When Are Financial Markets Perfectly Competitive?

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Abstract

We study competitiveness of financial markets in a one-period model, in which traders speculate on private information and hedge endowment shocks. Developing and fully characterizing a new measure of market competitiveness, we find that market becomes perfectly competitive if and only if the number of traders approaches infinity and speculation becomes negligible relative to hedging. While perfect competition – the key assumption for rational expectations equilibrium – is a strong condition, we show when it can be made innocuous and when it cannot. We discuss further implications of market competitiveness for the measurement of liquidity and the real-world financial market design.

Keywords: market competitiveness, strategic trading, price-taking, rational expectations equilibrium, private information, liquidity, market design.

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1 Introduction

There seems to be a gap between the market microstructure literature and the rest of finance. In the former, prices move because of trading; such impact on the price makes traders behave strategically; the strategic incentives of privately informed traders are crucial in determining equilibrium, e.g., Kyle (1985). In the latter, which uses the noisy rational expectations equilibrium (REE) concept of Grossman and Stiglitz (1980), strategic trading is unimportant. Traders take the price as given, i.e., markets are perfectly competitive; prices move because of information.

In this paper, we attempt to bridge this gap. To do so, we study what determines competitiveness of financial markets in which traders have private information. By providing a necessary and sufficient condition for perfect competition, we show when REE coincides with its strategic counterpart and when it does not. Even if markets are imperfectly competitive, we show when the perfect competition assumption is innocuous and when it is not. Furthermore, we find that market competitiveness has implications for the measurement of liquidity and the real-world financial market design.

Model Preview Our basic model adopts the setting of Diamond and Verrecchia (1981): risk averse agents trade to speculate on their private information and to hedge their random endowments; traders submit demand schedules (i.e., sets of price-contingent orders); the demand schedules are aggregated to find the market clearing price. We study both REE, which in this paper we refer to as a competitive equilibrium, and a strategic equilibrium. We then extend the model to incorporate two elements: (i) the liquidation value of the asset has residual uncertainty, about which no trader receives any information; (ii) the random endowments are correlated across the traders.

Results Preview The main result of this paper is the characterization of equilibrium market competitiveness. To this end, we develop a new measure for it. Although the literature often uses price impact, also known as Kyle's lambda, we show that it is inaccurate. Measuring the effect on the price per share traded, it does not take into account the traded quantity. The market may remain imperfectly competitive even as the price impact approaches zero.

Based on comparing a strategic equilibrium with its competitive counterpart, our measure χ is defined as the ratio of the quantity that a strategic trader optimally trades to the hypothetical quantity that she would have traded if she were a price taker. It is a constant between zero and one. The market is perfectly competitive if and only if χ equals one; the market becomes less and less competitive as χ decreases.

In equilibrium, market competitiveness is determined by the number of traders and relative informational efficiency φ, which is defined by the ratio of the precision revealed by the price to the precision available to the rest of the market à la Kyle (1989). It increases in the number of traders and decreases in φ. As φ increases so that the price reveals a larger fraction of private information, trading destroys a larger fraction of the possible profit, to which traders respond by further reducing the quantity, resulting in less competitive markets.

It follows that the market becomes perfectly competitive if and only if the number of traders approaches infinity and relative informational efficiency φ approaches zero. We show that φ approaches zero if and only if speculation becomes insignificant relative to hedging, i.e., either of the two holds: (i) the intensity in which traders speculate on their private information goes to zero relative to the intensity in which traders hedge their endowments; (ii) the amount of private information goes to zero relative to the amount of endowment shock. In requiring speculation to become insignificant relative to hedging, this result contrasts with the conventional wisdom that infinitely many traders are sufficient for achieving perfect competition. To put it differently, as long as speculation is meaningful, financial markets are imperfectly competitive.

The result has three implications – theoretical, empirical, and practical. The first implication is for the use of REE in theoretical models. Our result shows that perfect competition is a strong assumption: it is unlikely to hold even in large markets because we think speculation matters in financial markets. The REE approach focuses on endogenous information aggregation, taking market competitiveness exogenous. What we find, however, is that competitiveness crucially depends on information aggregation, i.e., competition is as endogenous as information.

Even if markets are imperfectly competitive, REE may be still used depending on the model's research questions. Since market competitiveness is captured by the quantity rather than the price, the perfect competition assumption can be made innocuous when studying the equilibrium price and information aggregation; it is less so when studying the equilibrium quantity and information acquisition.

A REE price can be supported by a strategic equilibrium because the price is unaffected by strategic trading, with traders reducing the quantity traded on information, endowments, and the price with the same proportion. Provided that the price reveals less than half of available information so that a strategic equilibrium with trade exists, REE may be used to study the price and thus the information that traders learn from the price. On the contrary, using REE can radically change the model's predictions on the equilibrium quantity and information acquisition. The perfect competition assumption would exaggerate the trading volume associated with private information and the incentive to acquire private information, which depends on the quantities that traders can trade upon acquiring it.

The second implication is for the measurement of liquidity. We show that our measure of market competitiveness χ is equivalent to the market's ability to fulfill the trading needs – one notion of market liquidity. Since the price impact does not accurately represent market (non)competitiveness as discussed above, it can also misrepresent market (il)liquidity. We show how to adjust the price impact for the traded quantity, which may help connecting theory to empirical measures of liquidity.

The third implication is for the market design of stock exchanges. Current stock exchanges require that traders submit one order at a time. Our result that strategic trading can be important even in large stock exchanges implies that this feature of stock exchanges would not work well for traders. Dynamic models of strategic trading show that the optimal strategy is to trade gradually over time; implementing such strategy in practice involves submitting a huge number of small orders. To overcome the limitation in the market design, traders devote their own resources, e.g., technology, which is unnecessary and can lead to further distortions. Thus, the market design of stock exchanges must incorporate market competitiveness.

Related Literature Broadly we can divide the literature closely related to this paper into five groups: the auctions literature with independent private values, the auctions literature with interdependent values, the models of strategic trading with noise trading, the literature on informational size, and Hellwig (1980).

The question of whether strategic traders become price-takers in a large market has long been studied in the auctions literature. When traders have independent private values, Wilson (1985), Satterthwaite and Williams (1989), and Rustichini, Satterthwaite and Williams (1994) show that markets become perfectly competitive as the number of traders grows to infinity. The independent private value setting is comparable to a special case of our model when each trader receives only one signal about her endowment shock. Although there is information asymmetry among traders, there is no adverse selection because trading is motivated by hedging only. Absent speculation, our result is consistent with the literature: infinitely many traders are sufficient for perfect competition.

When traders have interdependent values, Wilson (1977), Milgrom (1981), Pesendorfer and Swinkels (1997) and Reny and Perry (2006) show that traders become price-takers as the market becomes large. In these models traders are restricted to buy or sell up to one unit, which does not capture the trade-off between the price and the quantity important in financial markets. Allowing multi-unit demands, Vives (2011) finds that markets become perfectly competitive as the number of traders approaches infinity. Rostek and Weretka (2015) shows that perfect competition also requires that the traders's values do not become perfectly correlated. Contrary to this literature, we find that perfect competition requires that speculation becomes negligible relative to hedging. Even with some endowment shock, which prevents the traders's values from becoming perfectly correlated, markets remain imperfectly competitive unless hedging completely dominates speculation.

In a model of strategic speculation and noise trading, Kyle (1989) studies when a strategic equilibrium coincides with a competitive equilibrium. He shows that perfect competition requires both the number of traders and the size of noise trading to become infinity; the market becomes perfectly competitive with infinitely many traders when the per-capita noise trading is constant. Kovalenkov and Vives (2014) study the rate of convergence between a competitive and a strategic equilibria as the number of traders grows to infinity. Our result differs from this literature because the market remains imperfectly competitive with infinitely many traders when per-capita endowment shock, which some consider as tantamount to noise trading, is constant. Besides, noise traders are analogous to competitive traders in that they trade exogenous quantity without taking into account the price impact; they are unsuitable to study strategic behaviors. Our strategic model of speculation and hedging allows us to study the intensity in which traders endogenously hedge their endowments and compare it with the intensity in which they speculate.

