Game On: Social Networks and Markets

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Abstract

I present closed-form solutions for prices, portfolios, and beliefs in a model where four types of investors trade assets over time: naive investors who learn via a social network, "fanatics" possibly spreading fake news, rational short-term investors, and long-term investors. I show that fanatic and rational views dominate over time due to echo-chamber effects, and their relative importance depends on their following by influencers. Securities markets exhibit social network spillovers, large effects of influencers and thought leaders, bubbles, bursts of high volume, price momentum, fundamental momentum, and reversal. As an illustration of the model, I present empirical evidence on the GameStop stock.

Keywords: Echo chambers, networks, influencers, fake news, social media, pump-and-dump, bubbles

JEL Codes: D85, D91, G12, G14, G4, G5, L14

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Participants in financial markets have always relied on their social network as a source of information, but modern social media are changing the nature of these networks and are making them more observable to researchers. This paper presents a model of how investment ideas can propagate through a social network and affect market behavior and prices. When investors learn through their social network, echo-chamber effects and fake news can lead to disagreement for extensive time periods. This time-varying disagreement can generate a trading frenzy with a spike in turnover, high volatility, price momentum as a bubble builds, and a value effect as the bubble bursts. Rational investors may initially ride the bubble, but eventually bet on reversal to fundamentals. As an illustration of these insights of the model, the paper presents an empirical study of the dramatic events related to the GameStop stock in early 2021.

To study social network effects in the simplest possible way, I introduce rational agents and financial markets into an otherwise standard DeGroot (1974) model. In the DeGroot model, people update their beliefs by listening to other people in their social network. While this method of updating may initially be rational, the continued updating over multiple rounds of communication does not take into account that the same information may echo back many times (DeMarzo et al. (2003)). While standard network models assume that everyone behaves in this naive way (see surveys by Jackson (2010) and Golub and Sadler (2016)), I introduce rational learners into the model to capture the effects of sophisticated professional investors. Characterizing rational behavior in a network can be highly complex,[1] but I show that their updating becomes very tractable when they can use the rational strategy of listening to everyone.

I consider two types of investors with rational learning. Short-term traders try to predict sentiment changes among naive investors based on the social network dynamics, while long-term investors are focused on the asset's fundamental value. The tractability afforded by my setting allows me to solve all agents' beliefs at all times in closed form, and, further, allows me to derive closed-form solutions for portfolios and market prices, as well as their limits as time increases.

[1] E.g., DeMarzo et al. (2003) state that "We should emphasize that the calculations that agents must perform even in this simple case where the network is common knowledge can be very complicated" (page 927).

The model implies that stubborn agents – that is, agents who never change their opinion – become increasingly important in the long term. Rational agents are initially extremely flexible, listening to all available information, paying no special attention to their own initial view, and updating correctly based on their prior. However, once a rational agent has processed all the available information, the agent becomes completely stubborn in the sense that seeing a different view presented again and again does not sway the rational agent if the different view does not contain any new information. Therefore, rational agents quickly become stubborn, and repeating their rational view has an increasing influence on naive investors over time. However, naive agents who are stubborn about their own personal view, however irrational, can also have a large influence over time if others are willing to listen. In the long run, all agents' views are dominated by these stubborn views, but investors differ in their reliance on rational or fanatic views. I show that the aggregate importance (or "thought leadership") of each stubborn view is the sum-product of the attention it gets from its of followers and the "influencer values" of their followers. Further, I show how to compute influencer values in a simple way, and how these influencer values affect asset prices.

The resulting differences of opinions lead to trading activity, but the trading activity dies down over time as views stabilize (since trading arises from view changes). Nevertheless, investors' portfolios differ, even in the long run, in contrast to the prediction of the standard capital asset pricing model (CAPM). These network effects can further lead to high prices (bubbles), low prices (anti-bubbles or deep value), and large and prolonged price swings.

The model can therefore help explain pervasive market effects such as price momentum and reversal effects (see Asness et al. (2013) and references therein), large trading volume with poor performance of the retail investors who trade the most (Odean (1999)), the relation between volume and momentum (Lee and Swaminathan (2000)), and excess volatility (Shiller (1980)) driven by chat in social media (Antweiler and Frank (2004)). Also, while rational investors and efficient prices react almost immediately to earnings announcements and other news, investors learning via a social network react only gradually, so the model can also help explain post-earnings announcement drift (Ball and Brown (1968)) and other kinds of fundamental momentum and announcement effects.

While there already exist theories for several of these phenomena,[2] the key distinguishing feature of my theory is that it predicts that ideas and trading behavior spread via a social network. This specific prediction of the theory is confirmed by Bailey et al. (2018) who use Facebook data to show that people with friends who experienced recent house price gains increase their housing market expectations, are more likely to buy a home, and "buy larger houses and pay more for a given house." Investors' local social network can also help explain their local bias in their equity investments, and the resulting social network effects have an impact on firm values (Kuchler et al. (2020)), providing additional evidence consistent with the model's distinguishing predictions. Social networks have also been shown empirically to affect equity market participation of retail investors (Hong et al. (2004), Brown et al. (2008), Kaustia and Knüpfer (2012)), affect the portfolios of money managers (Hong et al. (2005)) and retail investors (Walden (2021)), and serve as a potentially useful source of information (Chen et al. (2014)). Moreover, social media facilitate pump-and-dump schemes in cryptocurrencies (Li et al. (2020)) and professional traders' discussions on social media have been central to several litigations on financial market misconduct (see, e.g., Financial Times, 11/12/14, "Traders forex chatroom banter exposed").

In the GameStop case, investors on social media signaled their stubborn commitment to buying and holding the stock via the meme "diamond hands." They spurred each other via Reddit, Twitter, YouTube, and other social media, and signaled an extreme view of the potential valuation via the "rocket" meme. The price increased when Elon Musk, an influencer on social media, tweeted a link to the Reddit site hosting the most fanatic GameStop traders, WallStreetBets. The stock increased 2500% over January, fell dramatically, and increased again, and the associated spikes in trading volume and volatility coincided with an increase in social media attention. Eaton et al. (2021) exploit platform outages for the broker used by many GameStop traders, Robinhood, to identify the causal effects of retail traders on financial markets. The paper finds that exogenous negative shocks to Robinhood

[2] Differences of opinions can generate bubbles (see Harrison and Kreps (1978) for a seminal of model) and turnover (Harris and Raviv (1993)). Theories of momentum and reversal based on psychological biases include Barberis et al. (1998), Daniel et al. (1998), and Hong and Stein (1999). Models of information percolation via random search (rather than a network) include Duffie et al. (2009), Andrei and Cujean (2017), and Hirshleifer (2020), where the latter paper assumes that the sender of information adds a bias in each random interaction termed a "social transmission bias."

participation leads to lower return volatility among stocks favored by Robinhood investors. Barber et al. (2020) also study trades by customers of Robinhood, reporting that "intense buying by Robinhood users forecast negative returns."

In summary, this paper contributes to the literature by developing a simple model of rational and naive investors who interact via a social network in order to trade assets.[3] The model yields closed-form beliefs, prices, portfolio decisions, and turnover; provides a new mechanism of a number of financial market properties such as social-network spillover effects, the effects of influencers and thought leaders, momentum, reversal, high trading volume, and large price variation; and sheds light on the events surrounding GameStop.

1 Model

The economy has N investors who communicate and trade an asset in discrete time indexed by $t = 0, 1, 2, ...$ I describe the asset, social network, and market in turn.

Asset and signals. The asset has a supply of shares given by s. Its fundamental value is given by $v + u(t) \in \mathbb{R}$, where $u(t)$ is a publicly observed random walk and v is an unobserved random variable that investors try to learn about. The random walk has innovations with constant variance given by $\sigma_u^2 = \mathrm{Var}(u(t) - u(t-1))$.

At time 0, each person i starts with a signal about the value v given by the random variable $x_i(0) = v_i$. This signal gives each agent a useful, but incomplete, piece of information about v. Collectively, all agents have full information about v, which is modeled via the relation $v = \sum_{i=1}^{N} \kappa_i v_i$, where the weights sum to one, $\sum_{i=1}^{N} \kappa_i = 1$. Each weight, κ_i, is a measure of the precision of agent i's signal; for example, if all agents receive signals of the same precision, then the weights are equal, $\kappa_i = \frac{1}{N}$. Agents have an incentive to communicate to learn from others, and a rational learner can eliminate all residual risk about v.[4]

[3] As cited earlier in the introduction, the paper is related to behavioral finance (see survey by Barberis and Thaler (2003)), the literature on networks (see surveys cited above), the literature on systemic risk via cascades of defaults in a network (see survey by Jackson and Pernoud (2020)), and the empirical finance literature on social networks (see survey by Kuchler et al. (2020)).

[4] The signals can also be written as a sum, $x_i(0) = \bar{v} + \varepsilon_i$, of a common random variable $\bar{v}$ and independent noise terms ε_i. Whereas the common $\bar{v}$ would be the true value in the standard information-theoretic framework (e.g., Hellwig (1980)), the information structure here is slightly different in that the value is $v = \bar{v} + \sum_{i=1}^{N} \kappa_i \varepsilon_i$. Either structure works, but my structure has two helpful implications: i) a rational

The unobserved value, v, is revealed each time period with probability π and remains unknown with probability $1-\pi$. When the asset's fundamental value is revealed at the random time τ, the price equals its total value, $v+u(\tau)$. For example, the fundamental value could be paid out as a final dividend at the time of revelation. The objective of the model is to understand how beliefs, trading, and prices evolve before the value is revealed.[5]

Naive and rational learning in a social network. People communicate with each other as follows. At each time t, everyone truthfully states their current views, collected in the vector $x(t)=(x_1(t),...,x_N(t))$, to everyone who listens. The economy consists of people with two methods of paying attention, "rational learners" and boundedly-rational ones denoted as "naive" for brevity. Each naive learner i selectively follows a subset of people that he views as most informative or most entertaining. Further, he uses the same method for updating each round. Specifically, he uses the vector $T_i \in \mathbb{R}^{1\times N}$ to make the update, such that his view in the next time period becomes

$$x_i(t+1)=T_i\,x(t) \tag{1}$$

where the weights add up to one, $\sum_j T_{ij}=1$ and people listen to themselves $T_{ii}>0$. The network is therefore characterized by T, which is called the transition matrix or trust matrix in the DeGroot (1974) model. The i'th row, T_i, defines the list of people that i "follows" and the amount of attention paid to each of them. Similarly, the i'th column contains the list of "followers." DeMarzo et al. (2003) show that the first-round updating can be seen as rational Bayesian updating given that an agent only listens to people in the trust vector. Hence, the naivety of this investor comes from his selective listening and from the fact that the agent keeps using the same method of updating. In particular, the naive agent does not take into account that the same information may be received many times, a form of "echo

update has weights on the agents' signals (x_i) that sum to 1 as seen in (2), a property that underlies all updating in the literature that follows DeGroot (1974); ii) the portfolio problem is simplified by the fact that there is no residual uncertainty about v.

