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Informational Frictions and Commodity Markets

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ABSTRACT

This paper develops a model with a tractable log-linear equilibrium to analyze the effects of informational frictions in commodity markets. By aggregating dispersed information about the strength of the global economy among goods producers whose production has complementarity, commodity prices serve as price signals to guide producers' production decisions and commodity demand. Our model highlights important feedback effects of informational noise originating from supply shocks and futures market trading on commodity demand and spot prices. Our analysis illustrates the weakness common in empirical studies on commodity markets of assuming that different types of shocks are publicly observable to market participants.

IN THE AFTERMATH OF THE DRAMATIC BOOM and bust cycle of commodity prices in 2007 to 2008, there has been renewed interest among academics and policy makers regarding the drivers of commodity price fluctuations, in particular, whether fundamental demand and supply shocks are sufficient to explain the observed price cycles and whether speculation in commodity futures markets exacerbated these cycles are subjects of debate. In this debate, it is common for academic and policy studies to treat different types of shocks (such as supply, demand, and financial market shocks) as observable to market participants.[1] In doing so, however, these studies ignore a key aspect of commodity markets, namely, severe informational frictions faced by market participants. The markets for major commodities, such as crude oil and copper, have become globalized in recent decades, with supply and demand now stemming from across the world. This globalization exposes market participants to heightened informational frictions regarding the global supply, demand, and inventory of these commodities.

The economics literature has developed an elegant theoretical framework to analyze how trading in centralized asset markets both facilitates information

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[1] See a recent review by Cheng and Xiong (2014).

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aggregation among market participants and helps them overcome the informational frictions they face (e.g., Grossman and Stiglitz (1980) and Hellwig (1980)). This framework, however, crucially relies on the combination of constant absolute risk aversion (CARA) utility functions for agents and Gaussian distributions for asset prices to ensure a tractable linear equilibrium, and thus one cannot readily adopt this framework to analyze commodity markets, in which both CARA utility and Gaussian distributions are unrealistic. It is challenging to analyze information aggregation in settings without the tractable linear equilibrium. This technical challenge is common in analyzing how asset prices affect real activity, such as firm investment and central bank policies, through an informational channel.[2]

In this paper, we aim to confront this challenge by developing a tractable model to analyze how informational frictions affect commodity markets. Our model integrates the standard framework of asset market trading with asymmetric information into an international macro setting (e.g., Obstfeld and Rogoff (1996) and Angeletos and La'O (2013)). In this global economy, a continuum of specialized goods producers whose production has complementarity—which emerges from their need to trade produced goods with each other—demand a key commodity, such as copper, as a common production input. Through trading the commodity, the goods producers aggregate dispersed information regarding unobservable global economic strength, which ultimately determines their commodity demand.

Our main model focuses on a centralized spot market through which the goods producers acquire the commodity from a group of suppliers, who are subject to an unobservable supply shock. The supply shock prevents the commodity price from perfectly aggregating the goods producers' information with respect to the strength of the global economy. Nevertheless, the commodity price provides a useful signal to guide the producers' production decisions and commodity demand. Despite the nonlinearity in the producers' production decisions, we derive a unique log-linear equilibrium in closed form. In this equilibrium, each producer's commodity demand is a log-linear function of its private signal and the commodity price, while the commodity price is a log-linear function of global economic strength and the supply shock. This tractable log-linear equilibrium builds on a combination of Cobb-Douglas utility functions for households, log-normal distributions for commodity prices, and a key aggregation property: the aggregate demand of a continuum of producers remains log-linear as a result of the Law of Large Numbers. We also extend the model to incorporate a futures market to further characterize the role of futures market trading.

It is common for empirical studies of commodity markets to rely on conventional wisdom generated from settings without any informational frictions (i.e., agents directly observing both supply and demand shocks). According to such wisdom, (1) a higher price leads to lower commodity demand as a result of the standard cost effect, (2) a positive supply shock reduces the commodity price, which in turn stimulates greater commodity demand, and (3) the futures

[2] See a recent review by Bond, Edmans, and Goldstein (2012).

price of the commodity simply tracks the spot price, and trading in the futures market does not affect either commodity demand or the spot price.

Our model allows us to contrast the effects of informational frictions with this conventional wisdom. First, through its informational role, a higher commodity price signals a stronger global economy and motivates each goods producer to produce more goods. This leads to greater demand for the commodity as an input, which offsets the usual cost effect. The complementarity in production among goods producers magnifies this informational effect through their incentives to coordinate production decisions. Under certain conditions, our model shows that the informational effect can dominate the cost effect and lead to a positive price elasticity of producers' demand for the commodity.

Second, our model illustrates a feedback effect of supply shocks. In the presence of informational frictions, supply shocks also act as informational noise, which prevents the commodity price from fully revealing the strength of the global economy. As goods producers partially attribute the lower commodity price caused by a positive supply shock to a weak global economy, this inference induces them to reduce their commodity demand. This feedback effect thus further amplifies the negative price impact of the supply shock and undermines its impact on commodity demand.

Third, futures markets serve as a useful platform, in addition to spot markets, for aggregating information regarding demand and supply of commodities. As futures markets attract a different group of participants from spot markets, the futures price is not simply a shadow of the spot price, and instead may have its own informational effects on commodity demand and the spot price.

Based on these results, our analysis offers important implications for the empirical analysis of commodity markets. In estimating the effects of supply and demand shocks in commodity markets, it is common for the empirical literature to adopt structural models that ignore informational frictions by simply assuming that agents can directly observe both demand and supply shocks. As highlighted by our analysis, this common practice is likely to understate the effect of supply shocks and overstate the effect of demand shocks. Our model provides the basic ingredients for expanding these structural models to account for how commodity prices impact agents' expectations.

Our analysis also cautions against a commonly used empirical strategy based on commodity inventory to detect speculative effects (e.g., Juvenal and Petrella (2012), Knittel and Pindyck (2013), and Kilian and Murphy (2014)). This strategy is premised on the widely held argument that, if speculators distort the price of a commodity upward, consumers will find the commodity too expensive and thus reduce consumption, causing inventory of the commodity to spike. By assuming that consumers are able to recognize the commodity price distortion, this argument again ignores realistic informational frictions faced by consumers, which are particularly relevant in times of great economic uncertainty. In contrast, our model shows that informational frictions may cause consumers to react to the distorted price by increasing rather than decreasing their consumption. In this light, the lack of any pronounced oil inventory spike before the peak of oil prices in July 2008, as highlighted by the recent empirical literature, cannot be taken as evidence to reject the presence of any speculative effect during the period.

Finally, by systematically illustrating that prices of key industrial commodities can serve as price signals for the strength of the global economy and that informational noise in commodity prices can feed back to commodity demand and spot prices,[3] our analysis provides a coherent argument for how the large inflow of investment capital to commodity futures markets during the 2000s might have amplified the boom and bust of commodity prices in 2007 to 2008. By interfering with the price signals, informational noise from the investment flow may have temporarily led market participants to increase their commodity demand despite a weakening global economy. This confusion helped sustain the commodity price boom until information arrived later to correct their expectations, which then caused commodity prices to collapse.

Our paper contributes to the emerging literature that analyzes the causes of the commodity market cycle of the 2000s, for example, Hamilton (2009), Stoll and Whaley (2010), Cheng, Kirilenko, and Xiong (2012), Hamilton and Wu (2015), Henderson, Pearson, and Wang (2012), Tang and Xiong (2012), Singleton (2014), and Kilian and Murphy (2014). The mechanism illustrated by our model echoes Singleton (2014), who emphasizes the importance of accounting for agents' expectations to explain this commodity market cycle. In particular, our analysis highlights the weakness common in empirical studies on the effects of supply and demand shocks and speculation in commodity markets of assuming that different types of shocks are publicly observable to market participants.

Our model complements the recent macro literature that analyzes the role of informational frictions on economic growth. Lorenzoni (2009) shows that, by influencing agents' expectations, noise in public news can generate sizable aggregate volatility. Angeletos and La'O (2013) focus on endogenous economic fluctuations that result from the lack of centralized communication channels to coordinate the expectations of different households. Our model adopts the setting of Angeletos and La'O (2013) to model goods market equilibrium and derive endogenous complementarity in goods producers' production decisions. We analyze information aggregation through centralized commodity trading, which is absent from their model, and the feedback effects of the commodity price.

The literature has long recognized that trading in financial markets aggregates information and the resulting prices can feed back to real activity (e.g.,

[3] Consistent with this notion, in explaining the decision of the European Central Bank (ECB) to raise its key interest rate in March 2008 on the eve of the worst economic recession since the Great Depression, ECB policy reports cite high prices of oil and other commodities as a key factor, suggesting the significant influence of commodity prices on the expectation of central bankers. Furthermore, Hu and Xiong (2013) provide evidence that, in recent years, stock prices across East Asian economies have displayed significant and positive reactions to overnight futures price changes of a set of commodities traded in the United States, suggesting that people across the world regard commodity futures prices as barometers of the global economy.

Bray (1981) and Subrahmanyam and Titman (2001)). Furthermore, recent literature points out that such feedback effects can be particularly strong in the presence of strategic complementarity in agents' actions. Morris and Shin (2002) show that, in such a setting, noise in public information has an amplified effect on agents' actions and thus on equilibrium outcomes. In our model, commodity prices serve such a role in feeding back noise to goods producers' production decisions. Similar feedback effects are also modeled in several other contexts, such as from stock prices to firm capital investment decisions and from exchange rates to policy choices of central banks (e.g., Ozdenoren and Yuan (2008), Angeletos, Lorenzoni, and Pavan (2010), and Goldstein, Ozdenoren, and Yuan (2011, 2013)). The log-linear equilibrium derived in our model accommodates the nonlinearity induced by goods producers' production decisions and, at the same time, is tractable for the analysis of feedback effects of commodity prices. This tractable log-linear equilibrium can be adapted by future studies to analyze feedback effects in settings outside commodity markets.[4]

The paper is organized as follows. We first present the model setting in Section I and derive the equilibrium in Section II. Section III analyzes the effects of informational frictions. Section I provides a brief summary of a model extension to include a futures market. We discuss the implications of our analysis in Section V and conclude the paper in Section VI. We relegate all the technical proofs to the Appendix and provide a separate online appendix (the Internet Appendix) to provide the details of the model extension summarized in Section IV.[5]

I. Model Setting

In this section, we develop a baseline model with two dates $t = 1, 2$ to analyze the effects of informational frictions on the market equilibrium related to a commodity. One can think of this commodity as crude oil or copper, which is used across the world as a key production input. We model a continuum of islands of total mass one. Each island produces a single good, which can be either consumed at "home" or traded for another good produced "away" by another island. A key feature of the baseline model is that the commodity market is not only a place for market participants to trade the commodity but also a platform to aggregate private information about the strength of the global economy, which ultimately determines the global demand for the commodity.

[4] It is also worth noting that our setting is different from existing settings adopted by the literature to analyze real consequences of asset prices. For example, Goldstein, Ozdenoren, and Yuan (2013) develop a model to analyze stock market trading with asymmetric information and the feedback effect from the equilibrium stock price to firm investment. The equilibrium derived in their model is also nonlinear. They ensure tractability by imposing a set of assumptions, including that each trader is risk-neutral and faces upper and lower position limits and that noisy stock supply follows a rigid functional form involving the cumulative standard normal distribution function. Our setting does not require these nonstandard assumptions.

[5] The Internet Appendix may be found in the online version of this article.

Table I
Timeline of the Model

	$t = 1$ Spot Market	$t = 2$ Goods Market
Households		Trade/Consume Goods
Producers	Observe Signals Acquire Commodity Produce Goods	
Commodity Suppliers	Observe Supply Shock Supply Commodity	

Table I summarizes the timeline of the model. There are three types of agents: households on the islands, goods producers on the islands, and a group of commodity suppliers. The goods producers trade the commodity with commodity suppliers at $t = 1$ and use the commodity to produce goods at $t = 2$. Their produced goods are distributed to the households on their respective islands at $t = 2$. The households then trade their goods with each other and consume.

