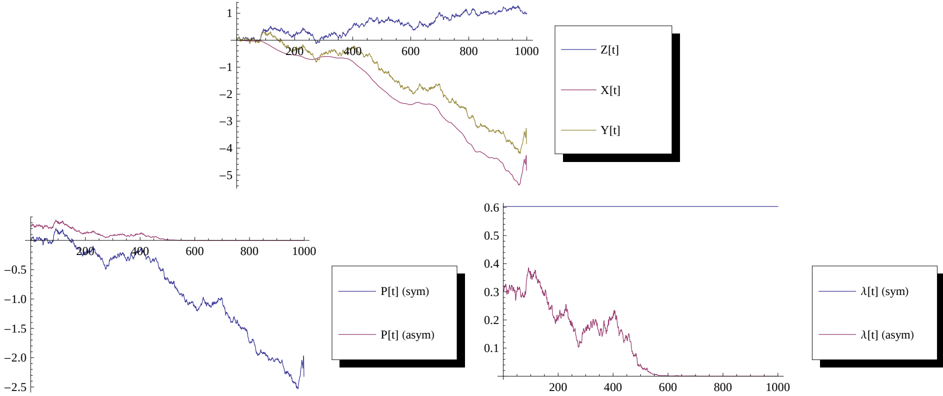
The price function at any time $t \leq T$ is given by:

$$P(y,t) = v_0 + \psi(\mu_x + \Lambda y)\mathrm{N}\left[\frac{\mu_x + \Lambda y}{\Lambda\sigma\sqrt{T-t}}\right] + \psi\Lambda\sigma\sqrt{T-t}\mathrm{n}\left[\frac{\mu_x + \Lambda y}{\Lambda\sigma\sqrt{T-t}}\right] \quad (8)$$

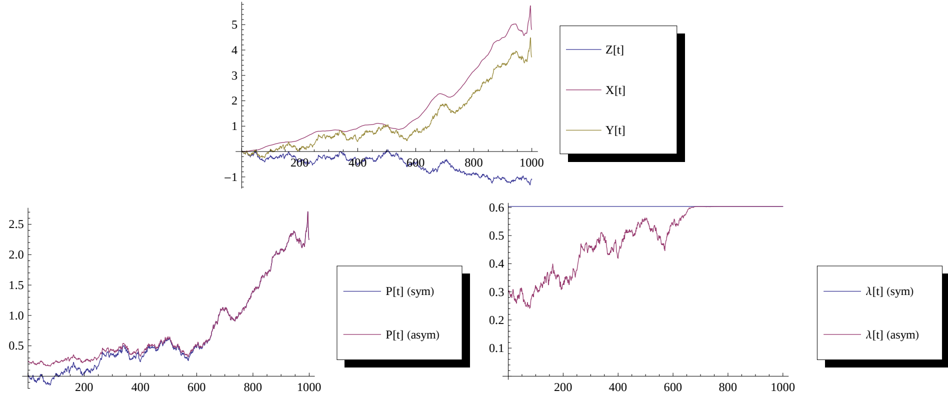
The price impact function is given by:

$$\lambda(y,t) = \psi\Lambda\mathrm{N}\left[\frac{\mu_x + \Lambda y}{\Lambda\sigma\sqrt{T-t}}\right] \quad (9)$$

Symmetric vs. asymmetric quadratic cost function



Symmetric vs. asymmetric quadratic cost function



Example

In the exponential model, $V(x) = v_0 e^{\psi x}$, so

$$h(y) = v_0 e^{\psi(\mu_x + \Lambda y)}$$

The price function at any time $t \leq T$ is given by:

$$P(y,t) = v_0 e^{\psi\left(\mu_x + \Lambda y + \frac{1}{2}\Lambda^2 \sigma^2 (T-t)\right)} \tag{10}$$

The price impact function is given by:

$$\lambda(y,t) = \Lambda\, P(y,t) \tag{11}$$

A Black-Scholes price process with a price-volume relationship.