In a general equilibrium model, McLean and Postlewaite (2002) introduce the concept of informational size: when agents are informationally small, their incentive issues regarding private information can be ignored for studying ex-post efficiency.[1] They find that agents are small if their private information "adds little to the aggregate information" and the number of agents grows to infinity. This literature is related to our paper because perfect competition is closely connected to ex-post efficiency. In fact, we find that if and only if markets are perfectly competitive, traders perfectly hedge their endowment shock, exactly because perfect competition requires hedging to completely dominate speculation. Our result, however, is different from this literature because we find that markets with infinitely many traders can remain imperfectly competitive although the precision of each trader's private information becomes a zero fraction of that of aggregate information.

The question of when financial markets are perfectly competitive is distinct from that of Hellwig (1980). (The discussion below is rather extensive because understanding the relationship between the two is delicate and important on its own.) He points out that competitive traders are "schizophrenic" because they perfectly anticipate the market clearing price, while they incorrectly assume the slope of their residual supply schedule (i.e., the price impact) to be zero. REE requires the conjectured price to be consistent with the market clearing price of the best response demand schedules; it does not require the conjectured slope to be consistent with the slope resulting from market clearing. When there are finitely many traders, the equilibrium price impact is always strictly positive, incompatible with the conjectured price impact of zero.

[1] See also Palfrey and Srivastava (1986), Blume and Easley (1990), Gul and Postlewaite (1992), McLean and Postlewaite (2004), and McLean, Peck and Postlewaite (2005).

Stating that "in order to avoid these difficulties, the present paper will study the aggregation of information in a large market," Hellwig (1980) takes the limit of a sequence of competitive equilibria as the number of traders approaches infinity. If the competitive equilibrium price impact goes to zero in the limit, which it does with some parameters, it conforms to the conjectured price impact; the schizophrenia vanishes.

His question of resolving the inconsistency of REE is thus different from our question of studying endogenous market competitiveness. The market may be imperfectly competitive even as the competitive equilibrium price impact, slightly different from its strategic counterpart, approaches zero. The fact that the limit of competitive equilibria is internally consistent does not imply that the limit of strategic equilibria coincides with a competitive equilibrium. On the other hand, the converse is true: if markets become perfectly competitive in the limit, a competitive equilibrium is free of schizophrenia because it coincides with a strategic equilibrium.

The same logic applies to the models with a continuum of traders. Since no trader can affect the price, REE with a continuum of traders is internally consistent. As Aumann (1964) explains, modeling agents as a continuum is a way to represent the ideal state of perfect competition. In other words, if the model assumes a continuum of traders, it also assumes perfect competition. Thus, modeling a continuum of traders is appropriate only if perfect competition is appropriate to assume, which depends on the model's questions as discussed in the theoretical implication.

The plan for this paper is as follows. Section 2 describes the setup and characterizes both a strategic and a competitive equilibria. Section 3 develops a measure of market competitiveness and provides a necessary and sufficient condition for perfect competition. Section 4 extends the basic model and shows robustness of the main result. Section 5 discusses various implications. Section 6 concludes.

2 Basic Model

This section studies a model in which traders speculate on their private information and hedge their random endowments. After describing the setup and definitions of equilibrium in 2.1, we solve for a strategic equilibrium in 2.2 and a competitive equilibrium in 2.3. We compare the two equilibria to study market competitiveness in Section 3, to which readers may proceed for main results.

2.1 Setup

The setup of the basic model follows a CARA-normal framework of Diamond and Verrecchia (1981). There is one round of trading in which traders exchange a risky asset against a safe asset whose return is normalized to one. The liquidation value of the risky asset is

$$v, \qquad \text{where} \qquad v \sim \mathrm{N}\left(0, \sigma_V^2\right). \tag{1}$$

There are N traders, each with constant absolute risk aversion A. Each trader receives two private signals before trading. First, trader n receives private information

$$i_n = v + e_n, \qquad \text{where} \qquad e_n \sim \mathrm{N}\left(0, \sigma_V^2 \tau_I^{-1}\right). \tag{2}$$

The ratio of the variance of the signal (σ_V^2) to the variance of the noise ($\sigma_V^2 \tau_I^{-1}$) is the precision parameter $\tau_I \in [0, \infty)$. The signal i_n is a pure noise if $\tau_I = 0$. It reveals the realization of v if $\tau_I \to \infty$.

Second, trader n receives random shares of the risky asset given by

$$s_n, \qquad \text{where} \qquad s_n \sim \mathrm{N}\left(0, \sigma_S^2\right). \tag{3}$$

The liquidation value v, the noise in the private information $e_1, \ldots, e_N$, and random endowments $s_1, \ldots, s_N$ are independently distributed.[2]

Trader n submits a demand schedule $X_n\left(p \mid i_n, s_n\right)$, where X_n is a function of the price p measurable with respect to the trader's private signals.[3] The price is determined by the market clearing condition[4]

$$\sum_{n=1}^{N} X_n\left(p\right) = 0. \tag{4}$$

There are five exogenous parameters: (i) N is the number of traders; (ii) A is the risk aversion; (iii) σ_V is the standard deviation of the liquidation value; (iv) τ_I is the precision of private information; (v) σ_S is the standard deviation of the endowment shock. We assume $A > 0$, $\sigma_V > 0$, and $N > 1$.

Definitions of Equilibrium We study two types of equilibria.

A *strategic equilibrium* is a set of demand schedules $X_1\left(p\right), \ldots, X_N\left(p\right)$ such that each trader chooses his strategy to maximize his expected utility, taking as given the strategies of the other $N-1$ traders and the market clears. I.e., a strategic equilibrium is a (Bayesian) Nash equilibrium.

A *competitive equilibrium* is a set of demand schedules $X_1\left(p\right), \ldots, X_N\left(p\right)$ such that each trader chooses his strategy to maximize his expected utility, taking the price as given and the market clears. A competitive equilibrium is also known as a noisy rational expectations equilibrium (REE). In this paper we call it a competitive equilibrium

[2]The model is extended to allow residual uncertainty in the liquidation value and a correlation among the endowments in Section 4.

[3]In Grossman and Stiglitz (1980) traders are assumed to choose a quantity after observing the realized price. In a competitive equilibrium this is essentially the same as traders choosing the demand schedule, with which, by design, traders know the price at which they would trade a certain quantity. This assumption also allows us to harmonize the setting for both equilibria that we study.

[4]If there is no market clearing price, nobody trades. If there are many market clearing prices, the smallest price which minimizes trading volume is chosen, with possible ties resolved by flipping a coin.

to clearly distinguish it from a strategic equilibrium by emphasizing its price-taking assumption. In both equilibria traders rationally learn from the price.

We focus on a symmetric linear (strategic or competitive) equilibrium. A *symmetric linear equilibrium* is an equilibrium in which all traders choose the same linear demand schedule

$$X_n\left(p \mid i_n, s_n\right) = \pi_C + \pi_I i_n - \pi_S s_n - \pi_P p. \tag{5}$$

Without loss of generality, we assume $\pi_C = 0$ since it can be shown to be zero in equilibrium regardless of the values of other parameters.[5]

If (and only if) $\pi_P = 0$, market clearing requires each trader's demand to be identically zero (i.e., $X_n \equiv 0$ for all n). Any price can support this allocation, and such an equilibrium always exist. We call this a *trivial no-trade equilibrium* and exclude it from the following analysis because the market clearing price is not uniquely determined.

2.2 Strategic Equilibrium

A strategic equilibrium is found like Kyle (1989). The difference is that in this model traders have two private signals and there are no noise traders or uninformed traders.