[5] The model is intended to be as simple as possible, but note that the model could easily be extended in several ways. For example, new signals about the fundamental value could arrive after the revelation of v, and so on, so that the whole game would repeat itself (possibly many times). Further, while the model is focused on a single asset, the analysis is straightforward to extend to the case of any number of assets (e.g., by letting $x_i(t)$ be a row vector with investor i's views about the different assets, implying that $x(t)$ becomes a matrix of all investors' views about all assets).

chamber." DeMarzo et al. (2003) denote this as "persuasion bias." Naive agents who only listen to themselves $T_{jj} = 1$ are denoted as fanatics, and they play a special role in the analysis of this paper.

In the literature that follows the standard DeGroot model, everyone is naive, but I also consider rational learners to capture the effects of sophisticated investors. A rational learner i listens to everyone in the first round of communication. Based on hearing everyone's views, the rational learner updates her view to $x_i(1) = x_r$, where the rational view x_r is given by:

$$x_r = E(v|x_1(0), \dots, x_N(0)) = (\kappa_1, \dots, \kappa_N)x(0) = v \tag{2}$$

Naturally, the rational view uses the information contained in all signals, and here the precision-weighted average of all signals in fact reveals the fundamental value v for simplicity. Further, the rational person never changes her view about v after the first round since no new information arrives, $x_i(t) = x_i = x_r$ for all $t \geq 1$. In other words, a rational agent i can also be seen as updating her views using the trust matrix as in (1), but she initially uses the row κ' and thereafter she uses $T_i = e_i$, where $e_i = (0, ..., 0, 1, 0, ..., 0)$ is the i'th unit vector.

Portfolios and prices. Trading starts at time 1, after the first round of communication. When the final value of the asset is revealed, the price naturally equals this final value. Before revelation, the endogenous price of the asset at time t is denoted by $p(t)$. All agents take this price as given and the equilibrium market price is determined such that the total demand equals the supply of shares s, that is, $s = \sum_{i=1}^{N} d_i(t)$, where $d_i(t)$ is the demand of agent i at time t, which depends on the price as described next.

The economy has two types of investors: long-term investors and short-term traders, where long-term investors think it terms of the long-term asset value at revelation, whereas short-term investors think in terms of the one-period expected gains each time period. Naive, fanatic, and some of the rational learners behave as long-term investors, while the remaining rational learners behave as short-term investors.

Specifically, any naive, fanatic, or rational long-term investor i chooses his investment demand to maximize his mean-variance utility at the time of revelation time, τ:

$$\max_{d_i} d_i E_t\left[x_i(t) + u(\tau) - p(t)\right] - \frac{1}{2\gamma}\mathrm{Var}_t\left[d_i(x_i(t) + u(\tau) - p(t))\right] \tag{3}$$

where γ is the investor's risk aversion. Recall that the final payoff is $v+u(\tau)$, but v is replaced by $x_i(t)$ in equation (3) because rational investors have learned the true value of v at time 1, and naive and fanatic investors behave as if their current belief is also the true value. Based on this expected final payoff and the corresponding risk given by $\mathrm{Var}_t(u_\tau) = \sigma_u^2/\pi$, the investor's optimal portfolio is[6]

$$d_i(t) = \frac{\pi}{\gamma\sigma_u^2}(x_i(t) + u(t) - p(t)) \tag{4}$$

Naturally, the demand increases in the perceived gap between the value and the price, and the price sensitivity is larger if the value is realized sooner (larger π) and if the risk (σ_u) or risk aversion (γ) is smaller.

Each rational short-term investor i maximizes her one-period mean-variance utility, which depends on the difference between the current price and the price in the next time period. The expected price in the next time period is the average of the price without and with revelation of the fundamental value, weighted by their respective probabilities, $1-\pi$ and π:

$$(1-\pi)E_t(p(t+1)) + \pi(x_r + u(t)) \tag{5}$$

Given this expected price and the one-period risk of $\sigma_u^2 = \mathrm{Var}_t(u(t+1) - u(t))$, the demand of any short-term investor is[7]

$$d_i(t) = \frac{1}{\gamma\sigma_u^2}\left[(1-\pi)E_t(p(t+1)) + \pi(x_r + u(t)) - p(t)\right] \tag{6}$$

[6] The risk is $\mathrm{Var}_t(u_\tau) = \mathrm{Var}(\Delta u_\tau + ... + \Delta u_{t+1}) = E(\Delta u_\tau + ... + \Delta u_{t+1})^2 = \sigma_u^2 E(\tau - t) = \sigma_u^2/\pi$, based on the optional stopping theorem and using $E(\tau - t) = \sum_{s=1}^{\infty} s(1-\pi)^s\pi = 1/\pi$.

[7] The price naturally has the following form: $\text{price}_t = u_t + \bar{p}_t + 1_{t\geq\tau}(v - \bar{p}_t)$, where $\bar{p}_t$ is a deterministic function of time conditional on the rational investor's time-1 information, as confirmed in Proposition 3. When computing the risk of price changes, short-term investors focus on $\mathrm{Var}_t(u(t+1) - u(t))$ and are risk-neutral with respect to uncertainty about the revelation time, for simplicity, as the model retains its tractability for any linear demand (and idiosyncratic event risk, τ, may be diversified away).

What is special about the short-term investors is that they seek to exploit predictable price changes due to the network spillover effects in the equilibrium price.

In summary, the economy has four types of agents who communicate to learn about an asset that they are trading: Naive learners, naive fanatics, long-term investors with rational learning, and short-term investors with rational learning. We use the term “hardheaded” investors for the latter three types, since these investors keep constant views after time 1. Agents are ordered such that the first N_n agents are naive, the next N_f are fanatics, the next N_l ones are long-term investors, and the last N_s are short-term investors, where $N = N_n + N_f + N_l + N_s$. The column vector of all views is written as $x(t) = (x_n(t); x_h)$, where $x_n(t) = (x_1(t), ..., x_{N_n}(t))$ contains the naive views and $x_h = (x_f; x_l; x_s)$ contains the hardheaded views. The hardheaded views are those of the fanatics, $x_f = (x_{N_n+1}; ...; x_{N_n+N_f})$, the long-term investors, $x_l = x_r 1_{N_l}$, and the short-term investors, $x_s = x_r 1_{N_s}$, where $1_M \in \mathbb{R}^M$ is a vector of ones. Note that x_h is not indexed by t since these views are constant after time 1, but we use the notation $x_h(0)$ for the initial views of these investors.

2 Market Behavior with a Social Network

2.1 Social influence when everyone is naive and connected

Suppose first that everyone is naive, which is the classic DeGroot model. In this case, agents' views after t rounds of communication is

$$x(t) = T^t x(0) \tag{7}$$

If everyone are strongly connected (everyone indirectly influences everyone), then it is well-known that a unique $z \in \mathbb{R}^N$ exits such that $z'T = z'$ and, as $t \to \infty$,[8]

$$x(t) = T^t x(0) \to 1_N z' x(0) \tag{8}$$

Hence, everyone ends up with the consensus view $z'x(0)$. The consensus view gives different weights to different agents, whereas the rational solution would weight all agents' signals by the precision ν_i rather than weighting by social influence z_i as in $z'x(0)$. In particular, person i's social influence, z_i, is the weighted-average of the social influence of everyone that listens to i, that is, $z_i = \sum_j T_{ij} z_j$. So a person becomes influential by having the ear of influential people. This eigenvector property is also the idea behind Google's page rank.

2.2 A fanatic in the echo chamber: Stubborn fake news

Consider next the case in which one agent is completely stubborn and all other agents are naive as above. The stubborn agent can be interpreted as a fanatic with an extreme view, someone who deliberately tries to shut down all alternative views, or someone spreading fake news. In particular, suppose that agent N is stubborn, meaning that N'th row of T is the N'th unit vector, $T_N = e'_N$. Mathematically, this corresponds to a Markov chain with an absorbing state, and opinions evolve as

$$x(t) = T^t x(0) \to 1_N e'_N x(0) = 1_N \, x_N(0) \tag{9}$$

meaning that everyone ends up having the same view as the stubborn person. In terms of the influencer analogy above, where influential people were those with influential listeners, what happens here is that the fanatic becomes influential by having an influential listener, namely himself. Since he is the only influential person and he listens to no one but himself, then no one else has any influence. This phenomenon does not arise with Google's pagerank,

[8] Strongly connected means that, for every i and j, i can influence j, that is, there exists t such that $(T^t)_{ij} > 0$. Aperiodic means that infinite cycles cannot arise, which is ensured by the assumption that people listen to themselves $T_{jj} > 0$. These properties correspond to an irreducible aperiodic Markov chain. Further, these properties ensure convergence of views, and w is the stationary distribution of the Markov chain. (As a counterexample that violates these properties, suppose that agent 1 only listens to agent 2 and vice versa – then these agents swap views in each round of communication, forever.)

because a website does not get credit for linking to itself, so, perhaps as a result, social influence via stubbornness is not the standard case in economics. Note that the limit is the same for a large class of networks, since the limit only depends on one agent being stubborn and the remaining agents being connected to one another and the stubborn one, but this property does not hold in the more general model.[9]

As an example, imagine people negotiating the price of a house. A buyer says: "I will pay \$300k," but the stubborn seller counters: "it is worth \$500k." Buyer: "let us meet half way, \$400k," but the seller insists on \$500k. "How about \$450?" says the buyer, but, in the end, the house goes for \$500k. Of course, this example is oversimplified, since investors' valuations depend on the quantities that they buy, there can be many stubborn views, and rational fundamentals play an important role.