A. Island Households

Each island has a representative household. Following Angeletos and La'O (2013), we assume a particular structure for goods trading between households on different islands. Each island is randomly paired with another island at $t = 2$. The households on the two islands trade their goods with each other and consume both goods produced by the islands. For a pair of matched islands, we assume that the preference of the households on these islands over the consumption bundle (C_i, C_i^*), where C_i represents consumption of the "home" good while C_i^* consumption of the "away" good, is determined by a utility function $U(C_i, C_i^*)$. The utility function increases in both C_i and C_i^*. This utility function specifies all "away" goods as perfect substitutes, so that the utility of the household on each island does not depend on the matched trading partner. The households on the two islands thus trade their goods to maximize the utility of each. We assume that the utility function of the island households takes the Cobb-Douglas form

$$U\left(C_i, C_i^*\right) = \left(\frac{C_i}{1-\eta}\right)^{1-\eta} \left(\frac{C_i^*}{\eta}\right)^{\eta}, \tag{1}$$

where $\eta \in [0, 1]$ measures the utility weight of the away good. A larger η means that each island values more of the away good and hence relies more on trading its good with other islands. Thus, η eventually determines the degree of complementarity in the islands' goods production.

B. Goods Producers

Each island has a locally owned representative firm to organize its goods production. We refer to each firm as a producer. Production requires use of the commodity as an input. To focus on the commodity market equilibrium, we exclude other inputs such as labor from production. Each island has the following decreasing-returns-to-scale production function:[6]

$$Y_i = AX_i^{\phi}, \tag{2}$$

where Y_i is the output produced by island i and X_i is the commodity input. Parameter $\phi \in (0, 1]$ measures the degree to which the production function exhibits decreasing returns to scale. When $\phi = 1$, the production function has constant returns to scale. The variable A is the common productivity shared by all islands. For simplicity, we assume that each island's productivity does not have an idiosyncratic component. This simplification is innocuous for our qualitative analysis of how information frictions affect commodity demand.

For an individual goods producer, A has a dual role—it determines its own output as well as other producers' output. To the extent that demand for the producer's good depends on other producers' output, A represents the strength of the global economy. We assume that A is a random variable, which becomes observable only when the producers complete their production at $t = 2$. This is the key informational friction in our setting. We assume that A has a log-normal distribution,

$$\log A \backsim \mathcal{N}\left(\bar{a}, \tau_A^{-1}\right),$$

where $\bar{a}$ is the mean of $\log A$ and τ_A^{-1} is its variance. At $t = 1$, the goods producer on each island observes a private signal about $\log A$,

$$s_i = \log A + \varepsilon_i,$$

where $\varepsilon_i \backsim \mathcal{N}(0, \tau_s^{-1})$ is random noise independent of $\log A$ and independent of noise in other producers' signals, and τ_s is the precision of the signal. The signal allows the producer to form its expectation of the strength of the global economy and determine its production decision and commodity demand. The commodity market serves to aggregate the private signals dispersed among the producers. As each producer's private signal is noisy, the publicly observed commodity price also serves as a useful price signal to form its expectation.

At $t = 1$, the producer on island i maximizes its expected profit by choosing its commodity input X_i,

$$\max_{X_i} \ \mathrm{E}\left[P_i Y_i \mid \mathcal{I}_i\right] - P_X X_i, \tag{3}$$

[6] One can also specify a Cobb-Douglas production function with both commodity and labor as inputs. The model remains tractable although the formulas become more complex and harder to interpret.

where P_i is the price of the good produced by the island. The producer's information set $\mathcal{I}_i = \{s_i, P_X\}$ includes its private signal s_i and the commodity price P_X. The goods price P_i, which one can interpret as the terms of trade, is determined at $t = 2$ based on the matched trade with another island.

C. Commodity Suppliers

We assume there is a group of commodity suppliers who face a convex labor cost

$$\frac{k}{1+k} e^{-\xi/k} (X_S)^{\frac{1+k}{k}}$$

in supplying the commodity, where X_S is the quantity supplied, $k \in (0, 1)$ is a constant parameter, and ξ represents random noise in the supply. As a key source of information frictions in our model, we assume that ξ is observable to the suppliers themselves but not by other market participants. We assume that, from the perspective of goods producers, ξ has Gaussian distribution $\mathcal{N}(\overline{\xi}, \tau_\xi^{-1})$, where $\overline{\xi}$ is its mean and τ_ξ^{-1} its variance. The mean captures the part that is predictable to goods producers, while the variance represents uncertainty in supply that is outside goods producers' expectations.

Based on the above, given a spot price P_X, suppliers face the following optimization problem:

$$\max_{X_S} \; P_X X_S - \frac{k}{1+k} e^{-\xi/k} (X_S)^{\frac{1+k}{k}}. \tag{4}$$

It is easy to determine from (4) that the suppliers' optimal supply curve is

$$X_S = e^{\xi} P_X^k, \tag{5}$$

where ξ is uncertainty in the commodity supply and k the price elasticity.

D. Joint Equilibrium of Different Markets

Our model features the joint equilibrium of a number of markets: the goods markets between each pair of matched islands and the market for the commodity. Equilibrium requires clearing of each of these markets:

- At $t = 2$, for each pair of randomly matched islands $\{i, j\}$, the households of these islands trade their produced goods and clear the market for each good,

$$C_i + C_j^* = AX_i^{\phi},$$

$$C_i^* + C_j = AX_j^{\phi}.$$

- At $t = 1$, in the commodity market, the goods producers' aggregate demand equals the supply,

$$\int_{-\infty}^{\infty} X_i(s_i, P_X)\, d\Phi(\varepsilon_i) = X_S(P_X),$$

where each producer's commodity demand $X_i(s_i, P_X)$ depends on its private signal s_i and the commodity price P_X. The demand from producers is integrated over the noise ε_i in their private signals.

II. The Equilibrium

A. Goods Market Equilibrium

We begin our analysis of the equilibrium with the goods markets at $t = 2$. For a pair of randomly matched islands, i and j, the representative household of island i possesses Y_i units of the good produced by the island while the representative household of island j holds Y_j units of the other good.[7] They trade the two goods with each other to maximize the utility function of each given in (1). The following proposition, which resembles a similar proposition in Angeletos and La'O (2013), describes the goods market equilibrium between these two islands.

PROPOSITION 1: *For a pair of randomly matched islands, i and j, their representative households' optimal consumption of the two goods is*

$$C_i = (1-\eta)Y_i,\ C_i^* = \eta Y_j,\ C_j = (1-\eta)Y_j,\ C_j^* = \eta Y_i.$$

The price of the good produced by island i is

$$P_i = \left(\frac{Y_j}{Y_i}\right)^{\eta}. \tag{6}$$

As a direct implication of the Cobb-Douglas utility function, each household divides its consumption between the home and away goods with fractions $1 - \eta$ and η, respectively. When $\eta = 1/2$, the household consumes the two types of goods equally. The price of each good is determined by the relative output of the two matched islands.[8] One island's good is more valuable when the other island produces more. This feature is standard in the international macroeconomics literature (e.g., Obstfeld and Rogoff (1996)) and implies that each goods producer needs to take into account the production decisions of producers of other goods.[9]

[7] Here, we treat a representative household as representing different agents holding stakes in an island's goods production, such as workers, managers, suppliers of inputs, etc. We agnostically group their preferences for the produced goods of their own island and other islands into the preferences of the representative household.

[8] The goods price P_i given in (6) is the price of good i normalized by the price of good j produced by the other matched island.

[9] Decentralized goods market trading is not essential to our analysis. This feature allows us to conveniently capture endogenous complementarity in goods producers' production decisions

B. Production Decision and Commodity Demand

By substituting the production function in (2) into (3), which gives the expected profit of the goods producer on island i, we obtain the following objective:

$$\max_{X_i} \mathrm{E}\left[AP_i X_i^{\phi} \middle| s_i, P_X \right] - P_X X_i.$$

In a competitive goods market, the producer will produce to the level that marginal revenue equals marginal cost:

$$\phi \mathrm{E}\left[AP_i | \, s_i, P_X \right] X_i^{\phi-1} = P_X.$$

By substituting in P_i from Proposition 1, we obtain

$$X_i = \left\{ \frac{\phi \mathrm{E}\left[AX_j^{\phi\eta} \middle| s_i, P_X \right]}{P_X} \right\}^{1/(1-\phi(1-\eta))}, \tag{7}$$

which depends on the producer's expectation $\mathrm{E}[AX_j^{\phi\eta}|s_i, P_X]$ regarding the product of global productivity A and the production decision $X_j^{\phi\eta}$ of its randomly matched trading partner, island j. This expression demonstrates the complementarity in the producers' production decisions. A larger η makes the complementarity stronger as the island households engage more in trading the produced goods with each other and the price of each good depends more on the output of other goods.

The commodity price P_X is a source of information for the producer to form its expectation of $\mathrm{E}[AX_j^{\phi\eta}|s_i, P_X]$, which serves as a channel for the commodity price to feed back into each producer's commodity demand. The presence of complementarity strengthens this feedback effect relative to standard models of asset market trading with asymmetric information.

C. Commodity Market Equilibrium

By clearing the aggregate demand of goods producers with the supply of suppliers, we derive the commodity market equilibrium. As is common in settings with real investment, equation (7) shows that each producer's commodity demand is a nonlinear function of the price. Despite the nonlinearity, we manage to derive a tractable and unique log-linear equilibrium in closed form. The following proposition summarizes the commodity price and each producer's commodity demand in this equilibrium.

PROPOSITION 2: *At $t = 1$, the commodity market has a unique log-linear equilibrium: (1) The commodity price is a log-linear function of* $\log A$ *and* ξ*,*

$$\log P_X = h_A \log A + h_\xi \xi + h_0, \tag{8}$$

with tractability. Alternatively, one can adopt centralized goods markets and let island households consume goods produced by all producers. See Angeletos and La'O (2009) for such a setting. We expect our key insight to carry over to this alternative setting.

with the coefficients h_A and h_ξ given by

$$h_A = -\frac{(1-\phi)\,b + (1-\phi\,(1-\eta))\,\tau_s^{-1}\tau_\xi b^3}{1+k\,(1-\phi)} > 0, \tag{9}$$

$$h_\xi = -\frac{1-\phi + (1-\phi(1-\eta))\tau_s^{-1}\tau_\xi b^2}{1+k(1-\phi)} < 0, \tag{10}$$

where $b < 0$ is given in equation (A19) and h_0 in equation (A20). (2) The commodity purchased by goods producer i is a log-linear function of its private signal s_i and $\log P_X$,

$$\log X_i = l_s s_i + l_P \log P_X + l_0, \tag{11}$$

with the coefficients l_s and l_P given by

$$l_s = -b > 0,\; l_P = k + h_\xi^{-1}, \tag{12}$$

and l_0 by equation (A21).

Proposition 2 shows that each producer's commodity demand is a log-linear function of its private signal and the commodity price, while the commodity price $\log P_X$ aggregates the producers' dispersed private information to partially reveal the global productivity $\log A$. The commodity price does not depend on any producer's signal noise as a result of the aggregation across a large number of producers with independent noise. This feature is similar to Hellwig (1980). The commodity price also depends on the supply shock ξ, which serves the same role as noise trading in the standard models of asset market trading with asymmetric information.

It is well known that asset market equilibria with asymmetric information are often intractable due to the difficulty in analyzing each agent's learning from the equilibrium asset price and in aggregating different agents' asset demands. Existing literature commonly adopts the setting of Grossman and Stiglitz (1980) and Hellwig (1980), which features CARA utility for agents and Gaussian distributions for asset fundamentals and noise trading. This setting ensures a linear equilibrium in which the asset price is a linear function of asset fundamental and noise trading, while each agent's asset demand is a linear function of the price and its own signal. One cannot directly adopt this setting, however, to analyze informational feedback effects of asset prices to real activity, such as firm investment and central bank policies, which typically involve asset fundamentals with non-Gaussian distributions and agents with non-CARA utility.