Taking as given the other traders's strategies, market clearing implies that trader n faces the residual supply schedule given by

$$p = p_n + \lambda x_n = \frac{\pi_I}{\pi_P}\frac{\sum_{n' \neq n} i_{n'}}{N-1} - \frac{\pi_S}{\pi_P}\frac{\sum_{n' \neq n} s_{n'}}{N-1} + \frac{1}{(N-1)\,\pi_P} x_n, \tag{6}$$

where x_n denotes the quantity that trader n trades ($X_n\left(p \mid i_n, s_n\right) = x_n$). It is linear, char-

[5]Linearity is not an assumption for strategies but an equilibrium property. When a trader conjectures that the other traders play linear strategies, the trader's response is also linear, not by assumption but by optimality. It is beyond the scope of this paper to study nonlinear or asymmetric equilibria, for which readers may refer to Breon-Drish (2015), Pálvölgyi and Venter (2015), and Glebkin (2015).

acterized by the intercept and the slope. The intercept p_n is the price that would prevail if trader n did not trade. Submitting a demand schedule, the trader knows the price p at which she would trade a certain quantity, which allows her to choose the quantity x_n as if she knows the realized p_n. The slope λ is the price impact, which measures the marginal effect on price of trading one additional share à la Kyle (1985).

The trader finds his best response in two steps: (1) learning from the price; (2) choosing the optimal quantity to trade.

First, to describe learning from the price, rewrite p_n as

$$\frac{\pi_P}{\pi_I} p_n = v + e_n^P = v + \frac{\sum_{n' \neq n} e_{n'}}{N-1} - \frac{\pi_S}{\pi_I} \frac{\sum_{n' \neq n} s_{n'}}{N-1} \tag{7}$$

to summarize all new information that a trader can learn about the other traders's private information from the price. This information is generally noisier than what the trader could have learned from directly observing (the sum of) the other signals.

Define φ as

$$\varphi := \frac{\sigma_V^2 \operatorname{var}^{-1}\left\{e_n^P\right\}}{(N-1)\,\tau_I}. \tag{8}$$

The numerator is the precision that a trader extracts from the price, which is the ratio of the variance of the signal (σ_V^2) to the variance of the noise ($\operatorname{var}\{e_n^P\}$). The denominator is the precision that a trader can extract from observing the sum of the other signals. Thus, φ is the ratio of the precision revealed by the price to the precision available to the rest of the market à la Kyle (1989).

Since the price is a noisy signal of the other traders's private information, φ lies between zero and one. If $\varphi = 0$, the price reveals no information. If $\varphi = 1$, the price reveals all available information. It measures how efficiently the price aggregates available information; we thus call φ relative informational efficiency.

Substituting (7) into (8) yields

$$\varphi = \left(1 + \left(\frac{\pi_S}{\pi_I}\right)^2 \left(\frac{\sigma_S^2}{\sigma_V^2 \tau_I^{-1}}\right)\right)^{-1}. \tag{9}$$

It is determined by the ratio between the two demand schedule coefficients (π_S/π_I) as well as the ratio between the two error variances ($\sigma_S^2/\left(\sigma_V^2\tau_I^{-1}\right)$). Intuitively, the price aggregates information more efficiently (i) when traders speculate on their information more aggressively than they hedge their endowments; (ii) when the variance from the other traders's endowments is smaller relative to that from the errors in their private information.

Using φ, we can express the trader's learning (Bayesian inference) as follows.

$$E\left\{v \mid i_n, s_n, p\right\} = \frac{1}{1 + \tau_I + (N-1)\tau_I\varphi}\left(\tau_I i_n + (N-1)\tau_I\varphi\frac{\pi_P}{\pi_I}p_n\right); \tag{10}$$

$$\operatorname{var}\left\{v \mid i_n, s_n, p\right\} = \frac{\sigma_V^2}{1 + \tau_I + (N-1)\tau_I\varphi}. \tag{11}$$

The second step in finding the best response is choosing the optimal quantity. Since all random variables are jointly normally distributed and trading strategies are linear, the optimal quantity x_n solves the quadratic maximization problem

$$\max_{x_n}\left[\left(E\left\{v \mid i_n, s_n, p\right\} - p_n - \lambda x_n\right)x_n - \frac{A}{2}\left(s_n + x_n\right)^2 \operatorname{var}\left\{v \mid i_n, s_n, p\right\}\right], \tag{12}$$

whose first and second order conditions are given by

$$x_n = \frac{E\left\{v \mid i_n, s_n, p\right\} - p_n - s_n A\operatorname{var}\left\{v \mid i_n, s_n, p\right\}}{2\lambda + A\operatorname{var}\left\{v \mid i_n, s_n, p\right\}} \tag{13}$$

and $2\lambda + A\operatorname{var}\left\{v \mid i_n, s_n, p\right\} \geq 0$ respectively. The residual supply schedule (6) can be substituted into the FOC to find the best-response demand schedule $X_n\left(p\right)$.

A strategic equilibrium is found when the best response is consistent with the conjectured demand schedules for the other traders. The following proposition fully characterizes a strategic equilibrium. All proofs are in the Appendix.

Proposition 1 (Strategic Equilibrium)**.** *There exists a symmetric linear strategic equilibrium, excluding trivial no-trade equilibria, if and only if*

$$(N-2)\,(A\sigma_V\sigma_S)^2 > N\tau_I. \tag{14}$$

An equilibrium is unique if it exists.

The equilibrium is the set of demand schedules

$$X_n\left(p \mid i_n, s_n\right) = \frac{1}{A\sigma_V^2}\left(\frac{N-2}{N-1} - 2\varphi\right)\left(\tau_I i_n - A\sigma_V^2 s_n - \left(\tau_I + \frac{1}{1+(N-1)\,\varphi}\right)p\right), \tag{15}$$

with

$$\varphi = \frac{\tau_I}{\tau_I + A^2\sigma_V^2\sigma_S^2}. \tag{16}$$

In equilibrium, the market clearing price p is

$$p = \frac{1+(N-1)\,\varphi}{1+\tau_I+(N-1)\,\tau_I\varphi}\left(\tau_I\frac{\sum_{n=1}^N i_n}{N} - A\sigma_V^2\frac{\sum_{n=1}^N s_n}{N}\right) \tag{17}$$

and the price impact λ is

$$\lambda = \frac{A\sigma_V^2}{1+\tau_I+(N-1)\,\tau_I\varphi}\,\frac{1+(N-1)\,\varphi}{N-2-2\,(N-1)\,\varphi}. \tag{18}$$

The existence of a strategic equilibrium with trade requires that the amount of endowment shock is sufficiently large relative to that of private information. Further discussion on the properties of this equilibrium is in 3.1.

2.3 Competitive Equilibrium

We solve for the competitive equilibrium of Diamond and Verrecchia (1981) to directly compare with its strategic counterpart.

Taking the price as given, market clearing implies

$$p = \frac{\pi_I}{\pi_P}\frac{\sum_{n=1}^{N} i_n}{N} - \frac{\pi_S}{\pi_P}\frac{\sum_{n=1}^{N} s_n}{N}, \tag{19}$$

i.e., trader n faces a flat residual supply schedule. The intercept is p and the slope is zero because the trader does not affect the price.

The trader finds his best response in the same two steps. The first is to learn from the price. To isolate new information, rewrite p as

$$\frac{N\pi_P}{(N-1)\,\pi_I}\left(p - \frac{\pi_I}{\pi_P}\frac{i_n}{N} + \frac{\pi_S}{\pi_P}\frac{s_n}{N}\right) = v + \frac{\sum_{n'\neq n} e_{n'}}{N-1} - \frac{\pi_S}{\pi_I}\frac{\sum_{n'\neq n} s_{n'}}{N-1}. \tag{20}$$

The RHS is identical to that of (7) in a strategic equilibrium. If the ratio π_S/π_I were the same in the two equilibria, the information from the price and the relative informational efficiency φ, defined by (8), would also be the same in the two equilibria.