2.3 Rationality in the echo chamber: The stubbornness of truth

Suppose alternatively that one or more agents are fully rational, while the rest are naive and strongly connected. Recall that, after one round of communication, the rational agents already know the "truth," that is, the best possible estimate of the value, v. Hence, from time 1 and onwards, the rational agents do not further update their views. Said differently, they behave as if they are completely stubborn. Therefore, by the same logic as above, views converge as follows

$$x(t) \to 1_N \, x_r \tag{10}$$

meaning that everyone ends up having the same rational view.

Note that fanaticism and rationality look similar to a casual observer since such agents behave similarly in all rounds of communication except the first one. Indeed, a rational person is completely flexible in the first round of communication, seeking out all sources of information, having no special attachment to her own information, and logically aggregates

[9] As a less extreme version, suppose that you have a strongly connected group of agents, but agent i becomes increasingly stubborn in the sense of a comparative static (the i'th row of T converges to the unit vector, all other rows are unchanged). Then the limit beliefs of all agents approach that of agent i, that is, the stubborn agent becomes increasingly important in the long run.

all of this information. However, once all the information is aggregated, the rational person has no interest in hearing the same views again, does not budge to hearing a particular view repeated many times, and simply sticks to the same opinion forever (or at least until truly new information arrives when the final value is revealed).

Golub and Jackson (2010) describe another way to ensure rationality, even when all agents are naive. They show "that all opinions in a large society converge to the truth if and only if the influence of the most influential agent vanishes as the society grows." While a mathematically beautiful benchmark, the condition that the most influential agent has a vanishingly small influence seems clearly violated in the modern world of social media where, for example, Kim Kardashian West has more than 200 million followers on Instagram as of this writing. Hence, for rationality to prevail, the mechanism developed here based on the stubbornness of truth may have a better chance of success.

2.4 Fanaticism vs. rationality in the echo chamber

In the most general and realistic case, the economy has some of all the types of agents: naive, fanatic, and rational. This framework is the focus of the rest of the paper. After time 1, all rational agents are completely stubborn since they know the truth, and fanatics are stubborn for other reasons. So a central ingredient to model is understanding the dynamics of an economy in which some agents are hardheaded ("h") and others are naive.

Recall that agents are ordered such that the first agents N_n are naive and the rest are hardheaded — specifically, the next N_f are fanatics, the last $N_l + N_s$ ones are rational. In this case, the transition matrix has the form:

$$T = \begin{pmatrix} T_{nn} & T_{nh} \\ 0 & I \end{pmatrix} \tag{11}$$

where $T_{nn} \in \mathbb{R}^{N_n \times N_n}$ is an irreducible matrix that determines how the naive agents listen to each other, and $T_{nh} \in \mathbb{R}^{N_n \times (N_f + N_l + N_s)} \backslash \{0\}$ is a matrix that determines how they listen to the hardheaded agents. The lower rows of T consists of a matrix of zeros and the identity matrix since each hardheaded agent only listens to himself.

Example 1. To illustrate how the model works, consider the following numerical example:

$$\begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_f \\ x_r \end{pmatrix} = \begin{pmatrix} 60\% & 5\% & 5\% & 20\% & 10\% \\ 35\% & 40\% & 5\% & 10\% & 10\% \\ 35\% & 5\% & 40\% & 10\% & 10\% \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1(t-1) \\ x_2(t-1) \\ x_3(t-1) \\ x_f \\ x_r \end{pmatrix} \tag{12}$$

As seen in the first row, agent 1 gives 60% weight to his own previous opinion, 5% weight to each of the other two naive agents, 20% to the fanatic agent, and 10% to the rational. Agents 2 and 3 also give 40% weight to their own respective views, 35% weight to agent 1, 5% to each other, and 10% on the fanatic and rational agents. Hence, agent 1 is an "influencer" in the sense that other agents pay strong attention to his views. The views of the fanatic and a rational learner (agents 4 and 5) do not change after time 1. ■

The first step in solving the model is to determine the vector of naive views, $x_n(t)$, at each point in time. Naturally, the vector of naive views depends on itself, on the vector of beliefs by fanatics and rational agents after the rational agents have learned the truth, $x_h = (x_f; x_l; x_s)$, and their views before rational agents have learned anything, $x_h(0) = (x_f; x_l(0); x_s(0))$. The following proposition shows that the hardheaded views dominate over time due to echo chamber effects. All proofs are in the appendix.

Proposition 1 (Belief spillover in a network and echo-chamber convergence) *The views of naive agents at time 1 is $x_n(1) = T_{nn}x_n(0) + T_{nh}x_h(0)$ and, for $t = 2, 3, ...,$ their views are*

$$x_n(t) = T_{nn}^{t-1}x_n(1) + \sum_{k=0}^{t-2} T_{nn}^k T_{nh} x_h \tag{13}$$

In the limit as $t \to \infty$, each naive agent's view is a convex combination of the views of *fanatics and rational agents*

$$x_n(t) \to (I - T_{nn})^{-1} T_{nh} x_h \tag{14}$$

The first part of the proposition shows how the naive agents are initially influenced by each other and by rational and fanatic views (equation (13)). The second part of the proposition shows how the naive investors end up with views that are a mixture of the fanatic and rational views, x_h. The relative weights given to the views different fanatic and rational people have a natural interpretation as discussed next.

2.5 Influencers versus thought leaders

Recall that the standard connected DeGroot model of section 2.1 has the property that everyone reaches a consensus in the long run, which implies a simple measure of social influence, namely the weight z_i of agent i's opinion in the consensus. Social influence works differently, however, in my setting of Section 2.4 with rational and fanatic agents. In this case, people never reach a consensus, but the focus on the consensus view can be replaced by a focus on the average naive opinion.

Proposition 1 shows that naive agents' views disappear in the long term (except in how they initially affect rational views). Nevertheless, the communication of naive agents still matters, because this communication determines the relative weight on the various fanatic and rational views. To capture this idea, I introduce the following definition of the "influencer value" of any naive agent.

Definition 1 (Influencer value) *The influencer value of naive agent i is μ_i, where*

$$\mu' = \frac{1}{N_n} 1'_{N_n} (I - T_{nn})^{-1} \tag{15}$$

To understand this definition, note that influencer values can also be written as

$$\mu' = \sum_{k=0}^{\infty} \frac{1}{N_n} 1'_{N_n} T_{nn}^k \tag{16}$$

In words, the influencer value of any agent i is his average direct following among naive investors (i.e., the average of the i'th column of T_{nn}), plus his indirect following (i.e., the average of the i'th column of T_{nn}^2), plus his next-order indirect following (i.e., the average of the i'th column of T_{nn}^3), and so on.

Based of these influencer values of naive agents, I define the "thought leadership" of hardheaded agents as follows.

Definition 2 (Thought leadership) *The thought leadership of a fanatic or rational agent j is the sum-product of her following, T_{ij}, and the influencer values of her followers, μ_i:*

$$\theta_j = \sum_{i=1}^{N_n} \mu_i T_{ij} \tag{17}$$

These thought leaderships can also be written in vector form as $\theta' = \mu' T_{nh}$. Using these measures of thought leadership, the next proposition characterizes how the overall naive opinion, $\bar{x}_n(t) = \sum_{i=1}^{N_n} \bar{x}_i(t)/N_n$, is formed in the long term.

Proposition 2 (Thought leaders) *As $t \to \infty$, the overall view of naive agents, $\bar{x}_n(t)$, converges to the weighted-average views of fanatics and rational agents, where the weights equal their thought leadership θ_j,*

$$\bar{x}_n(t) \to \theta' x_h = \sum_{j=N_{n+1}}^{N} \theta_j x_j \tag{18}$$

where thought leaderships add to one, $\sum_{j=N_{n+1}}^{N} \theta_j = 1$.

The intuition behind this result is that hardheaded agents become thought leaders due to their commitment to their views. Influencers, on the other hand, are not committed to any particular view, but affect which thought leaders get the most attention. Hence, a thought leader can increase his impact by acquiring the following of an "influencer," that is, an agent with large influencer value.

Interestingly, my framework implies separate roles for influences and thought leaders, while these are one and the same thing in the standard connected DeGroot model of section

2.1. In DeGroot, you become a thought leader by being an influencer, that is, you promote your own initial signal by having many influential followers. What is different here, is that the fluid opinion of an influencer changes over time and ends up being dominated by the views of thought leaders. Hence, over time, an influencer's impact comes from influencing the relative popularity of the various thought leading ideas.

An implication of Proposition 2 is that the economy is more rational if there are more rational people to begin with, if fanatics have views closer to the rational view, and if the naive people listen more to the rational ones, especially if the influential naive people do so. So education makes the economy more rational if it teaches naive people to be rational or to listen to those who are, and education of influencers is particularly effective.

As another example, religious beliefs may become thought leading partly by committing to a fixed text such as the bible. Simultaneously, science becomes thought leading by collecting as much data as possible and committing to a scientific understanding of the laws of nature. Continuously preaching these relatively fixed principles creates thought leadership.

Example 1, continued. Consider next the numerical example in equation (12). The influencer values of agents 1, 2, and 3 can be computed using equation (15) to be 2.3, 0.8, and 0.8, respectively. Naturally, agent 1 has the largest influencer value since other people pay most attention to him. The thought leadership of the fanatic is 61.3% based on (17), and the thought leadership of the rational agent is 38.7%. The fanatic has a larger thought leadership because of her larger following by the influencer.

It is interesting to consider what happens if agent 1 increases his following of the fanatic by 1 percentage point (i.e., from 20% to 21%) at the expense of a lower following of the rational agent (from 10% to 9%). This increases the thought leadership of the fanatic to 63.5%. The increase in the fanatic agent's thought leadership of 63.5%-61.3%=2.3% equals agent 1's influencer value, which helps explain the meaning of influencer value. If instead agent 2 increases his following of the fanatic by 1 percentage point (from 10% to 11%) at the expense of a lower following of the rational, then the fanatic's thought leadership increases to 62.1%. Again, the increase in thought leadership of 62.1%-61.3%=0.8% matches the influencer value of agent 2. ■

The next section shows that influencer values and thought leadership are also useful in characterizing the social network effects on prices.