Our model presents a tractable setting to analyze real consequences of asset prices. Despite the commodity price and each producer's commodity demand both having non-Gaussian distributions, the log-linear equilibrium derived in Proposition 2 maintains similar tractability to the linear equilibrium derived by Grossman and Stiglitz (1980) and Hellwig (1980). A key feature contributing to this tractability is that the producers' aggregate demand remains log-normal as a result of the Law of Large Numbers.

III. Effects of Informational Frictions

A. Perfect-Information Benchmark

To facilitate our analysis of the effects of informational frictions, we first establish a benchmark without any informational friction. Suppose that the global fundamental A and commodity supply shock ξ are both observable by all market participants. Then, the goods producers can choose their optimal production decisions without any noise interference. The following proposition characterizes this benchmark.

PROPOSITION 3: *When both A and ξ are observed by all market participants, there is a unique equilibrium. In this equilibrium, (1) the goods producers share an identical commodity demand curve, $X_i = X_j = (\frac{\phi A}{P_X})^{\frac{1}{1-\phi}}, \forall i$ and j, and (2) the commodity price is given by*

$$\log P_X = \frac{1}{1+k(1-\phi)}\log A - \frac{1-\phi}{1+k(1-\phi)}\xi + \frac{1}{1+k(1-\phi)}\log \phi,$$

while the goods producers' aggregate commodity demand is given by

$$\log X_S = \frac{k}{1+k(1-\phi)}\log A + \frac{1}{1+k(1-\phi)}\xi + \frac{k}{1+k(1-\phi)}\log \phi.$$

In the absence of any informational frictions, the benchmark features a unique equilibrium despite the complementarity in the goods producers' production decisions because competition between goods producers leads to a downward-sloping demand curve for the commodity. This demand curve intersects the suppliers' upward-sloping supply curve at the unique commodity price P_X given in the proposition. As a result, the complementarity between goods producers does not lead to multiple equilibria in which goods producers coordinate on certain high or low demand levels.

Proposition 3 derives the equilibrium commodity price and aggregate demand. Intuitively, the global fundamental $\log A$ increases both the commodity price and aggregate demand, while the supply shock ξ reduces the commodity price but increases aggregate demand.

The following proposition compares the equilibrium derived in Proposition 2 with the perfect-information benchmark.

PROPOSITION 4: *In the presence of informational frictions, the commodity price coefficients with the global fundamental $h_A > 0$ and the commodity supply shock $h_\xi < 0$, as derived in Proposition 2, are both lower than their corresponding values in the perfect-information benchmark, and converge to these values as $\tau_s \to \infty$.*

In the presence of informational frictions, the commodity price deviates from that in the perfect-information benchmark, with the supply shock having a greater price impact (i.e., h_ξ being more negative) and the global fundamental having a smaller impact (i.e., h_A being less positive). Through these price impacts, informational frictions eventually affect goods producers' production decisions and island households' goods consumption, which we analyze step-by-step below.

B. Price Informativeness

In the presence of informational frictions, the equilibrium commodity price $\log P_X = h_A \log A + h_\xi \xi + h_0$ serves as a public signal of the global fundamental $\log A$. This price signal is contaminated by the presence of the supply noise ξ. The informativeness of the price signal is determined by the ratio of the contributions to the price variance of $\log A$ and ξ:

$$\pi = \frac{h_A^2/\tau_A}{h_\xi^2/\tau_\xi}.$$

The following proposition characterizes how the price informativeness measure π depends on several key model parameters: τ_s, τ_ξ, and η.

PROPOSITION 5: *π is monotonically increasing in τ_s and τ_ξ, and is decreasing in η.*

As τ_s increases, each goods producer's private signal becomes more precise. The commodity price aggregates the goods producers' signals through their demand for the commodity and therefore becomes more informative. The parameter τ_ξ measures the amount of noise in the supply shock. As τ_ξ increases, there is less noise from the supply side interfering with the commodity price reflecting $\log A$. Thus, the price also becomes more informative.

The effect of η is more subtle. As η increases, there is greater complementarity in each goods producer's production decision. Consistent with the insight of Morris and Shin (2002), such complementarity induces each producer to put greater weight on the publicly observed price signal and lesser weight on its own private signal, which makes the equilibrium price less informative.

C. Price Elasticity

The coefficient l_P, derived in (12), measures the price elasticity of each goods producer's commodity demand. The standard cost effect suggests that a higher price leads to a lower demand. The producer's optimal production decision in equation (7), however, also indicates a second effect through the term in the numerator—a higher price signals a stronger global economy and greater production by other producers. This informational effect motivates each producer to increase its production and thus demand more of the commodity. The price elasticity l_P nets these two offsetting effects. The following proposition shows that, under certain necessary and sufficient conditions, the informational effect dominates the cost effect and leads to a positive l_P.

PROPOSITION 6: *Two necessary and sufficient conditions ensure that $l_P > 0$: first,*

$$\tau_\xi/\tau_A > 4k^{-1}(1-\phi+k^{-1}),$$

and, second, parameter η is within the range

$$1-\frac{1}{\phi}+\frac{k\tau_\xi\tau_s}{4\phi\tau_A^2}(1-\rho)^2 < \eta < 1-\frac{1}{\phi}+\frac{k\tau_\xi\tau_s}{4\phi\tau_A^2}(1+\rho)^2,$$

where $\rho = \tau_A^{1/2}\tau_\xi^{-1/2}\sqrt{\tau_\xi/\tau_A - 4k^{-1}(1-\phi+k^{-1})}$.

For the informational effect to be sufficiently strong, the commodity price has to be sufficiently informative. The conditions in Proposition 6 reflect this observation. First, the supply noise needs to be sufficiently small (i.e., τ_ξ sufficiently large relative to τ_A) so that the price can be sufficiently informative. Second, η needs to be within an intermediate range, which results from two offsetting forces. On the one hand, a larger η implies greater complementarity in producers' production decisions and thus each producer cares more about other producers' production decisions and assigns greater weight to the public price signal in its own decision making. On the other hand, a larger η also implies a less informative price signal (Proposition 5), which motivates each producer to be less responsive to the price. Netting out these two forces dictates that η needs to be in an intermediate range for $l_P > 0$.[10]

This second condition implies that, when $\eta = 0$, $l_P < 0$. Therefore, in the absence of production complementarity, the price elasticity is always negative, that is, the cost effect always dominates the informational effect.

D. Feedback Effect on Demand

In the perfect-information benchmark (Proposition 3), the supply shock ξ decreases the commodity price and increases the aggregate demand through the standard cost effect. In the presence of informational frictions, however, the supply shock, by distorting the price signal, has a more subtle effect on commodity demand.

By substituting equation (8) into (11), the commodity demand of producer i is

$$\log X_i = l_s s_i + l_P h_A \log A + l_P h_\xi \xi + l_P h_0 + l_0.$$

[10] Upward-sloping demand for an asset may also arise from other mechanisms even in the absence of informational frictions highlighted in our model, such as income effects, complementarity in production, and complementarity in information production (e.g., Hellwig, Kohls, and Veldkamp (2012)).

The producers' aggregate commodity demand is then

$$\log\left[\int_{-\infty}^{\infty} X_i\left(s_i, P_X\right) d\Phi\left(\varepsilon_i\right)\right] = l_P h_\xi \xi + \left(l_s + l_P h_A\right)\log A + l_0 + l_P h_0 + \frac{1}{2} l_s^2 \tau_s^{-1}.$$

Note that $h_\xi < 0$ (Proposition 2) and the sign of l_P is undetermined (Proposition 6). Thus, the effect of ξ on the aggregate demand is also undetermined.

Under the conditions given in Proposition 6, an increase in ξ decreases the aggregate demand, which is the opposite of the perfect-information benchmark. This effect arises through the informational channel. As ξ rises, the commodity price falls. Since goods producers cannot differentiate a price decrease caused by ξ from one caused by a weaker global economy, they partially attribute the reduced price to a weaker economy. This, in turn, motivates them to cut their commodity demand. Under the conditions given in Proposition 6, this informational effect is sufficiently strong to dominate the effect of a lower cost to acquire the commodity, leading to a lower aggregate commodity demand.

Furthermore, through its informational effect on aggregate demand, ξ can further push down the commodity price in addition to its price effect in the perfect-information benchmark. This explains why h_ξ is more negative in this economy than in the benchmark (Proposition 4): informational frictions amplify the negative price impact of ξ.[11]

E. Social Welfare

By distorting the commodity price and aggregate demand, informational frictions distort producers' production decisions and households' goods consumption. We now evaluate the unconditional expected social welfare at time 1:

$$W = \mathrm{E}\left[\int_0^1 \left(\frac{C_i}{1-\eta}\right)^{1-\eta}\left(\frac{C_i^*}{\eta}\right)^{\eta} di\right] - E\left[\frac{k}{1+k} e^{-\xi/k} X_S^{\frac{1+k}{k}}\right],$$

which contains two parts. The first part comes from aggregating the expected utility from goods consumption of all island households, and the second part comes from the commodity suppliers' cost of supplying labor.

The next proposition proves that informational frictions reduce the expected social welfare relative to the perfect-information benchmark.

PROPOSITION 7: *In the presence of informational frictions, the expected social welfare is strictly lower than that in the perfect-information benchmark.*

[11] One can also evaluate this informational feedback effect of the supply noise by comparing the equilibrium commodity price relative to another benchmark case, in which each goods producer makes his production decision based only on his private signal s_i without conditioning it on the commodity price P_X. In this benchmark, the commodity price $\log P_X$ is also a log-linear function of $\log A$ and ξ. Interestingly, despite the presence of informational frictions, the price coefficient on ξ is $-\frac{1-\phi}{1+k(1-\phi)}$, which is the same as that derived in Proposition 3 for the perfect-information benchmark. This outcome establishes the informational feedback mechanism as the driver for h_ξ to be more negative than that in the perfect-information benchmark.

IV. A Model Extension

Stimulated by the large inflow of investment capital to commodity futures markets in recent years, there is an ongoing debate about whether speculation in futures markets might have affected commodity prices.[12] In this debate, an influential argument posits that, as the trading of financial traders in futures markets does not directly affect the supply and demand of physical commodities, there is no need to worry about them affecting commodity prices. This argument ignores the informational role of futures prices. In practice, the lower cost of trading futures contracts compared with trading physical commodities encourages greater participation and facilitates aggregation of dispersed information among market participants.[13] To reduce this confusion, we extend our model to incorporate a futures market. For the sake of brevity, we briefly summarize the extended model and the key result in this section and relegate a more detailed model description and analysis to the Internet Appendix.

A. Model Setting

The objective of this extension is not to provide a general model of information aggregation with both spot and futures markets. Instead, we use a specific yet realistic setting to highlight the conceptual point that informational noise introduced by futures market trading can feed back to commodity demand and spot prices.

We introduce a new date $t = 0$ before the two dates $t = 1$ and 2 in the main model, and a centralized futures market at $t = 0$ for delivery of the commodity at $t = 1$. All agents can take positions in the futures market at $t = 0$, and can choose to revise or unwind their positions before delivery at $t = 1$. The ability to unwind positions before delivery reduces transaction costs and makes futures market trading appealing in practice.

We maintain all of the agents in the main model—island households, goods producers, and commodity suppliers—and add a group of financial traders. These financial traders take a position in the futures market at $t = 0$ and then unwind this position at $t = 1$ without taking delivery. We assume that there is no spot market trading at $t = 0$. A spot market naturally emerges at $t = 1$ through commodity delivery for the futures market.

Table II specifies the timeline of the extended model. The timing of information flow is key to our analysis. We assume that goods producers receive their respective private signals $\{s_i\}$ about the global productivity at $t = 0$ and commodity suppliers observe their supply shock ξ only at $t = 1$. This structure

[12] Since the mid-2000s, commodity futures has become a new asset class for portfolio investors such as pension funds and endowments, which regularly allocate a fraction of their portfolios to investing in commodity futures and swap contracts. As a result, capital on the order of hundreds of billions of dollars flowed to the long side of commodity futures markets. This process is also called the financialization of commodity markets (e.g., Cheng and Xiong (2014)).