The second step is choosing the optimal quantity, which we denote by x_n^{PT}. The FOC follows replacing the intercept and the slope in (13) with p and zero accordingly.

$$x_n^{PT} = \frac{E\left\{v \mid i_n, s_n, p\right\} - p - s_n \text{Avar}\left\{v \mid i_n, s_n, p\right\}}{\text{Avar}\left\{v \mid i_n, s_n, p\right\}}. \tag{21}$$

The SOC ($\text{Avar}\left\{v \mid i_n, s_n, p\right\} \geq 0$) is always satisfied. A competitive equilibrium is found when the trader's best response is consistent with the conjectured market clearing price.

Proposition 2 (Competitive Equilibrium)**.** *There exists a symmetric linear competitive equilibrium excluding trivial no-trade equilibria if and only if*

$$A\sigma_V\sigma_S > 0. \tag{22}$$

An equilibrium is unique if it exists.

The competitive equilibrium is the set of demand schedules

$$X_n^{PT}\left(p \mid i_n, s_n\right) = \frac{1-\varphi}{A\sigma_V^2}\left(\tau_I i_n - A\sigma_V^2 s_n - \left(\tau_I + \frac{1}{1+(N-1)\varphi}\right)p\right), \tag{23}$$

where φ *and the market clearing price* p *are the same as those in a strategic equilibrium given by* (16) *and* (17) *respectively.*

In equilibrium, the slope of the residual supply schedule is

$$\lambda^{PT} = \frac{A\sigma_V^2}{1+\tau_I+(N-1)\tau_I\varphi}\frac{1+(N-1)\varphi}{(N-1)\left(1-\varphi\right)}. \tag{24}$$

Any endowment shock is sufficient for the existence of a competitive equilibrium with trade. It is also necessary because private information alone cannot generate trade[6] Observe that the equilibrium residual supply schedule is upward-slopping ($\lambda^{PT} > 0$) although competitive traders take as given a flat residual supply schedule (19). Traders suffer from what Hellwig (1980) calls schizophrenia. Next, we analyze the two equilibria to study market competitiveness.

[6] See Aumann (1976) and Milgrom and Stokey (1982).

3 Market Competitiveness

This section addresses the main research question. We develop and characterize a measure of market competitiveness in 3.1, use the measure to show when financial markets are perfectly competitive in 3.2, and provide and discuss examples in which markets remain imperfectly competitive with infinitely many traders in 3.3.

3.1 How to Measure Market Competitiveness

To determine how competitive financial markets are, we need a way to measure it. The literature often uses the price impact λ as a measure of market power (or noncompetitiveness of the market). However, it is imperfect and can lead to incorrect conclusions, as we show in 3.3. We thus develop a new measure of market competitiveness by comparing the two equilibrium solutions from the previous section. Since the competitive traders's price-taking behavior represents the ideal state of perfect competition, we can quantify market competitiveness by analyzing how close (or far) a strategic equilibrium is to its competitive counterpart.

The difference between the two equilibria is captured by the traded quantities rather than by the price. The two equilibrium demand schedules (15) and (23) are the same except for one constant: replacing $\left(\frac{N-2}{N-1} - 2\varphi\right)$ in the strategic equilibrium with $1-\varphi$ would yield the competitive equilibrium. Taking into account their price impact, strategic traders choose to trade less than competitive traders do.[7] They reduce the quantity to the same extent for speculating on their private information, hedging their endowments, and responding to the price, since the price impact affects them all equally.[8]

[7] In the auctions literature this is called bid-shading; see Ausubel et al. (2014).

[8] See also Dávila and Parlatore (2017) who study the effect of trading costs on price informativeness. They show that with ex-ante symmetric traders various (quadratic, linear, fixed) trading costs do not affect information aggregation.

With the constant ratio between the two quantities being factored out when aggregating demand schedules, the market clearing price and thus relative informational efficiency φ are the same in the two equilibria.

Define χ as the ratio between the two quantities.

$$\chi := \frac{X_n\left(p \mid i_n, s_n\right)}{X_n^{PT}\left(p \mid i_n, s_n\right)}, \qquad \forall n = 1, \ldots, N. \tag{25}$$

The comparison between the two equilibria is meaningful when they both exist. Recall that we focus on equilibria with trade because the price is not uniquely determined in a trivial no-trade equilibrium. In (14) and (22) any endowment shock is sufficient to generate trade in a competitive equilibrium, while it is not for a strategic equilibrium.[9] Strategic traders trade only when the endowment shock is sufficiently large relative to private information so that relative informational efficiency φ is sufficiently low (i.e., $\varphi < \frac{N-2}{2(N-1)}$). They never trade when the price reveals more than half of their private information. Competitive traders, on the other hand, continue to trade as long as φ does not equal one.

Provided that a strategic equilibrium exists and thus both equilibria exist, χ lies between zero and one. If $\chi \to 1$, the market is perfectly competitive, with traders optimally taking the price as given. As χ decreases, the traders exercise their market power by progressively reducing the quantity they trade. If $\chi \to 0$, there is no trade.[10] We thus use χ as our measure of market competitiveness.

The following proposition characterizes the equilibrium market competitiveness.

Proposition 3 (Market Competitiveness)**.** *Assume $\frac{\tau_I}{(A\sigma_V\sigma_S)^2} < \frac{N-2}{N}$ so that φ, given by*

[9]This difference in the existence condition does not arise in the model of noise trading, like Kyle (1989) because noise traders by design ignore their price impact. In this sense, noise traders are similar to competitive traders; see also Banerjee and Green (2015).

[10]With risk-neutral traders ($A = 0$), competitive traders trade infinite quantities, and thus the trading volume can still be positive in the limit $\chi \to 0$.

(16), *satisfies* $\varphi < \frac{N-2}{2(N-1)}$. *Then we have*

$$\chi = \frac{N-2}{N-1} - \frac{N}{N-1}\left(\frac{\varphi}{1-\varphi}\right), \tag{26}$$

or equivalently,

$$\chi = \frac{N-2}{N-1} - \frac{N}{N-1}\frac{\tau_I}{(A\sigma_V\sigma_S)^2}. \tag{27}$$

Intuitively, the market becomes more competitive when there are more traders, i.e., χ increases in N. What is striking is that other than N, market competitiveness χ depends only on relative informational efficiency φ, or equivalently the ratio $\tau_I/(A\sigma_V\sigma_S)^2$. In other words, to understand how information asymmetry affects competitiveness of the market, it is sufficient to know how φ is determined.

To see how φ captures the effect of information asymmetry on χ, see

$$\underbrace{p - p_n}_{\text{profit erosion}} = \frac{1+(N-1)\varphi}{N} \times \left(\underbrace{E\{v \mid i_n, s_n, p\} - s_n \operatorname{Avar}\{v \mid i_n, s_n, p\} - p_n}_{\text{possible profit}}\right), \tag{28}$$

which follows substituting (13) and (18) into the residual supply schedule (6). If the trader were to have no effect on the price, trading any quantity at the prevailing price p_n, the expected profit per share would be $E\{v \mid i_n, s_n, p\} - s_n \operatorname{Avar}\{v \mid i_n, s_n, p\} - p_n$. The actual profit is lower than this because the price moves towards the trader's valuation (which is exactly how the price incorporates traders's private information). The fraction $\left(1+(N-1)\varphi\right)/N$ determines the extent of such profit erosion.

Thus, the market becomes less competitive when the price aggregates available information more efficiently, i.e., χ decreases in φ.[11] As the price reveals a larger fraction

[11] This conflict between relative informational efficiency and market competitiveness is related to Kawakami (2017) who studies the optimal market size which balances information aggregation and risk sharing.

of their private information, trading destroys a larger fraction of the possible profit and thus traders optimally trade less aggressively.[12]

Next, we use χ to analyze the condition for perfect competition.

3.2 When Are Financial Markets Perfectly Competitive?

The following proposition answers the question.