2.6 Prices with a social network

When the fundamental asset value is revealed, the price equals the fundamental, $v + u_t$, and, before that time, the equilibrium price is determined by equalizing the supply s and the total demand. The equilibrium condition can be written as:

$$p(t) = (1-c)\left(\frac{N_n}{N}\bar{x}_n(t) + \frac{N_f}{N}\bar{x}_f + \frac{(N_l + N_s)}{N}x_r + u(t) - \frac{s\gamma\sigma_u^2}{N\pi}\right) + cE_t(p(t+1)) \tag{19}$$

where

$$c = \frac{1}{1 + \frac{N}{N_s}\frac{\pi}{1-\pi}} \tag{20}$$

Iterating this equation forward yields the equilibrium price. If all agents are rational long-term or short-term investors, then the equilibrium price is

$$p_r(t) = x_r + u(t) - \frac{s\gamma\sigma_u^2}{N\pi} \tag{21}$$

The rational price, $p_r(t)$, is the expected fundamental value, $x_r + u(t)$, less a risk premium that depends on the supply of shares, s, the number of investors, N, and their risk aversion, γ. This rational price is a useful benchmark when considering the equilibrium price in the presence of naive investors and fanatics. As seen in the next proposition, the general price also depends on the average view among naive investors, $\bar{x}_n(t)$, and the average view among fanatics, $\bar{x}_f$.

Proposition 3 (Network effects on price) *The equilibrium price before the fundamental value is revealed is $p(t) = p_r(t) + p_n(t)$, that is, the sum of the rational price, $p_r(t)$, and the following social network component:*

$$p_n(t) = \frac{N_n(1-c)}{N}\sum_{s=0}^{\infty} c^s\left(\bar{x}_n(t+s) - x_r\right) + \frac{N_f}{N}(\bar{x}_f - x_r) \tag{22}$$

where c and p_r are given in (20)–(21). This equilibrium price is unique under the transversality condition that $p_t - u_t$ is bounded in t. As $t \to \infty$, the network component of the price converges to

$$p_n^{\infty} = \sum_{j=N_n+1}^{N_n+N_f} \frac{1 + N_n \theta_j}{N}(x_j - x_r) \tag{23}$$

which depends on the fanatics' mispricings, $x_j - x_r$, weighted by their thought leadership, θ_j.

The market price when investors learn through a social network can deviate significantly from the rational price. In particular, equation (22) shows how the price depends on all investors' views, which are determined via their social network interactions. These network effects are seen from the price dependence on the naive view $\bar{x}_n(t)$, which can be traced back to the investors' original views and their spillover through the network as seen from Proposition 1. The price also depends on expected future network effects, since the rational short-term investors anticipate these future network effects and adjust their asset demand accordingly, as seen from the terms depending on $\bar{x}_n(t+s)$ with $s > 0$ in equation (22).

Over the longer term, naive investors' views are completely dominated by the fanatic and rational views, and therefore the long-term prices depends on these hardheaded views and their thought leaderships, as seen in (23). These social dynamics can generate prices way above the fundamentals (bubbles) and way below fundamentals (anti-bubbles, or deep value). Naturally, the bubble gets larger if fanatics have more extreme valuations. Interestingly, this effect is larger if the fanatics have greater thought leadership.

Proposition 4 (Fanatic effect on price) *The price increases in the valuation, x_j, of fanatic agent j. This price sensitivity is larger if the agent has greater thought leadership θ_j:*

$$\frac{\partial p_n^{\infty}}{\partial x_j} = \frac{1 + N_n \theta_j}{N} \tag{24}$$

Another aspect of the social network dynamic is the effect of influencers.

Proposition 5 (Influencer effect on price) *If naive agent i with influencer value μ_i increases his following of fanatic agent j by ε at the expense of a lower following of a rational* *agent, then the effect on price is:*

$$\frac{\partial p_n^\infty}{\partial \varepsilon} = \frac{N_n}{N}\,\mu_i\,(x_j - x_r) \tag{25}$$

This proposition shows that a naive agent can affect the price by elevating the thought leadership of a fanatic. This effect is naturally larger if the naive agent has a greater influencer value. For example, when Elon Musk tweeted a link to WallStreetBets with its fanatically high valuation of GameStop, the price of GameStop increased.

Example 1, continued. In the numerical example in equation (12), suppose that the fanatic assigns a value to the asset of $x_f = 500$, the rational agent is a long-term investor with a valuation of $x_r = 400$, the random walk is $u(t) = 0$ for all t, and the supply is $s = 0$. In this case, the long-term price can be calculated from (23) to be $400 + p_n^\infty = 456.8$, which is closer to the valuation of the fanatic than the rational because the fanatic has greater thought leadership (as calculated in Section 2.5).

If the fanatic increases his valuation to 600, then the long-term price increases to 513.5, a price increase of 56.8, which is a 100 times the price sensitivity of 0.568 that can be computed from (24). If instead the rational view changes by 100, this leads to a price change of 43.2, which is a smaller price move due to the lower thought leadership of the rational agent.

To illustrate the importance of influencers, suppose that agent 1 increases his following of the fanatic by 10 percentage points. Then the price increases from 456.8 to 470.3, an increase of 13.5. In instead agent 2 increases his following of the fanatic by 10 percentage points, then the price increases only by 4.9. The price is more sensitive to the actions of agent 1 because of his larger influencer value. In fact, the relative effect on the price exactly equals the agents' relative influencer values, $13.5/4.9 = 2.3/0.8 = \mu_1/\mu_2$. ■

2.7 Portfolios and trading volume with a social network

It is also interesting to consider how short-term investors trade in light of the network dynamics driving the price. The demand (6) of short-term investors can be rewritten as

$$d_i(t) = \frac{(1-\pi)}{\gamma\sigma_u^2} E_t(p(t+1) - p(t)) + \frac{\pi}{\gamma\sigma_u^2}(x_r + u(t) - p(t)) \tag{26}$$

Here, the firm term shows that a short-term investor has an incentive to "ride" a bubble in the sense that the short-term investor looks at the expected price move if the fundamental is not revealed, $E_t(p(t+1) - p(t))$. The second term shows that a short-term investor also worries about the magnitude of the bubble, realizing that a revelation of the fundamental will burst the bubble, leading to an expected price move equal to the difference between the price and the fundamental, $x_r + u(t) - p(t)$.

When the short-term investors' positions have the same sign as the first term in (26), then they are "momentum traders," and when their positions have the same sign as the second term, then they are "value investors." In general, at any point in time, short-term traders act as either momentum traders, value investors, or both. However, for t large enough, they always become value investors since the momentum effect (the first term in (26)) dies down over time. The next proposition shows that short-term investors positions have a specific structure under symmetry.

Proposition 6 (Momentum and value) *Suppose that T_{nn} is symmetric. In case of a positive bubble, $p_n^\infty > 0$, short-term investors initially buy a rising undervalued asset (value and momentum investing), continue to hold when the asset becomes over-valued (momentum), and finally shorts when the over-valuation is large enough (value on the short side). Short-term investors only go through the latter one or two phases (momentum buying and value shorting) depending on the initial price $p(1)$ (i.e., depending on the initial signals).*

Conversely, in case of an anti-bubble, $p_n^\infty < 0$, the short-term investor initially shorts a falling over-valued asset (momentum and value shorting), continues to short as the asset becomes cheap (momentum), and eventually buys when the asset is cheap enough (value), or only goes through the latter one or two phases depending on $p(1)$.

Short-term traders face a trade off between riding a bubble and betting on its burst. Proposition 6 shows that this trade off leads them to initially ride the bubble when price moves driven by echo-chamber effects are large and the crash risk is limited. Eventually, however, naive investors' view converge as the echo resides, implying that further price moves become small, and, moreover, crash risk grows large over time, leading short-term traders to ultimately trade against any bubble. Further, social network effects increase the trading volume and creates another source of price variation.

Proposition 7 (Spike in volume and excess volatility) *With naive and fanatic agents, the trading volume is greater but dies down over time (until the fundamental is revealed when final trading may happen) and the valuation ratio (price minus fundamental $u(t)$) varies more, relative to when all agents are rational.*

Example 2. The economy has $N = 100$ investors, 96 of which are naive, 2 are fanatics, 1 is a long-term investor, and 1 is a short-term investor. The supply of shares is normalized to $s = 1$, the demand sensitivity is $a = 1$, and the asset's fundamental value is $v + u(t)$, where $u(t) = \Delta u(1)+...+\Delta u(t)$ with $\Delta u(t) \sim N(0, 3^2)$. The unobserved value is $v = \bar{v}+\sum_{i=1}^{100} \frac{1}{100}\varepsilon_i$, where $\bar{v} \sim N(100, 20^2)$, $\varepsilon_i \sim N(0, 40^2)$ and investors' initial beliefs are given by $x_i(0) = \bar{v}+\varepsilon_i$, except that the fanatics draw a signal of $x_f = 200$ and keep this view throughout. After receiving these initial signals, the agents communicate in a social network. Any naive agent i puts a weight of $T_{ii} = 30\%$ on his own previous view and splits the remaining 70% across all other agents. Rational agents initially listen to everyone, learn based on the average across all signals and their prior, and keep this updated view from time 1 and onwards.

The top panel of Figure 1 shows how investors' beliefs evolve over time. Naive investors start with a range of idiosyncratic beliefs, but are influenced by others in their social network. Over time, naive investors' beliefs converge to an average of the fanatic and rational views. In this example, all naive investors eventually agree with each other because of the symmetry in the example, but such agreement among naive investors need not happen more generally. To illustrate this point, the bottom panel of Figure 1 shows an example where the social network is randomly drawn. In this case, some naive investors end up with views closer to the rational view, while others converge to a view closer to the fanatic, depending on the social network. In particular, a naive agent who follows the rational agents more closely, or follows others who are influenced by rational views, end up closer to the rational belief.