[13] Roll (1984) systematically analyzes the role of the futures market of orange juice in efficiently aggregating information about weather in Central Florida, which produces more than 98% of the U.S. orange output. Garbade and Silber (1983) provide evidence that futures markets play a more important role in information discovery than cash markets for a set of commodities.

Table II
Timeline of the Extended Model

	$t=0$ Futures Market	$t=1$ Spot Market	$t=2$ Goods Market
Households			Trade/Consume Goods
Producers	Observe Signals Long Futures	Take Delivery Produce Goods	
Suppliers	Short Futures	Observe Supply Shock Deliver Commodity	
Fin Traders	Long/Short Futures	Unwind Position	

leads to two rounds of information aggregation: trading in the futures market at $t = 0$ serves as the first round with informational noise originating from the trading of financial traders, and trading in the spot market at $t = 1$ serves as the second round with financial traders unwinding their futures position and commodity suppliers observing their supply shock. We keep the same specification for the island households, who trade and consume both home and away goods at $t = 2$ as described in Section I.A.

We allow the goods producers to have the same production technology and private signals as specified in Section I.B. At $t = 1$, the producer optimizes its production decision X_i based on the objective function given in (3) and an expanded information set $\mathcal{I}_i^1 = \{s_i, F, P_X\}$, where F is the futures price traded at $t = 0$ and P_X is the spot price traded at $t = 1$, which is given by

$$X_i = \left\{\phi \mathrm{E}\left[AX_j^{\phi\eta}|\mathcal{I}_i^1\right] P_X\right\}^{1/(1-\phi(1-\eta))}.$$

At $t = 0$, the producer chooses a futures position $\tilde{X}_i$ to maximize the following expected production profit based on its information set $\mathcal{I}_i^0 = \{s_i, F\}$:

$$\max_{\tilde{X}_i} \mathrm{E}\left[P_i Y_i |\, \mathcal{I}_i^0\right] - F\tilde{X}_i.$$

In specifying this objective function, we adopt a simplification by assuming the producer is myopic at $t = 0$ (i.e., it treats $\tilde{X}_i$ as its production input at $t = 1$).[14] The producer's futures position is then

$$\tilde{X}_i = \left\{\phi \mathrm{E}\left[A\tilde{X}_j^{\phi\eta}|\mathcal{I}_i^0\right] F\right\}^{1/(1-\phi(1-\eta))}.$$

We assume that, in the futures market at $t = 0$, the aggregate long position of financial traders and goods producers is given by the aggregate position of

[14] In other words, at $t = 0$ each producer chooses a futures position as if it commits to taking full delivery and using the good for production, even though the producer can revise its production decision based on the updated information at $t = 1$. While this simplifying assumption affects each producer's trading profit, it is innocuous for our analysis of how the futures price feeds back to producers' later production decisions because each producer still makes good use of its information and the futures price is informative by aggregating each producer's information.

producers multiplied by a factor $e^{\kappa \log A+\theta}$: $e^{\kappa \log A+\theta}\int_{-\infty}^{\infty}\tilde{X}_i(s_i, F)d\Phi(\varepsilon_i)$, where the factor $e^{\kappa \log A+\theta}$ represents the contribution of financial traders. This multiplicative specification is useful for ensuring the tractable log-linear equilibrium of our model. The component $\kappa \log A$, where $\kappa > 0$, captures the possibility that the trading of financial traders is partially driven by their knowledge of the global fundamental $\log A$, while the other component $\theta \backsim \mathcal{N}(\overline{\theta}, \tau_\theta^{-1})$, a random Gaussian variable with mean $\overline{\theta}$ and variance τ_θ^{-1}, captures trading not related to the fundamental induced by diversification motives and is unobservable to other market participants.

We allow the commodity suppliers to have the same convex cost function specified in Section I.C. At $t = 1$, they observe their supply shock ξ and their marginal cost of supplying the commodity determines the spot price P_X. At $t = 0$, the suppliers take a short position in the futures market. To simplify the analysis, we assume that the suppliers are also myopic in believing that goods producers will take full delivery of their futures positions. Thus, the suppliers choose an initial short position to maximize the profit from making delivery of $e^{-(\kappa \log A+\theta)}\tilde{X}_S$ units of the commodity to goods producers:

$$\max_{\tilde{X}_S} \mathrm{E}\left[Fe^{-(\kappa \log A+\theta)}\tilde{X}_S \middle| \mathcal{I}_S^0\right] - \mathrm{E}\left[\frac{k}{1+k}e^{-\xi/k}\left(e^{-(\kappa \log A+\theta)}\tilde{X}_S\right)^{\frac{1+k}{k}} \middle| \mathcal{I}_S^0\right],$$

from which it follows that

$$\tilde{X}_S = e^{\bar{\xi}-\sigma_\xi^2/2k}\left\{\mathrm{E}\left[e^{-(\kappa \log A+\theta)}\middle| \mathcal{I}_S^0\right]/\,\mathrm{E}\left[e^{-\frac{1+k}{k}(\kappa \log A+\theta)}\middle| \mathcal{I}_S^0\right]\right\}^k F^k.$$

B. The Equilibrium and Key Result

We analyze the joint equilibrium of all markets: the goods markets between each pair of matched islands at $t = 2$, the spot market for the commodity at $t = 1$, and the futures market at $t = 0$. We derive a unique log-linear equilibrium of these markets in the Internet Appendix, and summarize only the key features of the equilibrium here.

During the first round of trading in the futures market at $t = 0$, the futures price aggregates the goods producers' private signals and is a log-linear function of $\log A$ and θ:

$$\log F = \tilde{h}_A \log A + \tilde{h}_\theta \theta + \tilde{h}_0, \tag{13}$$

where $\tilde{h}_A > 0$ and $\tilde{h}_\theta > 0$. The futures price does not fully reveal the global productivity $\log A$ because of the noise θ originated from the trading of financial traders.

The spot price that emerges from the commodity delivery at $t = 1$ represents another round of information aggregation by pooling together the goods producers' demand for delivery. As a result of the arrival of the supply shock ξ, the spot price $\log P_X$ does not fully reveal $\log A$ or θ, but instead reflects a linear combination of $\log A$, or $\log F$, and ξ:

$$\log P_X = h_A \log A + h_F \log F + h_\xi \xi + h_0, \tag{14}$$

where $h_A > 0$, $h_F > 0$, and $h_\xi < 0$.

Despite the updated information from the spot price at $t = 1$, the informational content of $\log F$ is not subsumed by the spot price, and still has an influence on goods producers' expectations of the global productivity. As a result of this informational role, the commodity consumed by producer i at $t = 1$ is increasing with $\log F$:

$$\log X_i = l_s s_i + l_F \log F + l_P \log P_X + l_0, \tag{15}$$

where $l_s > 0$ and $l_F > 0$. The coefficient on the spot price l_P has an undetermined sign, which reflects the offsetting cost effect and informational effect of the spot price, similar to our characterization of the main model.

While the trading of financial traders does not have any direct effect on commodity supply and demand, it affects the futures price, through which it can further impact commodity demand and the spot price. By substituting equation (13) into (14), we express the spot price $\log P_X$ as a linear combination of the primitive shocks $\log A$, θ, and ξ:

$$\log P_X = (h_A + h_F \tilde{h}_A) \log A + h_F \tilde{h}_\theta \theta + h_\xi \xi + h_F \tilde{h}_0 + h_0. \tag{16}$$

This expression shows that θ, the noise from financial traders' futures position, has a positive effect on the spot price. Furthermore, by substituting (16) and (13) into (15) and then integrating the individual producers' commodity demands, their aggregate demand is

$$\begin{aligned} \log\left[\int_{-\infty}^{\infty} X(s_i, F, P_X)\, d\Phi(\varepsilon_i)\right] &= [l_s + l_P h_A + l_F \tilde{h}_A + l_P h_F \tilde{h}_A] \log A \\ &\quad + (l_F + l_P h_F)\tilde{h}_\theta \theta + l_P h_\xi \xi + (l_F + l_P h_F)\tilde{h}_0 \\ &\quad + l_P h_0 + l_0 + \frac{1}{2} l_s^2 \tau_s^{-1}. \end{aligned} \tag{17}$$

We can further derive that the coefficient on θ in the aggregate commodity demand is

$$l_F + l_P h_F = k h_F > 0.$$

Thus, θ also has a positive effect on aggregate commodity demand.

The effects of θ on commodity demand and the spot price clarify the simple yet important conceptual point that traders in commodity futures markets, who never take or make physical delivery, can nevertheless impact commodity markets through the informational feedback channel of commodity futures prices. Information frictions in the futures market, originating from the unobservability of the positions of different participants, are essential for this feedback effect. In the Internet Appendix, we further derive that, as $\tau_\theta \to \infty$ (i.e., the position of financial traders becomes publicly observable), the spot market equilibrium converges to the perfect-information benchmark. This result highlights the importance of improving transparency in futures markets.

V. Implications

In this section, we discuss implications of our model for several empirical issues: estimating the effects of supply and demand shocks, detecting speculative effects in commodity markets, and understanding the puzzling commodity price boom in 2007 to 2008.

A. Estimating Effects of Supply and Demand Shocks

The feedback effect of commodity prices has important implications for studies of the effects of supply and demand shocks in commodity markets. For example, Hamilton (1983) emphasizes that disruptions to oil supply and resulting oil price increases can have a significant impact on the real economy, while Kilian (2009) argues that aggregate demand shocks have a bigger impact on the oil market than previously thought. As supply and demand shocks have opposite effects on oil prices, it is important to isolate their respective effects.

Existing literature commonly uses structural vector autoregressions (SVARs) to decompose historical commodity price dynamics. The premise of these structural models is that, while researchers cannot directly observe the shocks that hit commodity markets, agents in the economy are able to observe the shocks and optimally respond to them. As highlighted by our model, it is unrealistic to assume that agents can perfectly differentiate different types of shocks. In particular, our model shows that, in the presence of informational frictions, supply shocks and demand shocks can have effects in sharp contrast to standard intuition developed from perfect-information settings. These contrasts render the structural models that ignore informational frictions unreliable and potentially misleading.

We now use the popular SVAR model developed by Kilian (2009) for the global oil market as an example. This model specifies the dynamics for a vector $\mathbf{z}_t$ as

$$\mathbf{A}_0\mathbf{z}_t = \alpha + \sum_{i=1}^{24}\mathbf{A}_i\mathbf{z}_{t-i} + \varepsilon_t.$$

The vector $\mathbf{z}_t$ contains three variables: global crude oil production, a measure of real activity, and the spot price for oil. The vector ε_t contains serially uncorrelated and mutually independent structural shocks that hit the global oil market from different sources, such as an oil supply shock, a global demand shock, and an oil-specific demand shock. By imposing various restrictions on the matrix $\mathbf{A}_0$ (which is assumed to be invertible), the model recovers the structural shocks ε_t from shocks $\mathbf{e}_t$ estimated from a reduced-form VAR model for $\mathbf{z}_t$ according to $\varepsilon_t = \mathbf{A}_0\mathbf{e}_t$. Without going through the specific restrictions, the restrictions imposed in the literature are typically motivated by conventional wisdom regarding how supply, demand, and the spot price should react to the structural shocks under the implicit assumption that agents can directly observe them. Under this assumption, the structural shocks ε_t and the innovations to $\mathbf{z}_t$ (i.e., $\mathbf{e}_t$) are informationally equivalent.