Example

In the binary effort model,

$$V(x) = v_0 + \Delta \mathbf{1}_{[c/\Delta,\infty)}(x)\,,$$

so

$$\begin{aligned} h(y) &= v_0 + \Delta \mathbf{1}_{[c/\Delta,\infty)}\left(\mu_x + \Lambda y\right) \\ &= \begin{cases} v_0 & \text{if } y < \frac{(c/\Delta - \mu_x)}{\Lambda}\,, \\ v_0 + \Delta & \text{otherwise}\,. \end{cases} \end{aligned}$$

It follows that the price function at any time $t \leq T$ is given by:

$$P(y,t) = v_0 + \Delta \mathrm{N}\left[\frac{\mu_x + \Lambda y - c/\Delta}{\Lambda \sigma \sqrt{T-t}}\right] \tag{12}$$

The price impact is given by: $\lambda(y,t) = \frac{\partial P(y,t)}{\partial y} = \Delta \frac{\mathrm{n}\left[\frac{\mu_x + \Lambda y - c/\Delta}{\Lambda\sigma\sqrt{T-t}}\right]}{\sigma\sqrt{T-t}}$

Economic efficiency and market liquidity

- We measure economic efficiency by price (at time 0), which is the expected effort of the activist.
- We measure market liquidity by the expected average price impact ($\mathrm{E}[\frac{1}{T}\int_0^T \lambda_s ds] = \lambda_0$).
- Importantly, market liquidity (λ) can be affected by different channels:
  - Noise trading volatility (σ) $\sim$ Trading tax or length of disclosure window.
  - Prior uncertainty about insider's position (σ_X) $\sim$ Disclosure rules.
  - Initial block size (μ_x).
  - Productivity of the activist (Δ, ψ) $\sim$ Legal environment.
- These channels also have different implications for economic efficiency.

$\Rightarrow$ We consider separately the ex-ante impact at date 0 when $Y_0 = 0$ of a change in $\sigma, \mu_x, \sigma_x, \psi$ on price (economic efficiency) and price impact (market liquidity).

Productivity of the activist ψ, Δ (legal environment)

Example

In all (symmetric, asymmetric quadratic, exponential, binary) models:

$$\frac{\partial P}{\partial \psi} > 0 \quad \text{and} \quad \frac{\partial \lambda}{\partial \psi} > 0$$

- Variation in activism productivity generates negative cross-sectional relation between economic efficiency and market liquidity, because uncertainty about the activist's position creates more adverse selection when she is more productive.
- The causality activism $\rightarrow$ liquidity is reverse of what the literature has focused on.

Prior Uncertainty σ_x (disclosure rules)

Example

In the symmetric quadratic model: $\frac{\partial P}{\partial \sigma_x} = 0$ and $\frac{\partial \lambda}{\partial \sigma_x} > 0$

Example

In the asymmetric quadratic model: $\frac{\partial P}{\partial \sigma_x} > 0$ and $\frac{\partial \lambda}{\partial \sigma_x} > 0$

A general result obtains (since σ_x is mean-preserving spread for X_T):

Theorem

If $V(x)$ is convex then $\frac{\partial P}{\partial \sigma_x} \geq 0$ (and conversely if $V(x)$ is concave)

If $V(x)$ satisfies mild regularity conditions $\frac{\partial \lambda}{\partial \sigma_x} > 0$

- If $V(x)$ is convex then cross-sectional variation in μ_x, σ_x creates a negative relation between economic efficiency and market liquidity, because activism $\rightarrow$ liquidity.

Prior Uncertainty σ_x (disclosure rules)

Example

In the binary effort model, $\frac{\partial P}{\partial \sigma_x} \geq 0 \iff \mu_x \leq c/\Delta$ and $\frac{\partial \lambda}{\partial \sigma_x} > 0$

- If $V(x)$ is bounded above and below (not convex), then cross-sectional variation in μ_x, σ_x creates a negative relation between economic efficiency and market liquidity, if only if the expected initial take is too low to justify activism on its own.
- More uncertainty about the insider's position:
  - creates more adverse selection risk and makes markets less liquid.
  - increases the likelihood that actual stake X_0 is large enough to become active if $\mu_x \leq c/\Delta$.
- This lock-in effect is similar to Coffee (1991), Bhide (1993), Maug (1998).