Proposition 4 (Perfect Competition)**.** *The market becomes perfectly competitive if and only if there are infinitely many traders and the ratio between private information and relative informational efficiency φ approaches zero, i.e.,*

$$\chi \to 1 \qquad \textit{if and only if} \qquad N \to \infty \; \textit{ and } \; \varphi \to 0. \tag{29}$$

With φ given by (16)*, $\varphi \to 0$ is equivalent to $\frac{\tau_I}{(A\sigma_V\sigma_S)^2} \to 0$.*

Since χ increases in N and decreases in φ from (26), perfect competition requires that not only the number of traders approaches infinity but also the relative informational efficiency approaches zero.

Thus, infinitely many traders are necessary but insufficient for perfect competition. In the limit $N \to \infty$, each trader becomes small in their risk bearing capacity: the ratio of a trader's risk tolerance $1/A$ to that of the market N/A approaches zero.[13] Moreover, each trader becomes informationally small: the ratio of a trader's private information τ_I to that of the market $N\tau_I$ also approaches zero.[14] Why then do traders maintain their market power?

[12]Note that this is counter to the idea that the price becomes more informative as the market becomes more competitive because traders trade more aggressively. The distinction is that in our model both information and competition are endogenous.

[13]It is well known that the CARA preferences can be aggregated by summing up the risk tolerances.

[14]See McLean and Postlewaite (2002) and our discussion of related literature in Section 1.

The intuition is that traders take the price as given only when their trading does not diminish their profit, whose extent depends on N and φ in (28). In the limit $N \to \infty$, each trader loses φ fraction of the possible profit. Perfect competition thus requires φ to approach zero, in which case the profit erosion becomes negligible.

The condition $\varphi \to 0$ is equivalent to the ratio $\tau_I / (A\sigma_V\sigma_S)^2$ approaching zero. To interpret this, first suppose there is no private information ($\tau_I = 0$) and thus the condition is always satisfied. In this case, trading is motivated by hedging only. While there is information asymmetry about realized endowments, there is no adverse selection: the other traders's valuations of the asset have no effect on the trader's own valuation. Perfect competition is readily obtained in the limit $N \to \infty$.

The interesting case is when there is private information ($\tau_I > 0$). In this case, perfect competition requires not only N to approach infinity but also the endowment shock to explode (i.e., $(A\sigma_V\sigma_S)^2 \to \infty$) so that the ratio $\tau_I / (A\sigma_V\sigma_S)^2$ approaches zero. Traders may be speculating on their private information, but it would be completely dominated by hedging, i.e., speculation becomes negligible.

Summarizing, financial markets become perfectly competitive if and only if there are infinitely many traders *and* relative informational efficiency φ approaches zero, in which case hedging completely dominates speculation. As long as speculation remains important in financial markets, markets remain imperfectly competitive.[15]

3.3 Examples

Below we consider three examples of imperfect competition with infinitely many traders. These examples show that using price impact λ as a proxy for market power can lead to incorrect conclusions about market competitiveness.

[15]For discussion on how the result differs from the existing literature, see Section 1.

Example 1. Suppose that the number of traders N varies while the other parameters (A, σ_V, σ_S, and τ_I) are fixed and positive. Assume the ratio $\tau_I / (A\sigma_V\sigma_S)^2$ is less than one so that a strategic equilibrium exists for N sufficiently large. In the limit $N \to \infty$, both equilibria (15) and (23) are well-defined; φ, given by (16), is a positive constant. Thus, the market remains imperfectly competitive in the limit.

Now consider the price impact λ given by (18).

$$\lim_{N\to\infty} \lambda = \lim_{N\to\infty} \frac{A\sigma_V^2}{1+\tau_I+(N-1)\,\tau_I\varphi}\,\frac{1+(N-1)\,\varphi}{N-2-2\,(N-1)\,\varphi} = 0. \tag{30}$$

Why does the market remain imperfectly competitive when the price impact vanishes? The reason is that the price impact, measuring the effect on the price per share traded, does not take into account the quantity. As the number of traders increases, more information becomes available. With learning, the asset becomes less risky, which – other things being equal – makes competitive traders want to trade a larger quantity.[16]

Recall the FOC of a competitive trader (21). Compare this with that of a strategic trader: rewriting the FOC of a strategic trader (13) in terms of the market clearing price yields

$$x_n = \frac{E\left\{v \mid i_n, s_n, p\right\} - p - s_n \operatorname{Avar}\left\{v \mid i_n, s_n, p\right\}}{\lambda + \operatorname{Avar}\left\{v \mid i_n, s_n, p\right\}}. \tag{31}$$

Then market competitiveness χ, which is the ratio between the two quantities (21) and (31), is determined by the trade-off between λ and the riskiness of the asset.

$$\frac{1}{\chi} - 1 = \frac{\lambda}{\operatorname{Avar}\left\{v \mid i_n, s_n, p\right\}} = \frac{1+(N-1)\,\varphi}{N-2-2\,(N-1)\,\varphi}. \tag{32}$$

[16]Note that this does not imply that competitive traders would trade an infinite quantity in the limit as the asset becomes riskless. The reason is that as the asset becomes riskless, the expected profit also goes to zero. The same reason explains why traders continue to trade on their private information as the price fully reveals the liquidation value. As the expected profit of speculating goes to zero, the risk of speculating also goes to zero.

Thus, the market becomes perfectly competitive ($\chi \to 1$) if and only if the ratio of λ to the riskiness $\left(A \operatorname{var}\left\{v \mid i_n, s_n, p\right\}\right)$ approaches zero, i.e., the price impact becomes negligible relative to the risk. In this example, while both the price impact and the riskiness approach zero in the limit $N \to \infty$, the ratio remains positive $\left(\varphi / \left(1 - 2\varphi\right)\right)$ since $\varphi > 0$; the market remains imperfectly competitive.

Note that the price impact in a competitive equilibrium λ^{PT}, given by (24), also approaches zero in the limit.

$$\lim_{N \to \infty} \lambda^{PT} = \lim_{N \to \infty} \frac{A\sigma_V^2}{1 + \tau_I + (N-1)\,\tau_I \varphi} \frac{1 + (N-1)\,\varphi}{(N-1)\left(1 - \varphi\right)} = 0. \tag{33}$$

Although the market remains imperfectly competitive, a competitive equilibrium is free of schizophrenia in the limit because both the conjectured and the equilibrium residual supply schedules are flat.

Example 2. Suppose that as N varies, each trader's private information and endowment shock also vary so that aggregate information $\tau_E = N\tau_I$, aggregate endowment shock $\Sigma_S^2 = N\sigma_S^2$, and the other parameters (A and σ_V) are fixed and positive. Assume the ratio $\tau_E / \left(A\sigma_V \Sigma_S\right)^2$ is less than one so that a strategic equilibrium exists for N sufficiently large. Although the individual's private information and endowment shock decreases in N, the ratio $\tau_I / \left(A\sigma_V \sigma_S\right)^2$ is independent of N; φ is a positive constant. Thus the market remains imperfectly competitive in the limit $N \to \infty$.

In this case, the price impact remains positive.

$$\lim_{N \to \infty} \lambda = \frac{A\sigma_V^2}{1 + \tau_E \varphi} \frac{\varphi}{1 - 2\varphi} > 0. \tag{34}$$

Note that each trader's private information disappears in the limit (i.e., $\tau_I = \tau_E / N \to 0$). What matters for market competitiveness is not the amount of information but the amount of information relative to the endowment shock.

Example 3. Suppose that each trader's risk aversion $A = A_E N^{\epsilon}$ varies with N for some constant $A_E > 0$ and some exponent ϵ. While σ_V and τ_I are fixed, individual's endowment shock σ_S varies so that the ratio $\tau_I/(A\sigma_V\sigma_S)^2$ is a constant. Assume the constant is strictly between zero and one to ensure the existence of a strategic equilibrium for N sufficiently large. Then regardless of ϵ, φ is a positive constant and thus the market remains imperfectly competitive in the limit $N \to \infty$.