It is important to recognize that, in practice, agents observe neither the structural shocks ε_t nor the full vector $\mathbf{z}_t$. While agents can observe oil prices in a timely fashion, they observe quantity variables such as global oil production and GDP with a substantial delay on the order of several quarters.[15] This delay in observing the full vector $\mathbf{z}_t$ makes it impossible for agents to fully recover the structural shocks. Instead, they have to rely on what they can observe at the time to partially infer these shocks. Thus, by assuming that agents can directly observe the structural shocks, the model by Kilian (2009) ignores the realistic informational frictions that agents face in the global oil market. Without a systematic comparison using a correctly specified model, it is difficult to precisely determine the consequences of the misspecification. According to our model, since agents cannot disentangle supply and demand shocks, they partially attribute the observed price change caused by a positive supply shock to a weaker global economy. As a result, they reduce their own commodity demand, which amplifies the price impact of the initial supply shock. Therefore, by ignoring this learning effect induced by informational frictions, the misspecified SVAR model is likely to understate the effect of supply shocks and overstate of the effect of demand shocks.

Our model provides the basic ingredients for constructing more complete empirical models that account for informational frictions faced by economic agents. Ideally, one would want to build a full economic model that systematically accounts for how commodity prices aggregate agents' dispersed information and how each agent forms its expectations based on publicly observed commodity prices together with its own private signal. Even without such a model, one can still extend the more practical SVAR approach to explicitly account for the information set available to agents at the time they make their decisions. According to our analysis, the key is to account for how commodity prices impact agents' expectations.

B. Detecting Speculative Effects

In the ongoing debate on whether speculation has affected commodity prices during the commodity market boom and bust of 2007 to 2008, many studies (e.g., Juvenal and Petrella (2012), Knittel and Pindyck (2013), and Kilian and Murphy (2014)) adopt an inventory-based detection strategy. This strategy

[15] This delay results from the fact that it often takes several quarters for different countries to report both their GDP and their supply of and demand for crude oil, and some countries may even choose not to report at all. The measure of real activity used by Kilian (2009) builds on an index from dry bulk cargo freight rates. This index, while useful, is more a measure of expectations than a direct indicator of real activity.

builds on the widely held argument that, if speculators artificially drive up the commodity price, consumers will find consuming the commodity too expensive and thus reduce consumption, causing inventory of the commodity to spike.

Under this argument, price increases in the absence of inventory increases are explained by fundamental demand. Consequently, price effects induced by speculation should be limited to price increases that are accompanied by contemporaneous increases in inventory. Motivated by this argument, the literature, as reviewed by Fattouh, Kilian, and Mahadeva (2012), tends to use the lack of pronounced oil inventory spike before the July 2008 peak in oil prices as evidence ruling out any significant role played by speculation during the oil price boom.

Despite the intuitive appeal of this inventory-based detection strategy, it ignores important informational frictions faced by consumers in reality. Like the SVAR models we discussed earlier, it crucially relies on the assumption that oil consumers observe global economic fundamentals and are therefore able to recognize whether current oil prices are too high relative to fundamentals in making their consumption decisions. This assumption is unrealistic during periods with great economic uncertainty, especially during 2007 to 2008 when consumers faced severe informational frictions in inferring the strength of the global economy.

Our model illustrates a counterexample to this widely used detection strategy. Under the conditions specified by Proposition 6, the price elasticity of the goods producers' commodity demand is positive.[16] In such an environment, where goods producers have a positive demand elasticity, if speculation drives up the commodity price, the increased price will also cause goods producers to consume more rather than less of the commodity by influencing their expectations about the strength of the global economy. Our model therefore shows that, in the presence of severe informational frictions, speculation can drive up commodity prices without necessarily reducing commodity consumption and boosting inventory. This insight points to the weak power of the widely used inventory-based strategy in detecting speculative effects. In this light, the absence of a pronounced oil inventory spike before the July 2008 peak in oil prices cannot be taken as evidence rejecting the presence of a speculative effect during this period.

C. Understanding the Commodity Price Boom of 2007 to 2008

In the aftermath of the synchronized price boom and bust of major commodities in 2007 to 2008, the price boom has been attributed to the combination of rapidly growing demand from emerging economies and stagnant supply (e.g., Hamilton (2009)). This argument is popular for explaining the commodity price increases before 2008. However, oil prices continued to rise over 40%, peaking at $147 per barrel, from January to July 2008 at a time when the United States had already entered a recession (in November 2007 as dated

[16] We can also provide similar conditions for the extended model.

by the NBER), Bear Stearns had collapsed (in March 2008), and most other developed economies were already showing signs of weakness. While emerging economies remained strong at the time, it is difficult to argue, in hindsight, that their growth sped up so much as to be able to offset the weakness of the developed economies and cause oil prices to rise another 40%.

The informational frictions faced by market participants can help us understand this puzzling price boom. As a result of the lack of reliable data on emerging economies, it was difficult to precisely measure their economic strength in real time. The prices of crude oil and other commodities were regarded as important price signals.[17] This environment makes our model particularly appealing for linking the large commodity price increases in early 2008 to the concurrent large inflow of investment capital, motivated by many portfolio managers seeking to diversify their portfolios out of declining stock markets and into the more promising commodity futures markets (e.g., Tang and Xiong (2012)). By pushing up commodity futures prices and sending a wrong price signal, the large investment flow might have confused goods producers across the world into believing that emerging economies were stronger than they actually were. This distorted expectation could have prevented the producers from reducing their commodity demand despite the high commodity prices, which in turn made the high prices sustainable. Even though more information would eventually correct the producers' expectations, the high commodity prices persisted for several months before their collapse in the second half of 2008. Interestingly, after oil prices dropped from their peak of $147 to $40 per barrel at the end of 2008, oil demand largely evaporated and inventory piled up, despite the much lower prices.

Taken together, the commodity price boom of 2007 to 2008 was not necessarily a price bubble detached from economic fundamentals. Instead, it is plausible to argue that, in the presence of severe informational frictions in early 2008, the large inflow of investment capital might have distorted signals coming from commodity prices and led to confusion among market participants about the strength of emerging economies. This confusion, in turn, could have amplified the boom and bust of commodity prices, which echoes Singleton's (2014) emphasis on accounting for agents' expectations in explaining this price cycle. To systematically examine this hypothesis would require estimating a structural model that explicitly accounts for the informational feedback effect of commodity prices.

[17] Consistent with this notion, in explaining the decision of the European Central Bank (ECB) to raise its key interest rate in March 2008 on the eve of the worst economic recession since the Great Depression, ECB policy reports cite high prices of oil and other commodities as a key factor, suggesting the significant influence of commodity prices on the expectation of central bankers. Furthermore, Hu and Xiong (2013) provide evidence that, in recent years, stock prices across East Asian economies have displayed significant and positive reactions to overnight futures price changes of a set of commodities traded in the United States, suggesting that people across the world regard commodity futures prices as barometers of the global economy.

VI. Conclusion

This paper develops a tractable model to analyze effects of informational frictions in commodity markets. Our model shows that, through the informational role of commodity prices, goods producers' commodity demand can increase with the price, and supply shocks can have an amplified effect on the price and an undetermined effect on producers' demand. By further incorporating one round of futures market trading, our extended model shows that futures prices can also serve as important price signals, even when goods producers also observe spot prices. Thus, through the same informational channel, noise in futures market trading can also interfere with goods producers' expectations and affect their commodity demand. Our analysis highlights the weakness common in empirical and policy studies of assuming that different shocks are publicly observable to market participants. Our analysis also provides a coherent argument for how the large inflow of investment capital to commodity futures markets, by jamming commodity price signals and leading to confusion about the strength of emerging economies, might have amplified the boom and bust of commodity prices in the 2007 to 2008 period.

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Appendix: Proofs of Propositions

PROOF OF PROPOSITION 1: Consider the maximization problem of the household on island i:

$$\max_{\{C_i\}_{i\in[0,1]}} \left(\frac{H_i}{1-\eta_H}\right)^{1-\eta_H} \left\{\frac{1}{\eta_H}\left(\frac{C_i\,(i)}{1-\eta_c}\right)^{1-\eta_c}\left(\frac{\int_{[0,1]/i} C_j\,(i)\,dj}{\eta_c}\right)^{\eta_c}\right\}^{\eta_H}$$

subject to the budget constraint

$$P_H + \int_0^1 P_j C_j\,(i)\,dj = P_i A l_i. \tag{A1}$$

The first-order conditions with respect to C_i and C_i^* are

$$\left(\frac{C_i^*}{C_i}\right)^{\eta}\left(\frac{1-\eta}{\eta}\right)^{\eta} = \lambda_i P_i, \tag{A2}$$

$$\left(\frac{C_i}{C_i^*}\right)^{1-\eta}\left(\frac{\eta}{1-\eta}\right)^{1-\eta} = \lambda_i P_j, \tag{A3}$$

where λ_i is the Lagrange multiplier for his budget constraint. Dividing equations (A2) and (A3) leads to $\frac{\eta}{1-\eta}\frac{C_i}{C_i^*} = \frac{P_j}{P_i}$, which is equivalent to $P_j C_i^* = \frac{\eta}{1-\eta} P_i C_i$. By substituting this equation back to the household's budget constraint in (A1), we obtain $C_i = (1-\eta)Y_i$.

Market clearing of the island's produced goods requires $C_i + C_j^* = Y_i$, which implies that $C_j^* = \eta Y_i$. The symmetric problem of the household of island j implies that $C_j = (1-\eta)Y_j$, and market clearing of the goods produced by island j implies $C_i^* = \eta Y_j$.

The first-order condition in equation (A2) also gives the price of the goods produced by island i. Since the household's budget constraint in (A1) is entirely in nominal terms, the price system is only identified up to λ_i, the Lagrange multiplier. Following Angeletos and La'O (2013), we normalize λ_i to one. Then,

$$P_i = \left(\frac{C_i^*}{C_i}\right)^\eta \left(\frac{1-\eta}{\eta}\right)^\eta = \left(\frac{\eta Y_j}{(1-\eta)\,Y_i}\right)^\eta \left(\frac{1-\eta}{\eta}\right)^\eta = \left(\frac{Y_j}{Y_i}\right)^\eta.$$

□

PROOF OF PROPOSITION 2: We first conjecture that the commodity price and each goods producer's commodity demand take the following log-linear forms:

$$\log P_X = h_0 + h_A \log A + h_\xi \xi, \tag{A4}$$

$$\log X_i = l_0 + l_s s_i + l_P \log P_X, \tag{A5}$$

where the coefficients h_0, h_A, h_ξ, l_0, l_s, and l_P will be determined by equilibrium conditions.