Noise trading volatility (length of disclosure window)

Example

In the symmetric quadratic model: $\frac{\partial P}{\partial \sigma} = 0$ and $\frac{\partial \lambda}{\partial \sigma} < 0$

Example

In the asymmetric quadratic model: $\frac{\partial P}{\partial \sigma} > 0$ and $\frac{\partial \lambda}{\partial \sigma} < 0$

A general result obtains (since an increase in σ is mean-preserving spread for X_T):

Theorem

If $V(x)$ is convex then $\frac{\partial P}{\partial \sigma} \geq 0$ (and conversely if $V(x)$ is concave then $\frac{\partial P}{\partial \sigma} \leq 0$)

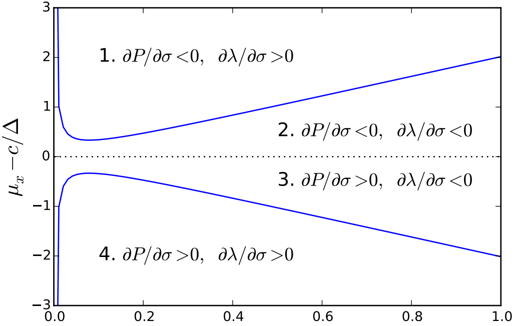
However, comparative statics for market liquidity λ are less straightforward.

Noise trading volatility (length of disclosure window)

Example

In the binary effort model,

$$\left\{\frac{\partial P}{\partial \sigma} \geq 0 \iff \mu_x \leq c/\Delta\right\} \quad \text{and} \quad \left\{\frac{\partial \lambda}{\partial \sigma} < 0 \iff |\mu_x - c/\Delta|^2 < T\sigma^2\Lambda^2(\Lambda - 1)\right\}$$



Conclusion

- Endogenize terminal value in Kyle-Back model to study link between efficiency and liquidity as parameters $\psi, \mu_x, \sigma_x, \sigma$ vary.
- Informs about consequences of policy change such as trading tax, changing disclosure rules, dislosure window, legal environment.
- Variation in activism productivity (ψ) creates a negative cross-sectional relation betwen efficiency and liquidity.
- Variation in (μ_x, σ_x) create a negative relation between efficiency and liquidity:
    - if $V(x)$ is convex, because more uncertainty about the insider's position increases both expected activism and adverse selection risk.
    - But if $V(x)$ is bounded above and below, then if and only if the initial stake is low enough that it does not justify activism on its own (lock-in effect).
- An increase in noise trading volatility (σ) raises economic efficiency if $V(x)$ is convex, or if $V(X)$ is bounded and the initial stake is sufficiently low.
- However, the effect on market liquidity can be ambiguous, because the realized amount of activism depends on the realized amount of liquidity trading. Thus, an increase in noise trading volatility can make markets more illiquid.

Extensions

- Stock-picking vs. activism: allow for privately known exogenous component of firm value
    - Prices are efficient: reflect fair value of security at T.
    - But, equilibrium is not fully revealing in that market cannot separate stock-picking ability from activism based on price and order-flow information.
- Risk-aversion and residual risk (incomplete information):
    - Trade-off between activism and diversification.
    - Multiple equilibria are possible in one-period model.
    - Price impact may be negative in one-period model.
    - Insider may not trade until maturity, but choose to reveal her position earlier.
    - Optimal trading strategy is not inconspicuous.
    - There may be a jump in position and return jumps at maturity (in continuous time).

Extensions

- Allow for stochastic noise trading volatility process (CDF (2016)). This gives the informed trader a liquidity timing option:
  - Trades more when uninformed volume is high.
  - Price volatilitiy is stochastic and positively correlated with uninformed volume.
  - Price impact is stochastic, increasing on average, and negatively correlated with volume
- Derivatives Trading by Activists (also part of 13D disclosure requirement):
  - Activists use derivatives in only 2.62% of all cases
  - When exchange-traded options are availble (20%) then use derivatives in 10% of cases.
  - Use derivative to increase their long-exposure (not to hedge) by 2.2% to achieve 8.5% total.
  - Options Implied Volatilities accurately forecast the move in realized volatility which drops 10% on average at announcement.
  - Option bid-ask spreads widen to reflect volatility component of private information.
  - Implied volatilities move on days when activists trade in the stock market even when they don't trade in derivatives.