Figure 2 shows the resulting equilibrium price. For the purpose of this figure, I assume that the fundamental value is revealed at time 35. The figures shows that the price tends to rise over time as naive investors influenced by the fanatics continue to buy more and more shares until the bubble finally bursts, leading to a price correction.

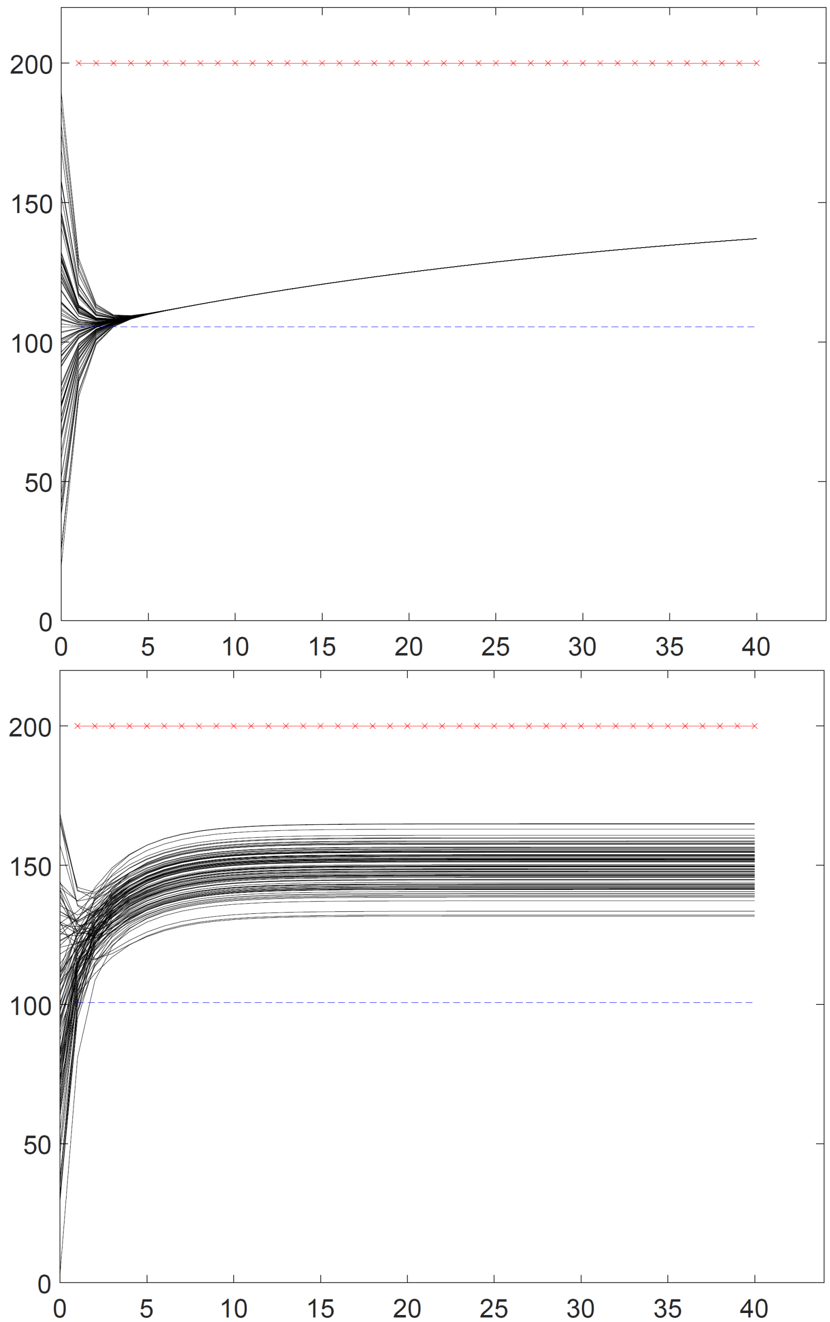

Figure 1: **Investor beliefs over time.** The top panel shows how investors' beliefs about the fundamental asset value (on the y-axis) change over time (the x-axis) in Example 2. The red line on top shows the fanatic belief of a \$200 valuation, the dotted line shows the rational view close to \$100, and the solid curves show how naive investors initially disagree, but over time converge to a view between the fanatic and the rational one. The bottom panel is similar, except that the network is drawn randomly such naive investor continue to disagree over the long term because of their different degrees of influence from rational vs. fanatic agents.

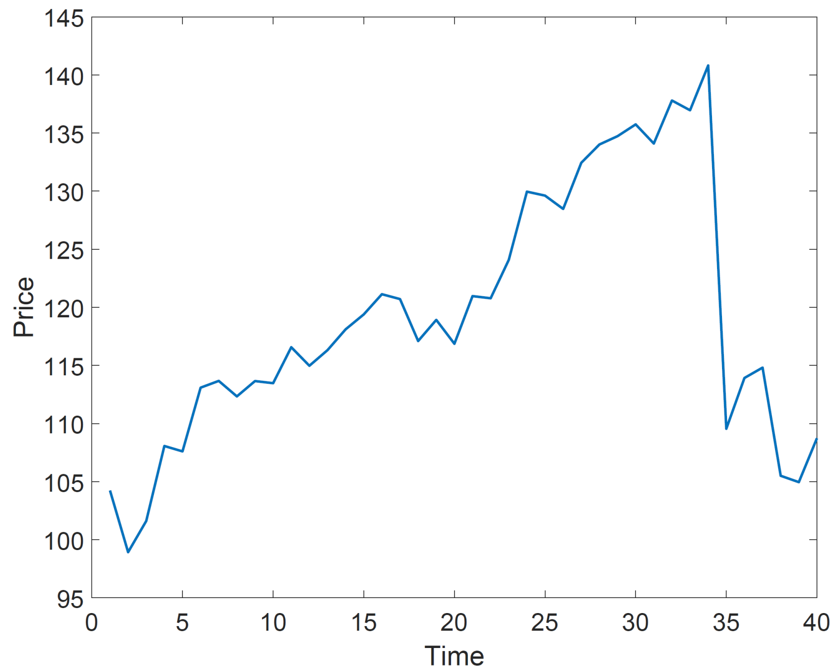

Figure 2: **Price bubble and burst.** This figure shows the evolution of the asset price over time in Example 2. The bubble builds until it bursts at time 35.

The network component of the price is rising monotonically until revelation in this example. However, many other price patterns can arise from this network framework. For example, if we change agents 97 and 98 from being complete fanatics to instead being just very self-reliant optimists (T_{jj} near, but strictly less than, 1), then the network component still rises in the beginning and still creates a bubble as the optimistic view initially spreads like a virus. But, the bubble peters out over time, even without revelation of the fundamental, as all investors eventually become convinced of the rational valuation.

Finally, Figure 3 shows the position of short-term investors. Initially, short-term investors have a positive asset position (i.e., a long position). This long position means that they are trading on the positive price momentum (despite the overvalued asset price) in the hope that the bubble will grow larger before it bursts. Later in time, short-term investor positions turn negative (i.e., short positions), which means that they are acting as value investors who bet that the price will revert toward fundamentals. Long-term investors (not shown) act as value investors at all times.

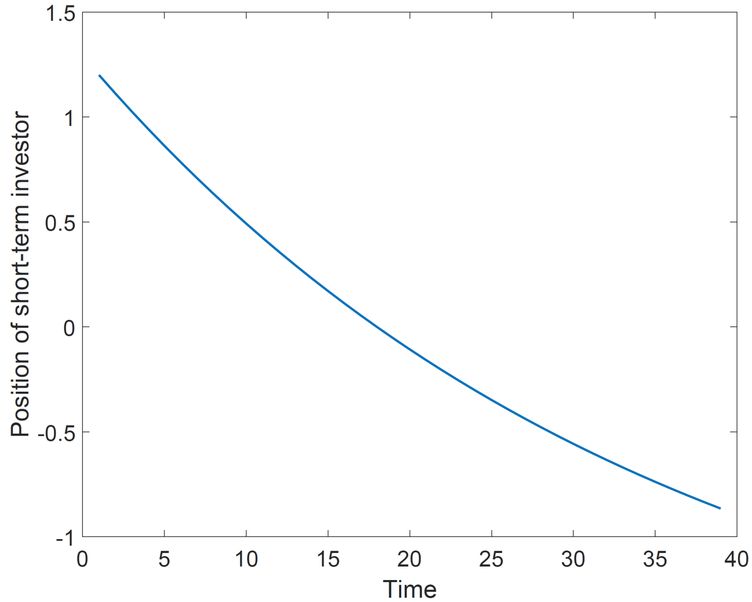

Figure 3: **Asset position of short-term investors.** This figure shows the asset position of short-term investors over time in Example 2. Short-term investors are initially long the asset, acting as momentum traders who ride the bubble. Later in time, short-term investors act as value investors who bet of a price reversion toward fundamentals when the bubble bursts.

2.8 Extensions and market implications of social networks

The model has a number of implications that may be of relevance to real markets.

Diamond hands. People who want to affect the opinion of others try to get attention, but may also enforce a fanatic stubbornness. In social media related to trading, such a stubborn willingness to keep buying and holding a stock is called "diamond hands" (often written with diamond and hand emojis). In the case of GameStop, Keith Gill – a trader who made an impact on social media – ended his testimony in the hearing of the U.S. House Committee on Financial Services by saying: "In short, I like the stock," seemingly liking the stock at almost any price. The model shows that a stubborn view can come to have a significant effect on naive investors, leading to bubbles.

Rocket and YOLO trades. A stubborn view has a larger effect if it is more extreme. Social-media investors sometimes signal such an extreme view of the potential rise in price via a rocket emoji and, the traders of GameStop on WallStreetBets had a special category for YOLO (you only live once) trades.

Gamification of trading. Some investors are trading with broker apps with game-like features while chatting on social media with their connections. Hence, another interpretation of the model is that it may capture such a gamification of trading.

Meme trading frenzy. A "meme" is an idea that becomes a fad and spreads by means of imitation in a social network, in the spirit of the model considered in this paper. The meme can lead to differences of opinions across variations of the meme (i.e., across different fanatics in the model) and relative to rational investors, leading to a trading frenzy (Proposition 7). This can happen even long after any news has arrived or a meme is originated, since the meme can take a long time to gather momentum in the echo chamber. See Shiller (2017) for other economic effects of viral "narratives" and Hirshleifer (2020) for the related idea of "social transmission bias."