Define

$$z \equiv \frac{\log P_X - h_0 - h_\xi \bar{\xi}}{h_A} = \log A + \frac{h_\xi}{h_A}(\xi - \bar{\xi}),$$

which is a sufficient statistic of information contained in the commodity price P_X. Then, conditional on observing its private signal s_i and the commodity price P_X, goods producer i's expectation of $\log A$ is

$$\mathrm{E}[\log A \mid s_i, \log P_X] = \mathrm{E}[\log A \mid s_i, z] = \frac{1}{\tau_A + \tau_s + \frac{h_A^2}{h_\xi^2}\tau_\xi}\left(\tau_A \bar{a} + \tau_s s_i + \frac{h_A^2}{h_\xi^2}\tau_\xi z\right),$$

and its conditional variance of $\log A$ is

$$\mathrm{var}[\log A \mid s_i, \log P_X] = \left(\tau_A + \tau_s + \frac{h_A^2}{h_\xi^2}\tau_\xi\right)^{-1}.$$

According to equation (7),

$$\log X_i = \frac{1}{1-\phi(1-\eta)}\{\log\phi + \log(\mathrm{E}[AX_j^{\phi\eta} \mid s_i, \log P_X]) - \log P_X\}. \quad \text{(A6)}$$

By using equation (A5), we obtain

$$\begin{aligned}
\mathrm{E}\big[AX_j^{\phi\eta} \mid s_i, \log P_X\big] &= \mathrm{E}\{\exp[\log A + \phi\eta(l_0 + l_s s_j + l_P \log P_X) \mid s_i, z]\} \\
&= \exp\left[\phi\eta\left(l_0 + l_P \log P_X\right)\right] \cdot \mathrm{E}\left[\exp\left((1+\phi\eta l_s)\log A + \phi\eta l_s \varepsilon_j\right) | s_i, \log P_X\right] \\
&= \exp\Big\{\phi\eta\left(l_0 + l_P \log P_X\right) + (1+\phi\eta l_s)\,\mathrm{E}\left[\log A \mid s_i, \log P_X\right] \\
&\quad + \frac{(1+\phi\eta l_s)^2}{2}\mathrm{var}\left[\log A \mid s_i, \log P_X\right] + \frac{\phi^2\eta^2 l_s^2}{2}\mathrm{var}\left[\varepsilon_j \mid s_i, \log P_X\right] \\
&\quad + (1+\phi\eta l_s)\,\phi\eta l_s \mathrm{cov}\left[\varepsilon_j \log A \mid s_i, \log P_X\right]\Big\}.
\end{aligned}$$

By recognizing that $\mathrm{cov}[\varepsilon_j \log A \mid s_i, \log P_X] = 0$ and substituting in the expressions of $\mathrm{E}[\log A \mid s_i, \log P_X]$, $\mathrm{var}[\log A \mid s_i, \log P_X]$, and $\mathrm{var}[\varepsilon_j \mid s_i, \log P_X]$, we can further simplify the expression of $\mathrm{E}[AX_j^{\phi\eta} \mid s_i, \log P_X]$. Equation (A6) then gives

$$\begin{aligned}
\log X_i &= \frac{1}{1-\phi(1-\eta)}\log\phi + \frac{\phi\eta}{1-\phi(1-\eta)}l_0 + \frac{1}{1-\phi(1-\eta)}(\phi\eta l_P - 1)\log P_X \\
&\quad + \left(\frac{1+\phi\eta l_s}{1-\phi(1-\eta)}\right)\left(\tau_A + \tau_s + \frac{h_A^2}{h_\xi^2}\tau_\xi\right)^{-1}\left(\tau_A\bar{a} + \tau_s s_i + \frac{h_A^2}{h_\xi^2}\tau_\xi\frac{\log P_X - h_0 - h_\xi\bar{\xi}}{h_A}\right) \\
&\quad + \frac{(1+\phi\eta l_s)^2}{2(1-\phi(1-\eta))}\left(\tau_A + \tau_s + \frac{h_A^2}{h_\xi^2}\tau_\xi\right)^{-1} + \frac{\phi^2\eta^2 l_s^2}{2(1-\phi(1-\eta))}\tau_s^{-1}.
\end{aligned}$$

For the above equation to match the conjectured equilibrium position in (A5), the constant term and the coefficients of s_i and $\log P_X$ have to match. We thus obtain the following equations for determining the coefficients in (A5):

$$\begin{aligned}
l_0 &= \left(\frac{1+\phi\eta l_s}{1-\phi(1-\eta)}\right)\left(\tau_A + \tau_s + \frac{h_A^2}{h_\xi^2}\tau_\xi\right)^{-1}\left(\tau_A\bar{a} - \frac{h_A}{h_\xi^2}\tau_\xi\left(h_0 + h_\xi\bar{\xi}\right)\right) \quad \text{(A7)} \\
&\quad + \frac{(1+\phi\eta l_s)^2}{2(1-\phi(1-\eta))}\left(\tau_A + \tau_s + \frac{h_A^2}{h_\xi^2}\tau_\xi\right)^{-1}\frac{\phi\eta}{1-\phi(1-\eta)}l_0 \\
&\quad + \frac{\phi^2\eta^2 l_s^2}{2(1-\phi(1-\eta))}\tau_s^{-1} + \frac{1}{1-\phi(1-\eta)}\log\phi,
\end{aligned}$$

$$l_s = \left(\frac{1+\phi\eta l_s}{1-\phi(1-\eta)}\right)\left(\tau_A + \tau_s + \frac{h_A^2}{h_\xi^2}\tau_\xi\right)^{-1}\tau_s, \quad \text{(A8)}$$

$$l_P = \frac{\phi\eta}{1-\phi(1-\eta)} l_P + \left(\frac{1+\phi\eta l_s}{1-\phi(1-\eta)}\right)\left(\tau_A + \tau_s + \frac{h_A^2}{h_\xi^2}\tau_\xi\right)^{-1} \frac{h_A}{h_\xi^2}\tau_\xi - \frac{1}{1-\phi(1-\eta)}. \quad \text{(A9)}$$

By substituting (A8) into (A9), we have

$$l_s = \frac{1+(1-\phi)l_P}{1-\phi(1-\eta)} \frac{h_\xi^2}{h_A} \tau_s \tau_\xi^{-1}. \quad \text{(A10)}$$

By manipulating (A8), we also have that

$$l_s = \left(\tau_A + \frac{1-\phi}{1-\phi(1-\eta)}\tau_s + \frac{h_A^2}{h_\xi^2}\tau_\xi\right)^{-1} \frac{\tau_s}{1-\phi(1-\eta)}. \quad \text{(A11)}$$

We now use the market-clearing condition for the commodity market to determine three other equations for the coefficients in the conjectured log-linear commodity price and demand. Aggregating (A5) gives the aggregate commodity demand of the goods producers:

$$\begin{aligned}
\int_{-\infty}^{\infty} X(s_i, P_X)\, d\Phi(\varepsilon_i) &= \int_{-\infty}^{\infty} \exp\left[l_0 + l_s s_i + l_P \log P_X\right] d\Phi(\varepsilon_i) \\
&= \int_{-\infty}^{\infty} \exp\left[l_0 + l_s(\log A + \varepsilon_i) + l_P\left(h_0 + h_A \log A + h_\xi \xi\right)\right] d\Phi(\varepsilon_i) \\
&= \exp\left[(l_s + l_P h_A)\log A + l_P h_\xi \xi + l_0 + l_P h_0 + \frac{1}{2} l_s^2 \tau_s^{-1}\right]. \quad \text{(A12)}
\end{aligned}$$

Equation (5) implies that $\log X_S = k \log P_X + \xi$.Thus, the market-clearing condition

$$\log\left[\int_{-\infty}^{\infty} X(s_i, P_X)\, d\Phi(\varepsilon_i)\right] = \log X_S(P_X)$$

requires that the coefficients on $\log A$ and ξ and the constant term be identical on both sides:

$$l_s + l_P h_A = k h_A, \quad \text{(A13)}$$

$$l_P h_\xi = 1 + k h_\xi, \quad \text{(A14)}$$

$$l_0 + l_P h_0 + \frac{1}{2} l_s^2 \tau_s^{-1} = k h_0. \quad \text{(A15)}$$

Equation (A14) directly implies that

$$l_P = k + h_\xi^{-1}. \quad \text{(A16)}$$

Equations (A13) and (A14) together imply that

$$l_s = -h_\xi^{-1} h_A. \quad \text{(A17)}$$

By combining this equation with (A11), and defining $b = -l_s = h_\xi^{-1} h_A$, we arrive at

$$b^3 + \left(\tau_A + \frac{1-\phi}{1-\phi(1-\eta)}\tau_s\right)\tau_\xi^{-1} b + \frac{\tau_\xi^{-1}\tau_s}{1-\phi(1-\eta)} = 0, \tag{A18}$$

where b is a real root of a depressed cubic polynomial of the form $x^3 + px + q = 0$, which has one real and two complex roots. As p and q are both positive, the left-hand side (LHS) is monotonically increasing in b while the right-hand side (RHS) is fixed. Thus, the real root b is unique and has to be negative: $b < 0$.

Following Cardano's method, the one real root of equation (A18) is given by

$$\begin{aligned} b = &\left(\frac{\tau_\xi^{-1}\tau_s}{2(1-\phi(1-\eta))}\right)^{1/3} \sqrt[3]{-1 + \sqrt{1 + \frac{4}{27}\left(\frac{\tau_\xi^{-1}\tau_s}{1-\phi(1-\eta)}\right)^{-2}\left(\tau_A + \frac{1-\phi}{1-\phi(1-\eta)}\tau_s\right)^3}} \\ &+ \left(\frac{\tau_\xi^{-1}\tau_s}{2(1-\phi(1-\eta))}\right)^{1/3} \sqrt[3]{-1 - \sqrt{1 + \frac{4}{27}\left(\frac{\tau_\xi^{-1}\tau_s}{1-\phi(1-\eta)}\right)^{-2}\left(\tau_A + \frac{1-\phi}{1-\phi(1-\eta)}\tau_s\right)^3}}. \end{aligned} \tag{A19}$$

Since $b = h_\xi^{-1} h_A$, we have $h_\xi = b^{-1} h_A$, which together with our expression for l_s and equations (A10) and (A16) implies that expressions for h_A and h_ξ be given as in (9) and (10). With h_A and h_ξ determined, l_s is then given by (A11), l_P by (A16), h_0 by (A7) as

$$\begin{aligned} h_0 = &\frac{1}{1+k(1-\phi)}\log\phi - \frac{1-\phi(1-\eta)}{1+k(1-\phi)} b\tau_s^{-1}\left(\tau_A\bar{a} - b\tau_\xi\bar{\xi}\right) \\ &+ \frac{1}{2}\frac{1-\phi(1-\eta)}{1+k(1-\phi)}\left(\left(\frac{1-\phi+\phi^2\eta^2}{1-\phi(1-\eta)} + \phi\eta\right)b - 1\right)\tau_s^{-1} b, \end{aligned} \tag{A20}$$

and l_0 by equation (A15) as

$$l_0 = (k - l_P)h_0 - \frac{1}{2} l_s^2 \tau_s^{-1}. \tag{A21}$$

□

PROOF OF PROPOSITION 3: We keep the same setting outlined in the main model, except we let A and ξ be observable by all market participants. We first derive the equilibrium. In this setting, each producer's private signal s_i becomes useless as A is directly observable. We can still use equation (7) to derive producer i's optimal commodity demand. As the producers now share the same information about A, they must have the same expectation about their future trading partners' production decisions. As a result, $X_i = X_j$ for any i and j. Equation (7) therefore implies that in equilibrium $X_i = (\frac{\phi A}{P_X})^{\frac{1}{1-\phi}}$.

Market clearing of the commodity market requires that the producers' aggregate demand equals the commodity supply, that is, $X_i = X_S$. From equation

(5), we must have that $\log X_i = k \log P_X + \xi$. We then obtain $\log P_X$ and $\log X_i$ stated in Proposition 3. It is clear that this equilibrium is unique. $\square$

PROOF OF PROPOSITION 4: As $\tau_s \to \infty$, equation (A18) implies that b goes to $-\frac{1}{1-\phi}$. Consequently, as $\tau_s \to \infty$, equation (9) gives that $h_A \to \frac{1}{1+k(1-\phi)}$, and equation (10) gives that $h_\xi \to -\frac{1-\phi}{1+k(1-\phi)}$. Therefore, both h_A and h_ξ converge to their corresponding values in the perfect-information benchmark.