The risk-averse Activist with incomplete information

- Risk-averse insider's maximization problem:

$$\max_{\theta_t \in \mathcal{A}, w} \mathrm{E}\left[\mathbf{U}\left\{(\upsilon + \epsilon + w)X_T - \int_0^T P_t \theta_t dt - C(w)\right\} \mid \mathcal{F}_t^Y, \upsilon, X_0\right] \qquad (13)$$

  where we add:
  - CARA utility $U\{x\} = -\exp(-ax)$
  - Incomplete information: Insider faces residual risk $\epsilon \sim N(0, \Sigma_\epsilon)$
  - Stock-picking Insider knows υ, but for MM $\upsilon \sim N(\mu_v, \Sigma_v)$
  - Activism: Insider has Quadratic cost: $C(w) = \frac{w^2}{2\psi}$, but for MM $X_0 \sim N(\mu_x, \Sigma_x)$.

- This model nests:
  - Kyle-Back when $(a, \psi, \Sigma_\epsilon) \to 0$.
  - Baruch-Subrahmanyam-Holden model with risk-aversion when when $(\psi, \Sigma_\epsilon) \to 0$
  - Symmetric Activism model when $(a, \Sigma_\epsilon) \to 0$

Insider's objective function

- As before Optimal 'effort' maximizes at T:

$$\max_{w} wX_T - C(w), \tag{14}$$

where $X_T = X_0 + \int_0^T \theta_t dt$

$\Rightarrow$ Leads to $w^* = \psi X_T$

- Plugging back into his objective function and integrating out residual risk we see that the insider is maximizing (set $X_0 = 0$ wlog):

$$\max_{\theta_t \in \mathcal{A}} \mathrm{E}\left[U\left\{ \int_0^T (\upsilon - P_t)\theta_t dt + \frac{(\psi - a\Sigma_\epsilon)X_T^2}{2} \right\} |\mathcal{F}_t^Y, \upsilon, X_0 \right] \tag{15}$$

$\Rightarrow$ trade-off between activism and residual risk.

- Market efficiency condition is:

$$P_t = \mathrm{E}[v + w \mid \mathcal{F}_t^Y] = \mathrm{E}[v + \psi X_t \mid \mathcal{F}_t^Y] \tag{16}$$

Main Results with a risk-averse insider

- Activism without residual risk:
  - Trading strategy is inconspicuous
  - Price impact is decreasing (because of risk-aversion)
  - Equilibrium where insider trades until T may fail to exist. Instead, there exists an equilibrium where activist 'announces' at an endogenous earlier date $\tau_0 < T$.
- Activism and stock-picking:
  - equilibrium may not be 'fully-revealing' as it reveals only $(\upsilon + \psi X_T)$ and not the separate components of stock-picking and activism (if $\Sigma_x(0)\Sigma_\upsilon(0) > 0$).
- Activism with residual risk (discrete time multi-period solution):
  - Trading strategy is **not** inconspicuous.
  - Price impact may turn negative.
  - There can be multiple equilibria (proof in one period case).
  - We conjecture that in the continuous time limit there may be a discrete jump at maturity in the position and an announcement return.

One period model with risk-aversion and residual risk $(\psi = 0)$

Insider

- the activist is endowed with an initial number of shares X_0
- the activist will purchase θ shares using a market order

Market Maker

- The market maker sets prices so as to break even.
- The market maker only observes total order flow $Y = \theta + u$ where $u \sim N(0, \sigma_u^2)$ is uninformed noise trading.
- We assume the market maker's initial prior about X_0 and v is Gaussian: $v \sim N(V_0, \sigma_v^2)$ and $X_0 \sim N(Q_0, \sigma_Q^2)$ and that their covariance is σ_{Xv}.

Activist's Problem

The activist solves:

$$\max_{\theta} \mathrm{E}\left[U\left\{(v-P_1)\theta - \frac{a\Sigma_\epsilon X_T^2}{2}\right\}|v, X_0\right] \tag{17}$$

We look for a linear equilibrium where price responds linearly to order flow:

$$P_1 = P_0 + \lambda(Y - \eta Q_0)$$

and the strategy of the insider is

$$\theta = \beta(v - P_0) + \gamma(X_0 - Q_0) + \eta Q_0$$

Note that unlike previous Kyle model we need $\mathrm{E}[\theta|Y] \neq 0$.