In this case, the limit of the price impact depends on ϵ:

$$\lim_{N\to\infty} \lambda = \begin{cases} \infty & \text{if } \epsilon > 1 \\ \text{constant} > 0 & \text{if } \epsilon = 1 \\ 0 & \text{if } \epsilon < 1 \end{cases} \cdot \tag{35}$$

The three examples highlight the role of φ in determining market competitiveness and a possible pitfall of using λ as a measure of market power.

After studying extensions in the next section, we discuss broader implications in Section 5, to which readers may proceed for a discussion on why the result matters.

4 Extensions

This section studies two extensions: residual uncertainty in 4.1 and correlated endowments in 4.2. These elements are frequently used in competitive equilibrium models.[17]

[17]For examples, see Ganguli and Yang (2009), Biais, Bossaerts and Spatt (2010), Manzano and Vives (2011), and Glebkin, Gondhi and Kuong (2018) among others.

4.1 Residual Uncertainty

We introduce residual uncertainty into the basic model in 2.1 by assuming that the exogenous liquidation value of the risky asset is

$$\hat{v} = v + y, \qquad \text{where} \qquad y \sim \mathrm{N}\left(0, \sigma_Y^2\right). \tag{36}$$

Traders continue to receive private signals about v given by (2). No one receives a signal about y, which is a pure noise, independently distributed from all the other random variables. When $\sigma_Y^2 > 0$, the asset has residual uncertainty: even if traders were to perfectly learn the realization of v, the asset would be still risky.

Residual uncertainty does not directly affect how traders learn from the price: since no traders have information about y, they do not learn about it from the price. It does, however, affect the optimal quantity by changing the riskiness of the asset. Adjusting the strategic trader's FOC (13) for this, we have

$$x_n = \frac{\mathrm{E}\left\{v \mid i_n, s_n, p\right\} - p_n - s_n A \mathrm{var}\left\{\hat{v} \mid i_n, s_n, p\right\}}{2\lambda + A \mathrm{var}\left\{\hat{v} \mid i_n, s_n, p\right\}}. \tag{37}$$

The expectation is unaffected since $\mathrm{E}\{y\} = 0$; the SOC is $2\lambda + A\mathrm{var}\left\{\hat{v} \mid i_n, s_n, p\right\} \geq 0$.

Still defining φ as the ratio of the precision of information about v revealed by the price to the precision available to the rest of the market as in (8), we have

$$\mathrm{var}\left\{\hat{v} \mid i_n, s_n, p\right\} = \frac{\sigma_V^2}{1 + \tau_I + (N-1)\,\tau_I \varphi} + \sigma_Y^2. \tag{38}$$

Learning from the price reduces the variance of v but has no effect on σ_Y^2.

The proposition below shows the effect of residual uncertainty on the equilibrium market competitiveness. The full characterization is found in the Appendix.

Proposition 5 (Residual Uncertainty)**.** *There exists a unique symmetric linear strategic equilibrium, excluding trivial no-trade equilibria, if and only if*

$$(N-2)\left(A\sigma_V\sigma_S\right)^2\left(1+\left(1+\frac{N}{2}\tau_I\right)\frac{\sigma_Y^2}{\sigma_V^2}\right)^2 > N\tau_I. \tag{39}$$

In equilibrium, φ is the unique solution to

$$\frac{\tau_I}{\left(A\sigma_V\sigma_S\right)^2}\left(\frac{1}{\varphi}-1\right)=\left(1+\left(1+\tau_I+(N-1)\,\tau_I\varphi\right)\frac{\sigma_Y^2}{\sigma_V^2}\right)^2 \tag{40}$$

and market competitiveness χ is

$$\chi=\frac{N-2}{N-1}-\frac{N}{N-1}\left(\frac{\varphi}{1-\varphi}\right). \tag{41}$$

Thus, we have

$$\chi\to 1 \qquad \textit{if and only if} \qquad N\to\infty \textit{ and } \varphi\to 0. \tag{42}$$

Market competitiveness χ is the same function of N and φ as before (26). Relative informational efficiency φ still captures the effect of information asymmetry on χ. The necessary and sufficient condition for perfect competition is identical to (29). The intuition that traders take the price as given if and only if the profit erosion from their trading is negligible continues to apply. The main result that as long as speculation remains important, the market remains imperfectly competitive is robust.

The only change is how φ is determined in equilibrium. In (40) the effect of σ_Y^2 on φ is summarized on the RHS. With $\sigma_Y^2>0$, φ is no longer determined by the ratio $\tau_I/\left(A\sigma_V\sigma_S\right)^2$ only. To see clearly how residual uncertainty affects φ and thus χ in the limit $N\to\infty$, consider the following two examples.

Example 4 Suppose that as N varies, the other parameters $(A, \sigma_V, \sigma_Y, \sigma_S$, and $\tau_I)$ are all fixed and positive. From (40), φ approaches zero in the limit $N \to \infty$.[18] This, from (9), implies that the ratio between the demand schedule coefficients π_S/π_I goes to infinity, i.e., the intensity in which traders hedge their endowments completely dominates the intensity in which traders speculate on their private information.

The intuition is that although the size of residual uncertainty σ_Y is fixed, its importance relative to the other uncertainty explodes to infinity: the effect of σ_Y, summarized in the the RHS of (40), goes to infinity as the price fully reveals the realization of v in the limit.[19] Since residual uncertainty makes speculation less desirable and hedging more necessary, the hedging motive completely dominates the speculative motive. Speculation becomes meaningless; the market becomes perfectly competitive.

Example 5 Suppose that as N varies, aggregate information $\tau_E = N\tau_I$, aggregate endowment shock $\Sigma_S^2 = N\sigma_S^2$, and the other parameters $(A, \sigma_V$ and $\sigma_Y)$ are fixed and positive. Assume the ratio $\tau_E/(A\sigma_V\Sigma_S)^2$ is less than one so that a strategic equilibrium exists for N sufficiently large. In this case, the available information τ_E is finite and traders do not learn v perfectly in the limit $N \to \infty$. The effect of residual uncertainty does not explode to infinity. In the limit φ is positive.[20] Both hedging and speculation remain meaningful; the market remains imperfectly competitive.

[18] If φ were to remain positive, the RHS would go to infinity while the LHS remained finite.

[19] Since $\varphi \to 0$ on the LHS, $1 + \tau_I + (N-1)\tau_I\varphi$ on the RHS must go to infinity in the limit $N \to \infty$.

[20] It is the solution to $\frac{\tau_E}{(A\sigma_V\Sigma_S)^2}\left(\frac{1}{\varphi} - 1\right) = \left(1 + (1 + \tau_E\varphi)\frac{\sigma_Y^2}{\sigma_V^2}\right)^2$.

4.2 Correlated Endowments

We introduce correlated endowments into the basic model in 2.1 by assuming that for all $m \neq n$,

$$\rho_S := \operatorname{corr}(s_n, s_m), \qquad \text{where} \qquad \rho_S \in \left(-\frac{1}{N-1}, 1\right). \tag{43}$$

The correlation ρ_S must be greater than $-\frac{1}{N-1}$ to ensure the covariance matrix is positive semi-definite. It must be less than one so that traders cannot perfectly infer the other traders's endowments.[21]

Correlated endowments affect how traders learn from the price because they can use their endowments to infer the other traders's endowments. The new information that trader n can learn from the intercept of the residual supply schedule (6) is

$$\frac{\pi_P}{\pi_I} p_n + \frac{\pi_S}{\pi_I} \rho_S s_n = v + e_n^P = v + \frac{\sum_{n' \neq n} e_{n'}}{N-1} - \frac{\pi_S}{\pi_I} \left(\frac{\sum_{n' \neq n} s_{n'}}{N-1} - \rho_S s_n \right). \tag{44}$$

Still defining φ by (8), i.e., the ratio of the precision revealed by the price to the precision available to the rest of the market, we have

$$\varphi = \left(1 + \left(\frac{\pi_S}{\pi_I} \right)^2 \left(\frac{\sigma_S^2 \left(1 - \rho_S\right) \left(1 + (N-1)\rho_S\right)}{\sigma_V^2 \tau_I^{-1}} \right) \right)^{-1}. \tag{45}$$

Comparing this with (9) shows the effect of ρ_S on the variance of the endowments. When endowments are correlated while the errors in private information are independent, the error variance from private information ($\frac{\sum_{n' \neq n} e_{n'}}{N-1}$) no longer decreases in N with the same proportion as the error variance from endowments ($\frac{\sum_{n' \neq n} s_{n'}}{N-1} - \rho_S s_n$) does.[22] The effect of ρ_S on the variance is non-monotonic: while the high correlation

[21] The assumption that all correlations are the same can be relaxed without affecting the equilibrium as long as the average pairwise correlation is the same across the traders, i.e., $\frac{1}{N-1} \sum_{m \neq n} \operatorname{corr}(s_n, s_m)$ is the same across all traders n; a version of equicommonality introduced by Rostek and Weretka (2012).