That $|h_\xi|$ is larger than it is in the perfect-information benchmark is apparent since the numerator of $|h_\xi|$ in equation (10) is positive and larger than $1-\phi$. That h_A is lower follows by substituting equation (A18) into equation (9) to arrive at

$$h_A = \frac{1 + \tau_A \tau_s^{-1} (1 - \phi(1-\eta))\, b}{1 + k(1-\phi)}.$$

Since $b < 0$, it follows that $h_A < \frac{1}{1+k(1-\phi)}$, which is the value of h_A in the perfect-information benchmark. $\square$

PROOF OF PROPOSITION 5: As $l_s = -h_\xi^{-1} h_A$ from (A17), $\pi = \frac{h_A^2/\tau_A}{h_\xi^2/\tau_\xi} = l_s^2 \frac{\tau_\xi}{\tau_A}$. Since $l_s > 0$, it is sufficient to study the behavior of how l_s varies with τ_s and η to understand how π changes with τ_s and η. To see that l_s is monotonically increasing in τ_s, we note that $l_s = -b$, where b is the only real and negative root of equation (A18). Then, by the Implicit Function Theorem it is apparent that

$$\frac{\partial b}{\partial \tau_s} = -\frac{\frac{1-\phi}{1-\phi(1-\eta)}\tau_\xi \tau_s b + \frac{\tau_s \tau_\xi}{1-\phi(1-\eta)}}{3b^2 + \left(\tau_A + \frac{1-\phi}{1-\phi(1-\eta)}\tau_s\right)\tau_\xi^{-1}}\tau_s^{-1} = \frac{b^3 + \tau_A \tau_\xi^{-1} b}{3b^2 + \left(\tau_A + \frac{1-\phi}{1-\phi(1-\eta)}\tau_s\right)\tau_\xi^{-1}}\tau_s^{-1} < 0.$$

Similarly, we have

$$\frac{\partial b}{\partial \eta} = \phi \tau_s \tau_\xi^{-1} \frac{(1-(1-\phi)l_s)\frac{1}{(1-\phi(1-\eta))^2}}{3l_s^2 + \left(\tau_A + \frac{1-\phi}{1-\phi(1-\eta)}\tau_s\right)\tau_\xi^{-1}} > 0.$$

Thus, l_s is increasing in τ_s and decreasing in η, which in turn implies that π is increasing in τ_s and decreasing in η.

To analyze the dependence of π on τ_ξ, we have

$$\frac{\partial \pi}{\partial \tau_\xi} = l_s^2 \frac{1}{\tau_A} + 2 l_s \frac{\tau_\xi}{\tau_A}\frac{\partial l_s}{\partial \tau_\xi} = \frac{1}{\tau_A} b\left(b + 2\tau_\xi \frac{\partial b}{\partial \tau_\xi}\right).$$

By applying the Implicit Function Theorem again, we obtain

$$\frac{\partial b}{\partial \tau_\xi} = \frac{\left(\tau_A + \frac{1-\phi}{1-\phi(1-\eta)}\tau_s\right)\tau_\xi^{-1} b + \frac{\tau_\xi^{-1}\tau_s}{1-\phi(1-\eta)}}{3b^2 + \left(\tau_A + \frac{1-\phi}{1-\phi(1-\eta)}\tau_s\right)\tau_\xi^{-1}}\tau_\xi^{-1} = \frac{-b^3}{3b^2 + \left(\tau_A + \frac{1-\phi}{1-\phi(1-\eta)}\tau_s\right)\tau_\xi^{-1}}\tau_\xi^{-1} > 0.$$

By substituting this into the above expression for $\frac{\partial \pi}{\partial \tau_\xi}$, we find that

$$\frac{\partial \pi}{\partial \tau_\xi} = \frac{b^2}{\tau_A}\left(\frac{b^2 + \left(\tau_A + \frac{1-\phi}{1-\phi(1-\eta)}\tau_s\right)\tau_\xi^{-1}}{3b^2 + \left(\tau_A + \frac{1-\phi}{1-\phi(1-\eta)}\tau_s\right)\tau_\xi^{-1}}\right) > 0.$$

Therefore, π is monotonically increasing in τ_ξ. □

PROOF OF PROPOSITION 6: Based on l_P and h_ξ given in equations (A16) and (10), $l_P > 0$ is equivalent to $b^2 > \frac{k^{-1}\tau_\xi^{-1}\tau_s}{1-\phi(1-\eta)}$, which, as $l_s = -b > 0$, is in turn equivalent to $l_s > l_s^* = \sqrt{\frac{k^{-1}\tau_\xi^{-1}\tau_s}{1-\phi(1-\eta)}}$. In words, this condition states that the commodity price has to be sufficiently informative. As b is the unique real and negative root of equation (A18), this condition is equivalent to the following condition on the LHS of equation (A18): $LHS(-l_s^*) > 0$. By substituting l_s^* into the LHS, we obtain the following condition:

$$-\frac{k^{-3/2}\tau_\xi^{-1}\tau_s}{1-\phi\left(1-\eta\right)} - \left(\tau_A + \frac{1-\phi}{1-\phi\left(1-\eta\right)}\tau_s\right)\tau_\xi^{-1}k^{-1/2} + \sqrt{\frac{\tau_\xi^{-1}\tau_s}{1-\phi\left(1-\eta\right)}} > 0,$$

which, as $1-\phi(1-\eta) > 0$ and defining $u = \sqrt{1-\phi(1-\eta)}$, can be rewritten as

$$u^2 - u\tau_A^{-1}\sqrt{k\tau_\xi\tau_s} + \left(1-\phi+k^{-1}\right)\tau_A^{-1}\tau_s < 0.$$

Note that the LHS of this condition $LHS(u)$ is a quadratic form of u, which has its minimum at $u^* = \frac{1}{2\tau_A}\sqrt{k\tau_\xi\tau_s}$. Thus, this condition is satisfied if and only if the following occurs. First, $LHS(u^*) < 0$, which is equivalent to $\tau_\xi/\tau_A > 4k^{-1}(1-\phi+k^{-1})$, the first condition given in Proposition 6. Second,

$$LHS\left(u\right) = \left(u-u^*\right)^2 - \left[\left(u^*\right)^2 - \left(1-\phi+k^{-1}\right)\tau_A^{-1}\tau_s\right] < 0,$$

which is equivalent to

$$\frac{u - \frac{1}{2\tau_A}\sqrt{k\tau_\xi\tau_s}}{\frac{1}{2}\tau_s^{1/2}\tau_A^{-1}\sqrt{k\tau_\xi - 4\left(1-\phi+k^{-1}\right)\tau_A}} \in \left(-1, 1\right).$$

This leads to the second condition given in Proposition 6. □

PROOF OF PROPOSITION 7: We begin by evaluating the first component of the social welfare from the island households' goods consumption. We denote this component by

$$W^C = \mathrm{E}\left[\int_0^1 \left(\frac{C\left(i\right)}{1-\eta}\right)^{1-\eta}\left(\frac{C^*\left(i\right)}{\eta}\right)^{\eta} di\right].$$

In the perfect-information benchmark, by substituting the symmetric consumption of all island households, the expected social welfare from consumption is

$$\log W_{bench}^{C} = \log \mathrm{E}\left[\int_0^1 AX_i^{\phi} di\right] = \log \mathrm{E}\left[AX_i^{\phi}\right].$$

Given $\log X_i$ derived in Proposition 3, we have

$$\log W_{bench}^{C} = \frac{\phi k}{1+k(1-\phi)}\log\phi + \frac{1+k}{1+k(1-\phi)}\bar{a} + \frac{1}{2}\left(\frac{1+k}{1+k(1-\phi)}\right)^2 \tau_A^{-1} + \frac{\phi}{1+k(1-\phi)}\bar{\xi} + \frac{1}{2}\left(\frac{\phi}{1+k(1-\phi)}\right)^2 \tau_\xi^{-1}.$$

Note that the total goods output in this economy is given by

$$\mathrm{E}\left[Y_{bench}^{aggr}\right] = \mathrm{E}\left[\int_0^1 Y_i di\right] = \mathrm{E}\left[\int_0^1 AX_i^{\phi} di\right] = W_{bench}^{C},$$

which indicates that, in this symmetric equilibrium with perfect information, the expected social welfare from consumption is equal to the expected aggregate goods output.

In the presence of informational frictions, by using Proposition 1, the expected social welfare from consumption is given by

$$\log W^{C} = \log \mathrm{E}\left[A\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} X(s_i, P_X)^{\phi(1-\eta)} X\left(s_j, P_X\right)^{\phi\eta} d\Phi(\varepsilon_i) d\Phi(\varepsilon_j)\right],$$

where, in the second line, an integral over ε_j, that is, noise in the signal of the goods producer of island j, is taken to compute expectation over uncertainty in ε_j. By substituting

$$\log X(\varepsilon_i) = l_0 + l_P \log P_X + l_s s_i = l_0 + l_P \log P_X + l_s(\log A + \varepsilon_i)$$

and $\log P_X = h_0 + h_A \log A + h_\xi \xi$ with our expressions for l_s, l_P, h_ξ, h_A, and b^3 from equation (A18), we obtain

$$\begin{aligned}\log W^{C} = {} & \frac{\phi k}{1+k(1-\phi)}\log\phi + \frac{1+k}{1+k(1-\phi)}\bar{a} + \frac{1}{2}(1+\phi k h_A)^2 \tau_A^{-1} + \frac{\phi}{1+k(1-\phi)}\bar{\xi} \\ & + \frac{1}{2}\phi^2\left(1+kb^{-1}h_A\right)^2 \tau_\xi^{-1} + \frac{1}{2}\left(1-\frac{1}{\phi}-2\eta(1-\eta)\right)\phi^2 b^2 \tau_s^{-1} \\ & - \frac{1}{2}\frac{\phi k(1-\phi(1-\eta))}{1+k(1-\phi)}\left(1-\left(\frac{1-\phi+\phi^2\eta^2}{1-\phi(1-\eta)}+\phi\eta\right)b\right) b\tau_s^{-1}.\end{aligned}$$

The logarithm of the expected total output in this economy is given by

$$\log \mathrm{E}\left[Y^{aggr}\right] = \log \mathrm{E}\left[\int_{-\infty}^{\infty} Y_i di\right] = \log \mathrm{E}\left[A\int_{-\infty}^{\infty} X(\varepsilon_i)^{\phi}\, d\Phi\left(\varepsilon_i\right)\right]$$

$$= \log \mathrm{E}\left[e^{\phi l_0+\phi l_P \log P_X+(1+\phi l_s)\log A}\int_{-\infty}^{\infty} e^{\phi l_s \varepsilon_i} d\Phi\left(\varepsilon_i\right)\right] = \phi l_0 + \frac{1}{2}\phi^2 l_s^2 \tau_s^{-1}$$

$$+\phi l_P h_0 + \phi l_P h_\xi \bar{\xi} + \frac{1}{2}\phi^2 l_P^2 h_\xi^2 \tau_\xi^{-1} + (1+\phi l_s + \phi l_P h_A)\bar{a}$$

$$+\frac{1}{2}\left(1+\phi l_s+\phi l_P h_A\right)^2 \tau_A^{-1}.$$

Again, by substituting the expressions for l_s, l_P, h_ξ, and h_A, we have

$$\log \mathrm{E}\left[Y^{aggr}\right] = \frac{\phi k}{1+k(1-\phi)}\log\phi + \left(\frac{1+k}{1+k(1-\phi)}\right)\bar{a} + \frac{1}{2}\left(1+\phi k h_A\right)^2 \tau_A^{-1}$$

$$+\phi\left(\frac{1}{1+k(1-\phi)}\right)\bar{\xi} + \frac{1}{2}\phi^2\left(1+kb^{-1}h_A\right)^2 \tau_\xi^{-1} - \frac{1}{2}\phi\left(1-\phi\right)b^2\tau_s^{-1}$$

$$-\frac{1}{2}\frac{\phi k\left(1-\phi\left(1-\eta\right)\right)}{1+k\left(1-\phi\right)}\left(1-\left(\frac{1-\phi+\phi^2\eta^2}{1-\phi\left(1-\eta\right)}+\phi\eta\right)b\right)b\tau_s^{-1}.$$

It is then easy to compute

$$\log \mathrm{E}\left[Y^{aggr}\right] - \log W = 2\phi^2\eta\left(1-\eta\right)b^2\tau_s^{-1} > 0.$$

We now compare expected aggregate goods output with and without informational frictions:

$$\log \mathrm{E}\left[Y^{aggr}\right] - \log \mathrm{E}\left[Y^{aggr}_{bench}\right] = \frac{1}{2}\left(\left(1+\phi k h_A\right)^2 - \left(1+\frac{\phi k}{1+k\left(1-\phi\right)}\right)^2\right)\tau_A^{-1}$$

$$+\frac{1}{2}\phi^2\left(\left(1+kb^{-1}h_A\right)^2 - \left(1-k\frac{1-\phi}{1+k\left(1-\phi\right)}\right)^2\right)\tau_\xi^{-1} - \frac{1}{2}\phi\left(1-\phi\right)b^2\tau_s^{-1}$$

$$-\frac{1}{2}\frac{\phi k\left(1-\phi\left(1-\eta\right)\right)}{1+k\left(1-\phi\right)}\left(1-\left(1-\phi+\phi\eta\left(1+\phi\eta\right)\right)b\right)b\tau_s^{-1}.$$