Equilibrium

We find that

$$P_1 = P_0 + \lambda(Y - \eta Q_0)$$

and

$$\theta^* = \beta(v - a\Sigma_e X_0 - P_0) + \gamma Q_0$$

where β, λ, η solve system of equations:

$$\begin{aligned} \lambda &= \beta \frac{\sigma_v^2 - a\Sigma_\epsilon \sigma_{Xv}}{\beta\omega^2 + \sigma_u^2} \\ \beta &= \frac{1}{2\lambda + a\lambda^2\sigma_u^2 + a\Sigma_\epsilon} \\ \eta &= -\frac{\beta\psi}{1 - \beta\lambda} \end{aligned}$$

where $\omega^2 \equiv \sigma_v^2 + 2a\Sigma_\epsilon\sigma_{Xv} + (a\Sigma_\epsilon)^2\sigma_X^2$ is total amount of Risk.

- Price impact is decreasing in risk-aversion and in residual uncertainty
- Trading strategy is not inconspicuous ($\eta < 0$), because of expected 'deleveraging'.
- Unique equilibrium if $\Sigma_{Xv} \geq a\Sigma_\epsilon\Sigma_x$ (else three equilibria may exist).
- Price impact can be negative.

Stock-picking vs. Activism: Model Setup

- Given a price process $P(t)$, the activist seeks to maximize

$$\max_{v,\theta} \mathrm{E}\left[v + w\, X_T - C(w) - \int_0^T P(t)\theta_t \,\mathrm{d}t \mid X_0 \right]. \tag{18}$$

  where
  - $C(w) = \frac{w^2}{2\psi}$ is quadratic effort cost paid by activist to achieve w.
  - v is fixed 'stock-picking' component
  - $X_t = X_0 + \int_0^t \theta_s ds$ is aggregate stock position of activist.

- Market Maker has prior $(X_0, v) \sim N(\mu, \Sigma)$ and observes total order flow Y_t:

$$dY_t = \theta_t dt + \sigma dZ_t$$

  where Z_t is standard Brownian motion.

- An equilibrium is a pair (P, θ) s.t. trading strategy θ maximizes (18) given P and

$$P(t) = \mathrm{E}\left[\psi X_T + v \mid \mathcal{F}_t^Y \right] = \psi Q_t + v_t \tag{19}$$

  for each t, given θ and where $\psi x = \mathrm{argmax}_w \{wx - C(w)\}$

Equilibrium with Stock-picking and Activism

- There exists an equilibrium characterized by deterministic functions λ_t, Λ_t such that

$$dP_t = (\lambda_t + \psi\Lambda_t)dY_t \tag{20}$$

- Total price impact is constant:

$$\lambda_t + \psi\Lambda_t = \hat{\Delta} := \psi + \sqrt{\psi^2 + \frac{\omega^2}{\sigma^2}} \quad \forall t, \tag{21}$$

where $\omega^2 = \sigma_\upsilon^2 + 2\psi\sigma_{\upsilon X} + \psi^2\sigma_X^2$.

- The optimal trading strategy for the insider is:

$$\theta_t^* = \frac{\hat{\Delta}\sigma^2}{\Omega_t}(\upsilon + \psi X_t - P_t), \tag{22}$$

where $\Omega_t := \Sigma_\upsilon(t) + 2\psi\Sigma_{X\upsilon}(t) + \psi^2\Sigma_X(t) = \omega^2(T-t)$.

- The equilibrium is revealing in that P_t converges to $\upsilon + \psi X_t$ at time T.
- In the filtration of the market maker, the stock price P_t is a Brownian martingale.

Parameter Choice

We fix $T = \sigma = 1$, and $\psi = 1$ and consider three cases for the initial two sources of adverse selection:

1. Initial position is known: $\sigma_X^2 = 0$, $\sigma_\upsilon^2 = 1$.
2. Exogenous component is known: $\sigma_X^2 = 1$, $\sigma_\upsilon^2 = 0$.
3. Both are unknown: $\sigma_X^2 = 0.5$, $\sigma_\upsilon^2 = 0.5$.

In all cases, we set $\sigma_{Xv} = 0$ so that the total signal to noise ratio $\frac{\omega}{\sigma}$ remains unchanged in all three cases.