Excess volatility. Since the echo-chamber dynamics are largely divorced from fundamentals, the resulting network effects can make asset prices vary for reasons unrelated to news about their fundamentals (Proposition 7), consistent with Shiller (1980), Shiller (1984). Volatility would be even larger in an extension of the model in which naive investors trade at random times, such that the price at any time would depend on the mix of investors trading at that moment.

Value and momentum. Social network effects can lead to price momentum and subsequent reversal toward fundamentals (also called a value effect) that are seen across many asset classes and global markets (Asness et al. (2013)). Indeed, the build-up of naive investor demand can lead to momentum effects, and the eventual price reversal as the fundamental value is revealed leads to a value premium.

Repeat news. In the model, repeating old news can move prices, especially if the repeat news is displayed prominently to many people, as naive investors keep reacting to the same information, consistent with the evidence of Huberman and Regev (2001) and Tetlock (2011).

Fake news. In the model, fanatics are spreading fake news, but they are assumed to do so because they truly hold this view. More broadly, all agents' truly report their views, consistent with the assumption that they are price-takers. An interesting an extension would consider agents' incentive to manipulate the price.

Pump and dump. An opportunistic investor could try to profit by first buying an asset, and then pretend to have a fanatic bullish view on the asset in order to create a wave of buying by his followers, leading to a price increase. If the opportunistic investor then sells despite talking up the asset, he is engaging in "pump and dump," an illegal form of market manipulation. In the model, the fanatics "pump" and the short-term investors initially ride this pump and eventually "dump"—so viewed as a team, these agents pump and dump.

Short squeeze. A bubble driven by social media effects can be greatly exacerbated if short-sellers are forced to close their positions due to share recalls or risk controls. This paper abstracts from this effect here for simplicity—see Brunnermeier and Pedersen (2005) for a model of short squeezes and other forms of predatory trading and Duffie et al. (2002) for a model of securities lending.

Social network effects and local bias. Social network effects mean that people are affected by those they are connected to, and, indirectly by the connections of their connections, and so on. Since people often interact with others in their local area and work place, network effects can contribute to investments having home bias (French and Poterba (1991)), local bias (Coval and Moskowitz (1999)), and own-firm bias. Kuchler et al. (2020) provide evidence that social network effects contribute to local bias and affect firm values.

Attention-grabbing profits. Fanatic views may spread more quickly when their proponents are seen profiting from their views. Hence, an interesting extension of the model would let the transition matrix T at each time depend on the past profits of the different agents. Given that fanatics tend to profit early on as their views start affecting the price, this extension could generate larger bubbles as the fanatic profits would grab attention, the attention would increase profits, and so on.

3 A Case Study of GameStop

The case of GameStop offers an interesting window into the effects of social networks on asset markets due to the extreme and observable effects. While the introduction cites scientific evidence in support of the model, GameStop offers an interesting illustration that may spur ideas for future research, hopefully ideas that also apply in less extreme cases such that they can help explain the general principles of market behavior.

Gamified trading of GameStop. In early 2021, the world's largest video game retailer, GameStop, was struggling as games were increasingly sold online and the retail industry was hit by the Covid-19 crisis. Nevertheless, a group of investors caught an almost fanatic liking to the stock, a trend that was reinforced through social networks, as illustrated in Figure 4. Keith Gill (better known under various social media aliases) became the most followed of these traders, so he can be represented in the model as one of the fanatics. As in the model, Gill expressed an unwavering belief in the stock at all observed price levels.

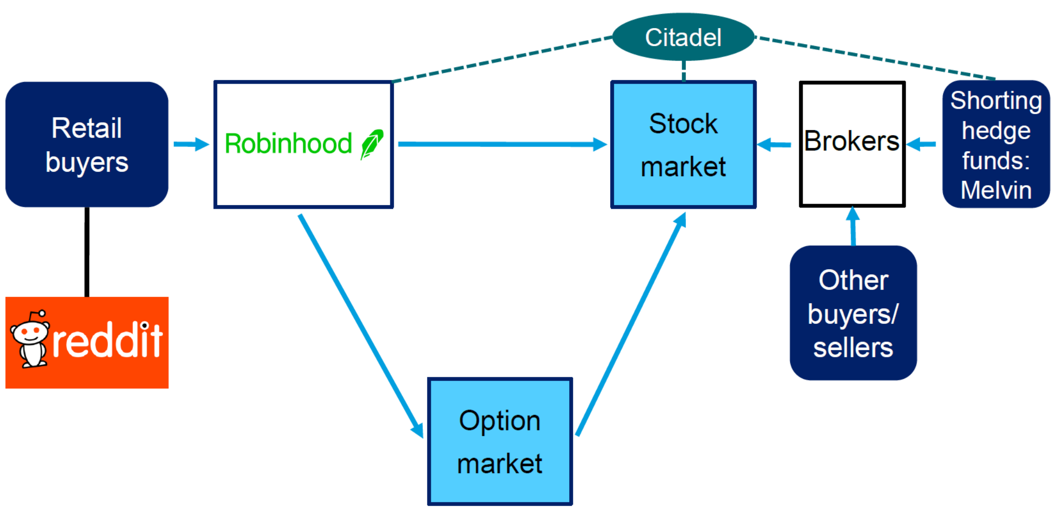

Figure 4: **Overview of the GameStop episode in early 2021.**

The most well-publicised group of retail investors interested in GameStop communicated through the platform Reddit, in the community (subreddit) called the WallStreetBets. Some of these investors had the view that GameStop was undervalued, others had a nostalgic feeling for the shop where they had purchased their first video games, and yet others believed that the business could be turned around by pivoting online with the help of recent block investor and board member, Ryan Cohen, and others. Another motivation for buying GameStop was its large short interest of just over 100% of shares outstanding at the beginning of the year (or around 140% of all floating shares). Some GameStop investors resented shortsellers in general and, in particular the largest shortseller of GameStop, the hedge fund Melvin Capital. Other GameStop investors expressed a view that pushing up the stock price could lead to a short squeeze, reinforcing the view that there was no limit to how high the stock price could go. The extreme view of the stock price was reinforced with rocket memes and by labelling GameStop a YOLO (you only live once) trade. Similarly in the model, the fanatics have a larger effect of the price if they express a more extreme valuation, $\bar{x}_f$, as seen in Proposition 3 and this effect is larger if the fanatics have more awareness or thought leadership (Proposition 4).

As another sign of the importance of social media, the interest in GameStop was spurred by a tweet by Elon Musk on January 26 with the single word "Gamestonk!!" along with a link to WallStreetBets. In the model, Elon Musk can be represented as an influencer who chooses to follow the fanatic. That is, the agent i representing Musk puts a positive weight, $T_{if} > 0$, on the fanatic's view, and Musk is an influencer in the sense that many other investors j follow Elon Musk, $T_{ji} > 0$. In the model, when a influencer follows a fanatic with a very bullish view, this increases the stock price (Proposition 5), as it did in the case of GameStop.

This bullish sentiment on GameStop communicated via social networks was translated into actual trading activity. Trading by inexperienced traders was facilitated by a competitive online brokerage industry offering zero-commission trading, lead by the new broker, Robinhood, which uses a business model based on payment for order flow from market makers such as Citadel. Robinhood and other brokers sought to make trading more broadly available to a wide range of investors in a fun way, which lead to the accusation that trading became "gamified."

Price, volume, volatility, and social media interest. As seen in Figure 5.A, GameStop had been trading at less than $20 per share through 2020, but increased dra-