[22] $\operatorname{var}\left\{ \frac{\sum_{n' \neq n} s_{n'}}{N-1} - \rho_S s_n \right\} = \frac{\sigma_S^2}{N-1} \left(1 + (N-2)\rho_S - (N-1)\rho_S^2\right).$

naturally increases the variance, it makes the inference easier, lowering the variance.

From (44), we have

$$\mathrm{E}\left\{v \mid i_n, s_n, p_n\right\} = \frac{1}{1+\tau_I+(N-1)\,\tau_I\varphi}\left(\tau_I i_n + (N-1)\,\tau_I\varphi\left(\frac{\pi_P}{\pi_I}p_n + \frac{\pi_S}{\pi_I}\rho_S s_n\right)\right). \tag{46}$$

Substituting this into the FOC (13) leads to the trader's best-response demand schedule. It is similar to that in the basic model except for the hedging intensity (π_S) because the trader's endowment s_n directly affects the conditional expectation. Since the best-response is consistent with the conjectured strategies in equilibrium,

$$\frac{\pi_S}{\pi_I} = \frac{1}{1+(N-1)\,\rho_S\varphi}\frac{A\sigma_V^2}{\tau_I}. \tag{47}$$

If (and only if) $\rho_S \neq 0$, the ratio π_S/π_I depends on φ. With correlated endowments, the asset valuation depends on s_n from learning. The extent to which the valuation depends on the endowment increases in φ. The valuation then affects how aggressively the trader hedges his endowment.

The equilibrium φ is jointly determined by (45) and (47), subject to the SOC. There may exist multiple equilibria due to strategic complementarity in the hedging intensity, analogous to the competitive models of Ganguli and Yang (2009) and Manzano and Vives (2011). Below we characterize the equilibrium effect of correlated endowments on market competitiveness. The full characterization of equilibrium is in the Appendix.

Proposition 6 (Correlated Endowments)**.** *The set of symmetric linear strategic equilibria, excluding trivial no-trade equilibria, is characterized by the set of all endogenous variables φ that solve*

$$\frac{\tau_I}{(A\sigma_V\sigma_S)^2}\left(\frac{1}{\varphi}-1\right) = \frac{\left(1-\rho_S\right)\left(1+(N-1)\,\rho_S\right)}{\left(1+(N-1)\,\rho_S\varphi\right)^2} \tag{48}$$

and satisfy

$$\varphi < \frac{N-2}{2(N-1)}. \tag{49}$$

In any equilibrium, market competitiveness χ is

$$\chi = \frac{N-2}{N-1} - \frac{N}{N-1}\left(\frac{\varphi}{1-\varphi}\right). \tag{50}$$

Thus,

$$\chi \to 1 \qquad \textit{if and only if} \qquad N \to \infty \textit{ and } \varphi \to 0. \tag{51}$$

Again, χ is the same function of N and φ as in the basic model; the condition for perfect competition is the same. The main result on market competitiveness is thus robust to allowing correlated endowments. The only change is how φ is determined, with the effect of ρ_S being summarized on the RHS of (48). To understand how correlated endowments affect φ in the limit $N \to \infty$, consider these last two examples.

Example 6. Suppose that as N varies, the other parameters (A, σ_V, σ_S, τ_I, and ρ_S) are all fixed and positive.[23] Then there are two solutions to (48) in the limit $N \to \infty$: $\varphi \to 0$ and $\varphi \to 1$.[24] The solution $\varphi \to 1$, however, does not constitute an equilibrium because the SOC (49) is violated: recall that strategic traders never trade when the price reveals more than half of their private information.

Thus the unique equilibrium solution is $\varphi \to 0$. This is because in the limit $N \to \infty$ the variance of endowment shock explodes relative to the variance of information in (45): the error variance from private information vanishes, while the correlation keeps the error variance from endowments positive. In this case, π_S/π_I stays finite, and thus

[23]Note that ρ_S cannot be negative because (43) is violated for N sufficiently large.

[24]Manipulate (48) to obtain $\frac{\tau_I}{(A\sigma_V\sigma_S)^2}\varphi(1-\varphi) = \frac{(1-\rho_S)\left(1+\frac{1}{(N-1)\rho_S}\right)}{\left(\frac{1}{\sqrt{(N-1)\rho_S\varphi}}+\sqrt{(N-1)\rho_S}\right)^2} \to 0.$

traders continue to trade on their information as well as their endowments.[25] However, the size of aggregate endowments completely overwhelms that of aggregate information. Trading is flooded with hedging; the market becomes perfectly competitive.

Example 7. Suppose that as N varies, ρ_S also varies so that the aggregate correlation $\Lambda = (N-1)\,\rho_S$ (where $\Lambda > -1$) stays constant, while the other parameters (A, σ_V, σ_S, and τ_I) are fixed and positive. With the RHS of (48) staying finite in the limit $N \to \infty$, any solution φ is strictly positive. In fact, the solution is unique.[26] Thus the unique equilibrium exists, provided that the solution satisfies (49). Since the variance of endowment shock relative to that of private information does not blow up, both hedging and speculating stay significant. Thus the market remains imperfectly competitive.

Summarizing, while the residual uncertainty and correlated endowments affect how φ is determined in equilibrium, the main result is essentially unchanged. As long as speculation is important, financial markets remain imperfectly competitive. We next explain why the result matters.

5 Implications

This section discusses three implications: a theoretical implication for modeling approaches in 5.1, an empirical implication for measuring liquidity in 5.2, and a practical implication for the market design of stock exchanges in 5.3.

[25] Multiply φ to both sides of (48) and obtain $(N-1)\,\varphi < \infty$. From (47) it follows that π_S/π_I is finite.

[26] Take the limit $N \to \infty$ of (48) and rewrite it as $(1-\varphi)(1+\Lambda\varphi)^2 = k\varphi$ for some $k > 0$. Then at $\varphi = 0$, the LHS > the RHS; the LHS cuts the RHS from above for the smallest φ. If there were another solution, the LHS should cut the RHS from below. But this cannot happen $\forall \varphi \in [0,1]$ because the derivative of the LHS has at most one solution if $\Lambda > 0$, in which case the LHS is increasing at $\varphi = 0$, and none otherwise.

5.1 Theoretical Implication: Modeling Approaches

What does our result imply for REE as a modeling approach? Our result shows that its key assumption, perfect competition is highly restrictive. It is implausible even for large markets with numerous traders because it requires that speculation is insignificant; we think speculation is generally important in financial markets. Thus the key assumption of REE is not without loss of generality.

The REE approach tends to focus on how the price aggregates available information, taking market competitive as exogenous to be perfectly competitive. On the contrary, in characterizing equilibrium market competitiveness, we find that the competitiveness of markets, in which traders have private information, is determined by endogenous information aggregation, captured by relative informational efficiency φ. That is to say, market competitiveness is as endogenous as information aggregation.