Flow of private information into prices

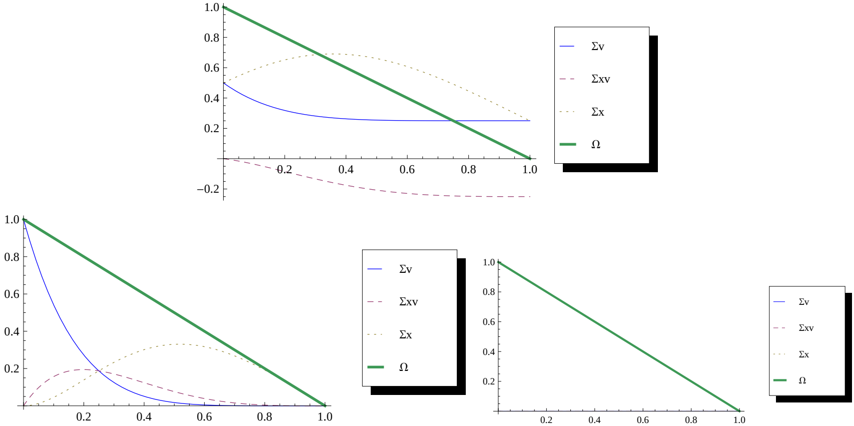


FIGURE: **Flow of private information into Prices.** The top panel corresponds to $\Sigma_X(0) = \Sigma_\upsilon(0) = 0.5$. The bottom left panel to the case with $\Sigma_v(0) = 1$ and $\Sigma_X(0) = 0$. The right panel to $\Sigma_X = 1$ and $\Sigma_v = 0$

Components of price impact

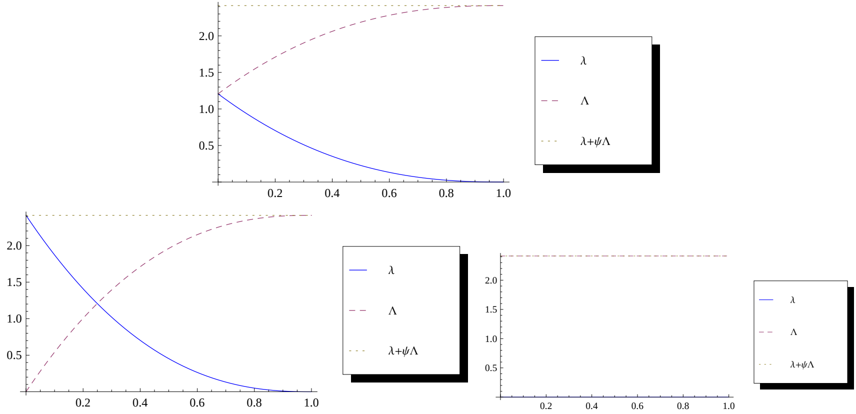


FIGURE: **Components of price impact.** The top panel corresponds to $\Sigma_X(0) = \Sigma_\upsilon(0) = 0.5$. The bottom left panel to the case with $\Sigma_v(0) = 1$ and $\Sigma_X(0) = 0$. The right panel to $\Sigma_X = 1$ and $\Sigma_v = 0$.

Optimal trading strategy

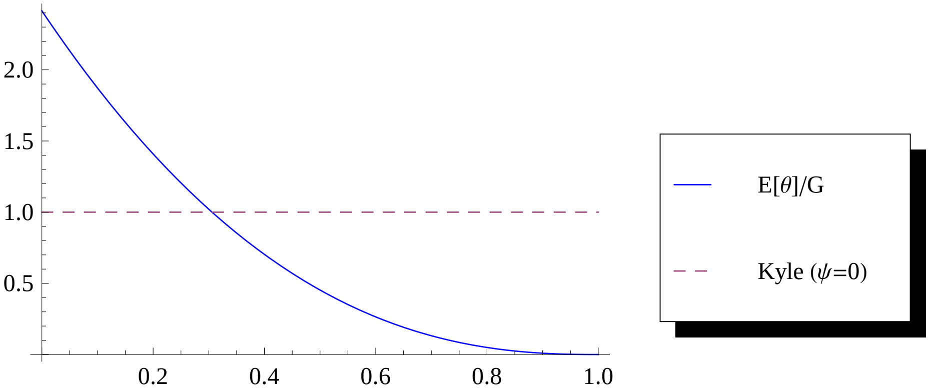


Figure: **Optimal trading strategy.** This figure shows the unconditional expected trading rate of the activist shareholder normalized by the initial valuation gap $\mathrm{E}[\theta_t|\mathcal{F}_0, \upsilon, X_0]/G_0$ with $G_0 = (\upsilon + \psi X_0 - P_0)$ as a function of time and compares that to the expected trading rate in