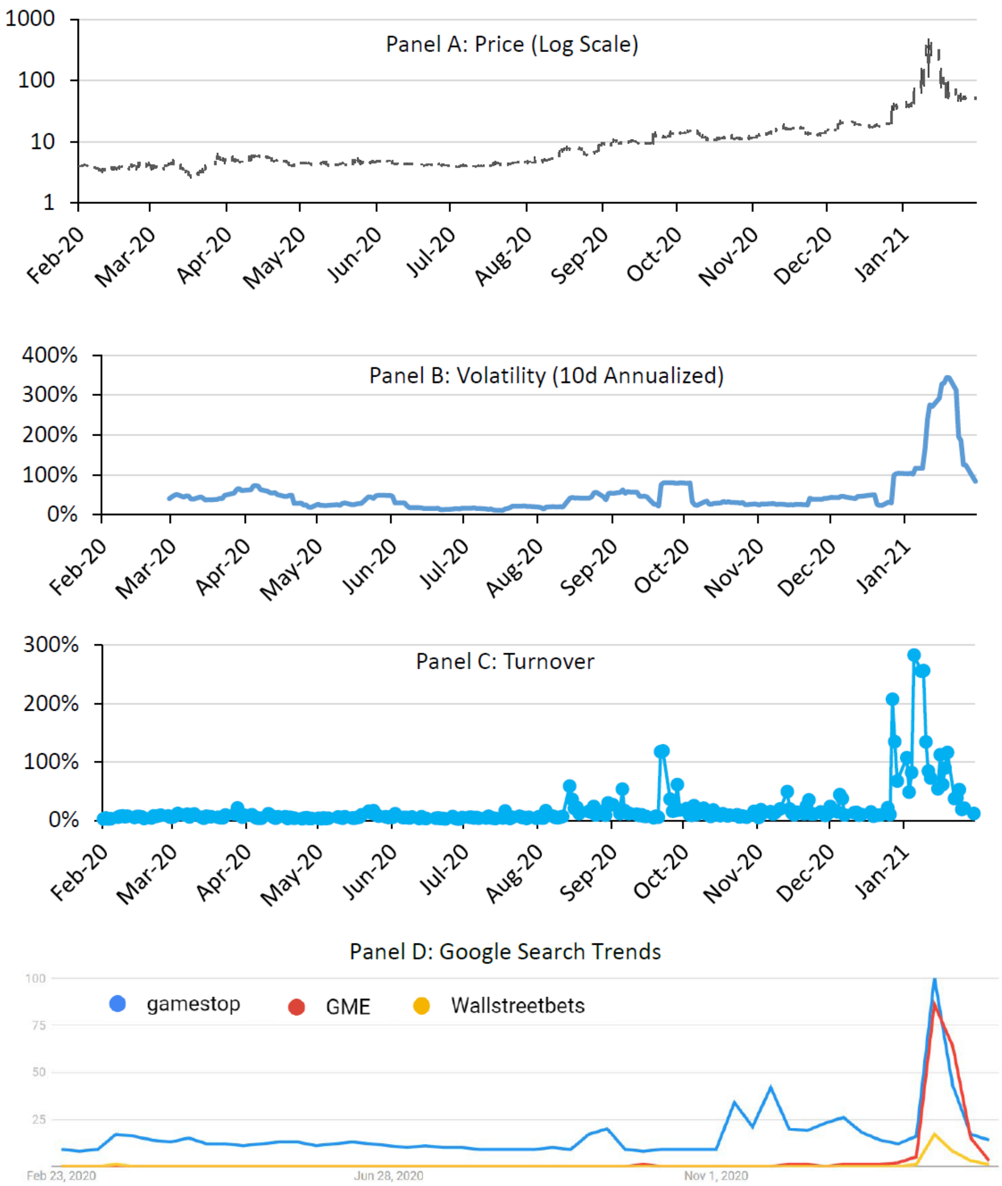

Figure 5: **GameStop price, volatility, turnover, and interest, 2/2020-2/2021.** Panel A shows the GameStop prices over time (plotted open-high-low-close on a log-scale), Panel B shows the realized volatility, Panel C shows the daily turnover, and Panel D shows the social media interest as proxied by Google search trends for GameStop, GME, and WallStreetBets.

matically in the beginning of 2021. The stock started the year at \$19 per share and hit an intra-day high of \$483 on January 28, 25 fold increase with little news. The price dropped to \$40 in February, but it started increasing significantly again in late February (not shown).

The extraordinary volatility visible in Figure 5.A is shown more explicitly in Figure 5.B, which plots the 10-day close-to-close realized volatility, annualized by multiplying by $\sqrt{250/10}$.

Realized volatility peeked at over 300%, an exceptionally large price volatility. The high volatility coincided with an enormous trading volume as GameStop temporarily became one of the most traded stocks in the world despite the modest size of the company. Figure 5.C shows the daily turnover computed as the daily trading volume in shares divided by the number of shares outstanding. The daily trading volume peaked at over 200% on January 22, meaning that all the company's shares were traded more than twice each day.[10] The spikes in trading volume and volatility also coincided with an increase in social media interest in GameStop as proxied by Google searches for GameStop, its ticker GME, and WallStreetBets as seen in Figure 5.D.

Other effects. The real world is almost always more complex than any stylized theoretical model, and the case of GameStop is no exception. The trading frenzy almost surely started in social media similarly to the model, but ultimately several effects played a role in the meteoric rise in the price. First of all, retail investors bet on GameStop both by buying the stock and by buying call options. Call options have embedded leverage, allowing investors to multiply their gains or losses many times for the same dollar investment (Frazzini and Pedersen (2021)). When end-investors buy call options, they are sold by market makers who hedge their risk by buying the stock. The hedge ratio (called the "delta") increases when the stock price increases (and this change in the delta is called the "gamma"). Therefore, an increasing stock price leads to buying from option market makers, leading to further stock price increases, and so on. In other words, buying call options is similar to a pre-programmed trading strategy, where the end-investor buys more and more shares as the price rises. In the model, this strategy corresponds to letting the demand-sensitivity of naive investors depend on the stock price (or their trading gains), an interesting extension of the model.

[10] On January 27, Robinhood increased their clients' margin requirements for GameStop to 100%, meaning that their clients could no longer borrow money to buy the stock. On the next day, Robinhood and other brokers imposed temporary trading restrictions, disallowing their clients to buy GameStop and other meme stocks, fueling speculation that they had been pushed to help shortsellers that were losing money due to the rise of GameStop. This speculation turned out to be wrong, since Robinhood's trading restrictions were only enforced to prevent Robinhood from running out of money, according to CEO Vladimir Tenev's testimony to the U.S. House Committee on Financial Services, 2/18/2021. Indeed, Robinhood had to post margin to their clearing houses, and the increased volatility and large and concentrated positions in GameStop by Robinhood investors meant that Robinhood's margin requirements increased more than 10 times over a few days.

As the stock price of GameStop rose in late January, some shortsellers were simultaneously forced to close their positions, buying shares that they earlier borrowed. This reduction in shortselling further pushed the price upward. Such a cycle of increasing prices and buying by option market makers and shortsellers is called a "gamma squeeze." This version of a short squeeze likely played a role in the price spike in January as illustrated in Appendex B, but it was not a factor in the subsequently price rise in March, suggesting an importance of social network effects.

Further, some newspapers reported possible buying by institutional investors, similar in spirit to the short-term investors in the model. As the short-term investors in the model, these investors may have chosen to ride the bubble, thus contributing to the increase in price. Other sophisticated investors were focused on the long-term value, shorting the stock or simply selling their positions.

In summary, GameStop started the year 2021 with a market capitalization of \$1.2B and reached a high on Jan. 28 of \$34B. While certainly an economically meaningful rise in value, GameStop's peak market capitalization was only 0.07% of US equities, still a small corner of the overall equity market, even when including the other meme stocks that also rose in value at the time. GameStop did not issue any shares in January, so arguably the event did not have real effects for the company, at least not as of this writing. One of the other meme stocks, AMC, did manage to raise about \$300m in an at-the-market offering in which it sold shares directly into the market, perhaps seeking to sell to naive investors rather than selling to institutional investors via a conventional bookbuilding.

Link to the model. The GameStop saga had many of the elements included in the model: an investment thesis spreads via a social network (Proposition 1), fanatic views gain prominence over time (Proposition 2), the contagious investment idea leads to network effects on prices (Propositions 3), which starts a bubble (Propositions 4), prices rise further as influencers link to the fanatics (Proposition 5), sophisticated momentum investors ride the bubble while value investors bet against it (Proposition 6), and high trading volume and volatility ensue (Proposition 7). Further, this episode may represent a magnified version of more common dynamics, one that allows us to more easily observe how social networks affect markets.

4 Conclusion: Social Network Effects Everywhere?

I present a simple model of a financial market in which some investors are rational and others learn through a social network with echo-chamber effects. The model can help explain a number of observed phenomena such as social network effects in equity ownership, social network effects in portfolio holdings, excess volatility, momentum and reversal effects, meme trading, the effects of repeated news, and spillover of expectations and transaction prices across people with social links. I study the events of GameStop in early 2021 in light of the model, showing how extreme price moves were related to extreme trading volume and social media attention.

Social networks have been prevalent throughout history, but modern information technology and social media are changing the nature of these networks. Social networks are becoming more observable, which opens up new research possibilities to test the model's predictions using data on networks and market behavior.

If social network effects are a force behind pervasive dynamics throughout global equity, bond, currency, commodity, cryptocurrency, and housing markets such as value, momentum, and excess volatility, then the GameStop affair is only the tip of the iceberg, a very extreme tip that is currently seen more clearly than the part hidden under the surface.

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A Appendix: Proofs

Proof of Proposition 1. Beliefs evolve as follows for $t > 1$

$$x(t) = T^{t-1}x(1) = \begin{bmatrix} T_{nn}^{t-1} & \sum_{k=0}^{t-2} T_{nn}^k T_{nh} \\ 0 & I \end{bmatrix} x(1) \to \begin{bmatrix} 0 & (I - T_{nn})^{-1} T_{nh} \\ 0 & I \end{bmatrix} x(1) \tag{A.1}$$

Here, the second equality can be seen via induction. The convergence result follows from summing the geometric series in the upper-right block matrix, and the upper-left block matrix converges to zero, $T_{nn}^{t-1} \to 0$, because all of its eigenvalues are strictly less than one.

To see this property of the eigenvalues, let $A = T_{nn}$, which is an irreducible matrix with $\sum_j A_{i,j} \leq 1$ for all i, with strict inequality for at least one i since $T_{nh} \neq 0$. The Perron-Frobenius Theorem shows that A has a strictly positive left eigenvector ξ corresponding to the largest eigenvalue λ, that is, $\xi' A = \lambda \xi'$ with $\xi_i > 0$ for all i. Therefore, $\lambda \sum_i \xi_i = \sum_i (\xi' A)_i = \sum_i \sum_j \xi_j A_{j,i} = \sum_j \xi_j \sum_i A_{j,i} < \sum_j \xi_j$, implying that $\lambda < 1$.

Finally, the naive views are convex combinations of the hardheaded views in the limit because $\lim T^t 1_N = 1_N$, implying that $(I - T_{nn})^{-1} T_{nh} 1_{N-N_n} = 1_{N_n}$ using (A.1). ■

Proof of Proposition 2. As $t \to \infty$, the overall view of naive agents, $\bar{x}_n(t)$, converges to

$$\bar{x}_n(t) \to \frac{1}{N_n} 1'_{N_n} (I - T_{nn})^{-1} T_{nh} x_h = \mu' T_{nh} x_h = \theta' x_h \tag{A.2}$$

using that thought leaderships can be written as $\theta' = \mu' T_{nh}$. The sum of the thought leaderships is therefore

$$\theta' 1_{N-N_n} = \mu' T_{nh} 1_{N-N_n} = \frac{1}{N_n} 1'_{N_n} (I - T_{nn})^{-1} T_{nh} 1_{N-N_n} = \frac{1}{N_n} 1'_{N_n} 1_{N_n} = 1 \tag{A.3}$$

where the second-to-last equality follows from (A.1) and the fact that $T 1_N = 1_N$. ■

Proof of Proposition 3. The equilibrium price is determined by equalized the supply s

and the total demand:

$$
\begin{aligned}
s\gamma\sigma_u^2/\pi =& (1'_{N_n}x_n(t) + N_n u(t) - N_n p(t)) + (1'_{N_f}x_f + N_f u(t) - N_f p(t)) \\
&+ N_l(x_r + u(t) - p(t)) + \frac{1}{\pi}N_s[(1-\pi)E_t(p(t+1)) + \pi(x_r + u(t)) - p(t)]
\end{aligned} \tag{A.4}
$$