Substituting with equations (9) and (A18), we arrive at

$$\log \mathrm{E}\left[Y^{aggr}\right] - \log \mathrm{E}\left[Y^{aggr}_{bench}\right] = \frac{1}{2}\frac{\phi k\left(1-\phi\left(1-\eta\right)\right)\tau_s^{-1}b}{\left(1+k\left(1-\phi\right)\right)^2}\left(1+k-\phi b\right)$$

$$-\frac{1}{2}\phi\left(1-\phi\right)b^2\tau_s^{-1} + \frac{1}{2}\frac{\phi k\left(1-\phi\left(1-\eta\right)\right)\tau_s^{-1}b^2}{1+k\left(1-\phi\right)}\left(\frac{1-\phi}{1-\phi\left(1-\eta\right)}\right.$$

$$\left.+\phi^2\eta^2 + \left(\frac{\phi^2\eta^2}{1-\phi\left(1-\eta\right)}-1\right)\phi\left(1-\eta\right)\right).$$

Notice that $\frac{\phi^2\eta^2}{1-\phi(1-\eta)} < 1$ and the first term is negative since $b < 0$. Further note that

$$\frac{1}{2}\frac{\phi k(1-\phi(1-\eta))\tau_s^{-1}b^2}{1+k(1-\phi)}\frac{1-\phi}{1-\phi(1-\eta)}-\frac{1}{2}\phi(1-\phi)b^2\tau_s^{-1}$$
$$=-\left(\frac{1-k\phi}{1+k(1-\phi)}\right)\frac{1}{2}\phi(1-\phi)b^2\tau_s^{-1}<0,$$

and

$$\frac{1}{2}\frac{\phi k(1-\phi(1-\eta))\tau_s^{-1}b^2}{1+k(1-\phi)}\left(\phi^2\eta^2-\frac{\phi-(1+k)b^{-1}}{1+k(1-\phi)}\right)<0,$$

because $\phi^2\eta^2 < \phi$ and, since $\phi < 1$ and $0 > b > -\frac{1}{1-\phi}$, $k(1-\phi)\phi^2\eta^2 + (1+k)b^{-1} < 0$. To see that $b > -\frac{1}{1-\phi}$, we rewrite equation (A18) as

$$b^3+\tau_A\tau_\xi^{-1}b+((1-\phi)b+1)\frac{\tau_s\tau_\xi^{-1}}{1-\phi(1-\eta)}=0,$$

from which it follows that $b > -\frac{1}{1-\phi}$.Therefore, we see that

$$\log \mathrm{E}[Y^{aggr}]-\log E\left[Y^{aggr}_{bench}\right]<0.$$

Given that the expected social welfare from consumption W^C_{bench} is equal to the expected aggregate output $E[Y^{aggr}_{bench}]$ in the perfect-information benchmark, and in the presence of informational frictions the expected social welfare from consumption W^C is strictly less than the expected aggregate goods output $\mathrm{E}[Y^{aggr}]$, the expected social welfare from consumption is lower in the presence of information frictions than in the perfect-information benchmark.

Now we return to the second part of the expected social welfare from commodity suppliers' disutility of labor. We denote this part by

$$W^L=\mathrm{E}\left[\frac{k}{1+k}e^{-\xi/k}X_S^{\frac{1+k}{k}}\right].$$

In the perfect-information benchmark, by using $\log X_S$ derived in Proposition 3, we have

$$\log W^L_{bench}=\log\frac{k}{1+k}+\frac{1+k}{1+k(1-\phi)}\log\phi+\frac{1+k}{1+k(1-\phi)}\bar{a}+\frac{\phi}{1+k(1-\phi)}\bar{\xi}$$
$$+\frac{1}{2}\left(\frac{1+k}{1+k(1-\phi)}\right)^2\tau_A^{-1}+\frac{1}{2}\left(\frac{\phi}{1+k(1-\phi)}\right)^2\tau_\xi^{-1}.$$

In the presence of informational frictions, aggregate demand X_S is given by

$$\log X_S=k\log P_X+\xi=kh_A\log A+\left(kh_\xi+1\right)\xi+kh_0,$$

and therefore the suppliers' disutility of labor reduces to

$$\log W^L = \log \frac{k}{1+k} + (1+k)\,h_0 + (1+k)\,h_A\bar{a} + \left(1+(1+k)\,h_\xi\right)\bar{\xi} + \frac{1}{2}\,(1+k)^2\,h_A^2\tau_A^{-1} + \frac{1}{2}\left(1+(1+k)\,h_\xi\right)^2\tau_\xi^{-1}.$$

We now analyze the overall social welfare $W = W^C - W^L$. We can express the relative welfare in the two economies as

$$\frac{W}{W_{bench}} = \frac{W^C - W^L}{W^C_{bench} - W^L_{bench}} = \frac{W^C}{W^C_{bench}}\frac{1 - W^L/W^C}{1 - W^L_{bench}/W^C_{bench}} < \frac{1 - W^L/W^C}{1 - W^L_{bench}/W^C_{bench}},$$

where the last inequality follows from $W^C < W^C_{bench}$, as proved above.

Note that, in the perfect-information benchmark,

$$\log W^L_{bench} - \log W^C_{bench} = \log \frac{\phi k}{1+k}.$$

Thus, $1 - W^L_{bench}/W^C_{bench} = 1 - \frac{\phi k}{1+k} > 0$. Therefore, it is sufficient to show that $W^L/W^C \geq W^L_{bench}/W^C_{bench}$ to establish that $\frac{W}{W_{bench}} < 1$.

With some manipulation of our expressions for $\log W^L$ and $\log W^C$, and by substituting our expressions for h_A and h_ξ and making use of equation (A18), we arrive at

$$\log(W^L/W^C) = \log \frac{\phi k}{1+k} + \frac{1}{2}b\tau_s^{-1}((1-\phi^2+\phi^2\eta^2+\phi\eta+\phi^2\eta(1-\eta))b - (1-\phi(1-\eta))) - (1-\phi(1-\eta))\tau_s^{-1}b^2 - \frac{1}{2}\left(1-\phi\left(1-\eta\right)\right)^2\tau_s^{-2}b^2\left(\tau_A + \tau_\xi b^2\right).$$

Finally, by invoking equation (A18) to rewrite the last term, we find that

$$\log(W^L/W^C) = \log \frac{\phi k}{1+k}.$$

Thus, $\log(W^L/W^C) = \log(W^L_{bench}/W^C_{bench})$, which in turn establishes the proposition. □

REFERENCES

Angeletos, George-Marios, and Jennifer La'O, 2009, Noisy business cycles, *NBER Macroeconomics Annual* 24, 319–378.

Angeletos, George-Marios, and Jennifer La'O, 2013, Sentiment, *Econometrica* 81, 739–780.

Angeletos, George-Marios, Guido Lorenzoni, and Alessandro Pavan, 2010, Beauty contests and irrational exuberance: A neoclassical approach, Working paper, MIT.

Bond, Philip, Alex Edmans, and Itay Goldstein, 2012, The real effects of financial markets, *Annual Review of Financial Economics* 4, 39–60.

Bray, Margaret, 1981, Futures trading, rational expectations, and the efficient markets hypothesis, *Econometrica* 49, 575–596.

Cheng, Ing-haw, Andrei Kirilenko, and Wei Xiong, 2012, Convective risk flows in commodity futures markets, *Review of Finance* (2014) pp. 1–49 doi:10.1093/rof/rfu043.

Cheng, Ing-haw, and Wei Xiong, 2014, The financialization of commodity markets, *Annual Review of Financial Economics* 6, 419–441.

Fattouh, Bassam, Lutz Kilian, and Lavan Mahadeva, 2012, The role of speculation in oil markets: What have we learned so far? Working paper, University of Michigan.

Garbade, Kenneth, and William L. Silber, 1983, Price movements and price discovery in futures and cash markets, *Review of Economics and Statistics* 65, 289–297.

Goldstein, Itay, Emre Ozdenoren, and Kathy Yuan, 2011, Learning and complementarities in speculative attacks, *Review of Economic Studies* 78, 263–292.

Goldstein, Itay, Emre Ozdenoren, and Kathy Yuan, 2013, Trading frenzies and their impact on real investment, *Journal of Financial Economics* 109, 566–582.

Grossman, Sanford, and Joseph Stiglitz, 1980, On the impossibility of informationally efficient markets, *American Economic Review* 70, 393–408.

Hamilton, James, 1983, Oil and the macroeconomy since World War II, *Journal of Political Economy* 91, 228–248.

Hamilton, James, 2009, Causes and consequences of the oil shock of 2007-2008, *Brookings Papers on Economic Activity*, Spring, 215–259.

Hamilton, James, and Cynthia Wu, 2015, Effects of index-fund investing on commodity futures prices, *International Economic Review* 56, 189–205.

Hellwig, Christian, Sebastian Kohls, and Laura Veldkamp, 2012, Information choice technologies, *American Economic Review Papers & Proceedings* 102, 35–40.

Hellwig, Martin, 1980, On the aggregation of information in competitive markets, *Journal of Economic Theory* 22, 477–498.

Henderson, Brian, Neil Pearson, and Li Wang, 2012, New evidence on the financialization of commodity markets, Working paper, University of Illinois at Urbana-Champaign.

Hu, Conghui, and Wei Xiong, 2013, The informational role of commodity futures prices, in E. Glen Weyl, Edward L. Glaeser, and Tano Santos, eds.: *Apres le Deluge: Finance and the Common Good after the Crisis*, University of Chicago Press, http://www8.gsb.columbia.edu/rtfiles/finance/misc/Information_Commodity4_c.pdf.

Juvenal, Luciana, and Ivan Petrella, 2012, Speculation in the oil market, Working paper, Federal Reserve Bank of St. Louis.

Kilian, Lutz, 2009, Not all oil price shocks are alike: Disentangling demand and supply shocks in the crude oil market, *American Economic Review* 99, 1053–1069.

Kilian, Lutz, and Daniel Murphy, 2014, The role of inventories and speculative trading in the global market for crude oil, *Journal of Applied Econometrics* 29, 454–478.

Knittel, Christopher, and Robert Pindyck, 2013, The simple economics of commodity price speculation, Working paper, MIT.

Lorenzoni, Guido, 2009, A theory of demand shocks, *American Economic Review* 99, 2050–2084.

Morris, Stephen, and Hyun Song Shin, 2002, The social value of public information, *American Economic Review* 92, 1521–1534.

Obstfeld, Maurice, and Ken Rogoff, 1996, *Foundations of International Macroeconomics* (MIT Press, Cambridge, MA).

Ozdenoren, Emre, and Kathy Yuan, 2008, Feedback effects and asset prices, *Journal of Finance* 63, 1939–1975.

Roll, Richard, 1984, Orange juice and weather, *American Economic Review* 74, 861–880.

Singleton, Kenneth, 2014, Investor flows and the 2008 boom/bust in oil prices, *Management Science* 60, 300–318.

Stoll, Hans, and Robert Whaley, 2010, Commodity index investing and commodity futures prices, *Journal of Applied Finance* 20, 7–46.

Subrahmanyam, Avanidhar, and Sheridan Titman, 2001, Feedback from stock prices to cash flows, *Journal of Finance* 56, 2389–2413.

Tang, Ke, and Wei Xiong, 2012, Index investment and financialization of commodities, *Financial Analysts Journal* 68, 54–74.

Supporting Information

Additional Supporting Information may be found in the online version of this article at the publisher's website:

Appendix S1: Internet Appendix.