Isolating $p(t)$ on the left-hand side gives

$$
\begin{aligned}
p(t) =& \frac{N_n}{N + \frac{1-\pi}{\pi}N_s}\bar{x}_n(t) + \frac{N_f}{N + \frac{1-\pi}{\pi}N_s}\bar{x}_f + \frac{N_l + N_s}{N + \frac{1-\pi}{\pi}N_s}x_r \\
&+ \frac{\frac{1-\pi}{\pi}N_s}{N + \frac{1-\pi}{\pi}N_s}E_t(p(t+1)) + \frac{N}{N + \frac{1-\pi}{\pi}N_s}u(t) - \frac{s\gamma\sigma_u^2/\pi}{N + \frac{1-\pi}{\pi}N_s}
\end{aligned} \tag{A.5}
$$

which proves (19) with $c = \frac{\frac{1-\pi}{\pi}N_s}{N + \frac{1-\pi}{\pi}N_s}$. Iterating this equation forward yields the equilibrium price using standard difference equation methods and the sum of a geometric series. In particular, one can eliminate the future price by discounting the future versions of the rest of the right-hand side by c. Note the discounted future price converges to zero because of the premise that (the network part of) the expected price is bounded and because this bounded value is discounted by $c \in (0, 1)$, thus yielding a unique solution (i.e., using the standard transversality condition). To see this, note that (19) has the following structure, where I collect several terms under the umbrella called $b(t)$:

$$
\begin{aligned}
p(t) &= b(t) + cp(t+1) \\
&= b(t) + c[b(t+1) + cp(t+2)] \\
&= b(t) + cb(t+1) + c^2[b(t+2) + p(t+3)] \\
&= b(t) + cb(t+1) + c^2 b(t+2) + c^3 b(t+3) + ...
\end{aligned}
$$

Using that $1 + c^2 + ... = 1/(1-c)$, the equilibrium price is calculated as:

$$
p(t) = \frac{N_n(1-c)}{N}\sum_{s=0}^{\infty} c^s \bar{x}_n(t+s) + \frac{N_f}{N}\bar{x}_f + \frac{N_l + N_s}{N}x_r + u(t) - \frac{s\gamma\sigma_u^2}{N\pi} \tag{A.6}
$$

When $N_n = N_f = 0$, the above expression yields the equation for the rational price since $N = N_l + N_s$. To get the network part of the price, one subtracts the rational price from (A.6), using that $\frac{N_n(1-c)}{N}\sum_{s=0}^{\infty} c^s + \frac{N_f}{N} + \frac{N_l+N_s}{N} = 1$.

The convergence result for the network part of the price as $t \to \infty$ now follows from (18):

$$p_n(t) \to \frac{N_n}{N}\left(\sum_{j=N_{n+1}}^{N} \theta_j x_j - x_r\right) + \frac{N_f}{N}(\bar{x}_f - x_r) = \sum_{j=N_{n+1}}^{N_n+N_f} \frac{1+N_n\theta_j}{N}(x_j - x_r) \tag{A.7}$$

using that $\sum_{j=N_{n+1}}^{N} \theta_j = 1$ and $x_j = x_r$ for $j > N_n + N_f$. ■

Proof of Proposition 4. The price sensitivity to each fanatic value is seen by differentiating (23). ■

Proof of Proposition 5. If naive agent i with influencer value μ_i increases his following T_{ij} of fanatic agent j by ε at the expense of a lower following of a rational agent, then the effect on price is:

$$\frac{\partial p_n^\infty}{\partial \varepsilon} = \frac{\partial}{\partial \varepsilon}\left(\frac{N_n}{N}\sum_{j=N_n+1}^{N_n+N_f}(1+\theta_j)(x_j - x_r)\right) = \frac{N_n}{N}\,\mu_i\,(x_j - x_r)$$

where the last equality uses that the fanatic agent's thought leadership increases as follows:

$$\frac{\partial \theta_j}{\partial \varepsilon} = \frac{\partial}{\partial \varepsilon}\mu_i(T_{ij}+\varepsilon) = \mu_i$$

based on (17). ■

Proof of Proposition 6. The general statement before the proposition that short-term investors always end up being value investors follows from the fact that the price absent revelation converges to a limit as seen in Proposition 3. This convergence means that momentum profits, captured in the first part of equation (26), converge to zero over time while profits from trading on reversal, captured in the second part of equation (26), are bounded away from zero.

When the naive network is symmetric, it holds that $1'T_{nn} = \lambda 1'$, for some $\lambda \in (0, 1)$.

Hence, the average view of naive investors is:

$$\bar{x}_n(t) = \frac{1}{N_n}1'x_n(t) = \frac{1}{N_n}1'T_{nn}^{t-1}x_n(1) + \frac{1}{N_n}1'\sum_{k=0}^{t-2}T_{nn}^kT_{nh}x_h \tag{A.8}$$
$$= \lambda^{t-1}\bar{x}_n(1) + \frac{1}{N_n}\sum_{k=0}^{t-2}\lambda^k 1'T_{nh}x_h = \lambda^{t-1}\bar{x}_n(1) + (1-\lambda^{t-1})\bar{x}_n^\infty$$

where $\bar{x}_n^\infty = \frac{1}{N_n}1'T_{nh}x_h/(1-\lambda)$. Therefore, the first term in the price equation (A.6) can be rewritten as follows, leaving out the constant term $\frac{N_n(1-c)}{N}$,

$$\sum_{s=0}^{\infty}c^s\bar{x}_n(t+s) = \sum_{s=0}^{\infty}c^s\left(\lambda^{t+s-1}\bar{x}_n(1) + (1-\lambda^{t+s-1})\bar{x}_n^\infty\right) = \frac{\lambda^{t-1}}{1-c\lambda}\left(\bar{x}_n(1)-\bar{x}_n^\infty\right) + \frac{\bar{x}_n^\infty}{1-c}$$

so the price can be written as

$$p(t) = \frac{\lambda^{t-1}(1-c)}{1-c\lambda}\frac{N_n}{N}\left(\bar{x}_n(1)-\bar{x}_n^\infty\right) + \frac{N_n}{N}\bar{x}_n^\infty + \frac{N_f}{N}\bar{x}_f + \frac{N_l+N_s}{N}x_r + u(t) - \frac{s\gamma\sigma_u^2}{N\pi}$$

Hence, if $(\bar{x}_n(1)-\bar{x}_n^\infty) < 0$, then, before revelation, the price is rising monotonically in time t and price changes $p(t+1)-p(t)$ are monotonically falling over time, and vice versa if $(\bar{x}_n(1)-\bar{x}_n^\infty) > 0$. Therefore, equation (26) shows that the position of short-term investors is falling in the first case and rising in the latter case. ■

Proof of Proposition 7. Regarding trading volume, when all agents are rational, then all agents establish their positions at time 1 and keep these positions until the payoff is revealed, so volume is minimal. With naive and fanatic agents, trading volume is generally positive as naive agents continue to update their views, but, as $t \to \infty$, their views converge so that view changes approach zero, leading to a diminishing volume.

Regarding price variability, with only rational agents, the price is $p(t) = x_r + u(t) - \frac{s\gamma\sigma_u^2}{N\pi}$ so the valuation ratio defined (because the model is additive, rather then multiplicative) as the price minus $u(t)$ is constant over time until the revelation, $x_r - \frac{s\gamma\sigma_u^2}{N\pi}$. With naive and fanatic agents, the valuation ratio varies over time due to changes in the network component of the price. This network effect creates another source of price variation in addition to the price variation arising from changes in u. ■

B Appendix: GameStop Short Squeeze

The top panel of Figure 6 shows that the estimated amount of shortselling decreased significantly over January as the price rose, and remained at a low level even as the price started to fall. While a short squeeze can happen for technical reasons as shares are recalled such that shortsellers become forced to close their positions, this did not happen in the case of GameStop. The securities lending markets for GameStop was affected by the high turnover, but largely remained "open" as seen in the bottom panel Figure 6 (and confirmed via discussions with several institutional investors and lending agents). Hence, well-capitalized institutional investors could continually short the GameStop stock at an annualized cost of around 30% while some investors, mostly retail investors, faced shorting costs as high as 200%. These shorting costs are high, but one must remember than the cost of shorting for a short time period is a small fraction of these annualized amounts. Since institutional shortsellers were not forced to close their positions for technical reasons, they likely did so because they could not sustain further losses and because their short positions had increased substantially in dollar value and volatility.

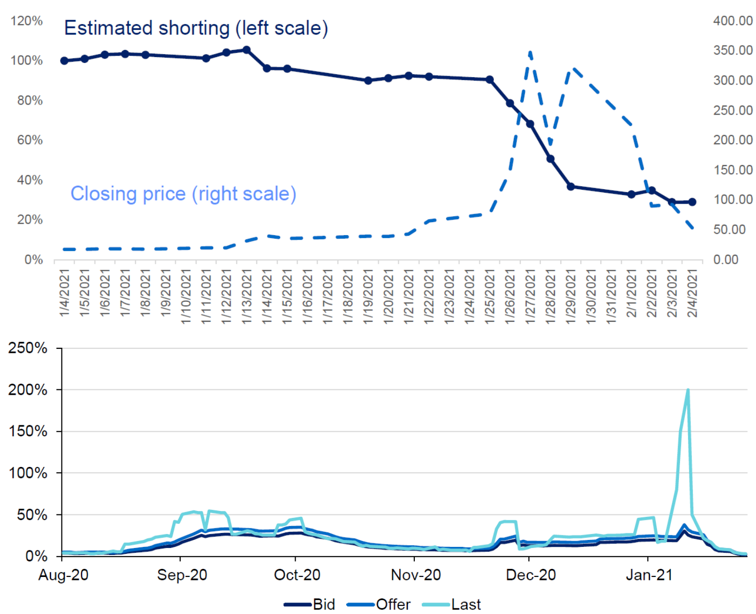

Figure 6: **Shorting amounts and costs for GameStop.** The top panel shows an estimate of the total amount of shortselling (left scale) and the closing price of GameStop (right scale). The bottom panel shows three measures of the annual shorting cost for GameStop, the bid, ask, and last price. The top panel uses estimates based on shorting demand from Markit and the bottom panel is based on data from S3.