Even if markets are imperfectly competitive, REE may be used to study the price and thus the information that traders learn from the price. Recall that our measure of market competitiveness centers around the quantities traded in a competitive and a strategic equilibria because the price and relative informational efficiency φ are the same in the two equilibria. Because the optimal exercise of market power manifests in reducing the quantity traded on information, endowments, and the price with the same proportion, the price is unaffected by strategic trading. Regardless of whether markets are perfectly competitive or not, the REE price can be thought of as the strategic equilibrium price.

One important catch, however, is that not all REE prices can be supported by a strategic equilibrium. Remember that the price is the same in the two equilibria provided that a strategic equilibrium – with trade – exists because the price is not uniquely determined in a trivial no-trade equilibrium. Under imperfect competition, traders completely refrain from trading when the price reveals more than half of their private information because a strategic equilibrium with trade exists if and only if $\varphi < \frac{N-2}{2(N-1)}$. Unless this inequality holds, the REE approach can lead to a meaningless price. Thus, with proper caution the REE approach may be used to study the equilibrium properties of the price and information aggregation.

When studying equilibrium quantities and endogenous information acquisition, however, assuming perfect competition and thus using REE can fundamentally alter the model's predictions. The traded quantities directly depend on market competitiveness. Assuming perfect competition in imperfectly competitive markets exaggerates the trading volume accompanied by private information. Since the value of private information depends on the quantities that traders can trade upon acquiring it, assuming perfect competition inflates the value of private information and the incentive for traders to acquire private information. Thus, the REE must be revised to take into account strategic trading to study the equilibrium properties of the quantity and information acquisition.

Aside: Is Nash Equilibrium Suitable For a Large Market? In a strategic equilibrium, traders correctly conjecture the other $N-1$ traders's strategies. This may seem an increasingly onerous task for the trader as the number of traders grows, especially in comparison with competitive traders who only need to conjecture the price correctly. A natural question is whether the Nash equilibrium concept is appropriate for studying a large market. While the concern is reasonable in general, it is less applicable to the trading environment that we study.

The intuition is that in this model all interactions among traders are captured by the residual supply schedule, whose intercept and slope are all traders need to conjecture. No matter how many traders there are, the only difference between the two equilibria is the slope. (Relatedly, Weretka (2011) defines a slope-taking equilibrium to show that it naturally extends the competitive framework in a general environment.) While strategic traders conjecture a correct slope, competitive traders, often incorrectly, assume a zero slope. Thus, the difference in complexity of the traders's problems does not grow in N; the generic concern does not apply. Furthermore, the property that traders only interact through the quantity and the price that they can trade makes this trading environment more relevant for large, anonymous, and centralized financial markets like stock exchanges.

5.2 Empirical Implication: Measuring Liquidity

The price impact λ is often used as a measure of market power in the literature. However, we show that it does not accurately characterize market power. What does this mean for measuring liquidity? To answer this question, we first submit that market competitiveness is equivalent to one concept of market liquidity. Since liquidity is a nebulous notion, we specify market liquidity to be the market's ability to fulfill trading needs of its traders.

Denote by s_n^{TI} the trader n's target inventory. It corresponds to s_n, with which the optimal quantity equals zero in the FOC (13). Then

$$s_n^{TI} := \frac{\mathrm{E}\left\{v \mid i_n, s_n, p\right\} - p}{\mathrm{Avar}\left\{v \mid i_n, s_n, p\right\}}. \tag{52}$$

The competitive trader's FOC (21) implies that competitive traders reach the target inventory after trading. Using the definition of χ, we have

$$s_n + x_n = \left(1 - \chi\right) s_n + \chi s_n^{TI}. \tag{53}$$

The trader's inventory after trading is a weighted average between the initial endowment and the target inventory. If and only if $\chi \to 1$, traders reach their target inventory; the market fulfills all trading needs. More competitive markets are more liquid, and vice versa.[27] Thus our market competitiveness χ is also a measure of liquidity.

While χ may be difficult to observe in the data, it is closely related to the usual price impact λ. Recall (32).

$$\frac{1}{\chi} - 1 = \frac{\lambda}{\text{Avar}\{v \mid i_n, s_n, p\}}. \tag{54}$$

Market (il)liquidity can therefore be measured by the trade-off between the price impact and the riskiness of the asset. For a given λ, the market for riskier assets is more liquid: other things being equal, traders would be less constrained from trading the risky asset because they want to trade a smaller quantity of it in the first place. Again for a given λ, the market with large asset managers is less liquid: thinking of risk tolerance $1/A$ as proportional to the assets under management, large asset managers are more constrained because they want to trade a larger quantity.[28]

There is a large empirical literature on the measurement of liquidity.[29] As Chordia, Huh and Subrahmanyam (2009) and Lou and Shu (2017) show, there has been a mixed success in linking empirical measures to the price impact. By accounting for the quantity as well as the price impact, the trade-off in (54) may help reconcile our

[27]This concept of market liquidity is related to Grossman and Miller (1988). Their market liquidity is determined by the supply and demand for immediacy. Customers are willing to pay whatever price the market makers charge for immediate execution of their desired quantity. This corresponds to the behavior of our price takers, immediately reaching their target inventory.

Our notion of market liquidity is thus different from theirs. Unless market is perfectly liquid, traders do not reach their target inventory. They demand immediacy if and only if it is free. In general traders optimally trade off their desire to obtain a certain quantity with the cost of doing so. Such trade-off, absent in customers of Grossman and Miller (1988), is what determines our market liquidity.

[28]In our model traders have the same risk aversion. If traders have different risk aversion, liquidity will have to be measured from the perspective of a representative trader of the market. Note that in such an asymmetric model, traders will face a different level of price impact as well.

[29]See Amihud (2002), Brennan and Subrahmanyam (1996), Pástor and Stambaugh (2003), Hasbrouck (2009), and Kyle and Obizhaeva (2016) among others.

understanding of market liquidity with its measurement.

5.3 Practical Implication: Market Design

What does imperfect competition mean for the market design of stock exchanges?

One relevant aspect of current stock exchanges is that they require traders to submit one order at a time: every time that traders want to make a trade they must submit a separate order. This would work well if markets were perfectly competitive. Since traders want to reach their target inventory immediately, they need to submit one order to do so until they receive a new shock.

If markets were imperfectly competitive, this does not work well. Dynamic models of strategic trading of Vayanos (1999), Du and Zhu (2017), and Kyle, Obizhaeva and Wang (2018) show that the optimal strategy is to trade gradually over time. This is consistent with what we see in practice. Many institutional traders shred their orders into small pieces and trade them consecutively.

Because current exchanges do not offer traders ways to directly implement their optimal strategy, traders devote their own resources to overcome it. This is unnecessary. Since the extent to which traders can spread out their trade depends on their technology, a pecking order results, with high frequency traders at the top and retail traders at the bottom. The unintended pecking order can lead to further distortions and inefficiencies.[30] Thus, stock exchanges should be designed to incorporate implications of imperfect competition; understanding market competitiveness matters for the real world financial market design.

[30]See Budish, Cramton and Shim (2015) and Kyle and Lee (2017) for more discussions and proposals for alternative market designs.

6 Conclusion

We study how market competitiveness is determined when traders have private information. In a classic model of speculation and hedging, we develop and characterize a new measure of market competitiveness to show that market competitiveness increases in the number of traders and decreases in relative informational efficiency, the fraction of private information revealed in the price. Thus market competitiveness, often considered as exogenous, is jointly determined with information aggregation in equilibrium. We find that achieving perfect competition requires that the number of traders approaches infinity and speculation becomes negligible relative to hedging, i.e., markets remain imperfectly competitive as long as speculation remains meaningful. This result, which differs from the existing literature and the conventional wisdom that infinitely many traders are sufficient for perfect competition, has implications for the use of REE as a modeling approach; the measurement of liquidity; the market design of stock exchanges. Therefore, understanding market competitiveness and strategic trading incentives is generally important for studying financial markets.

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A Proofs

Proof of Proposition 1. Substituting (10), (11), and the residual supply schedule (6) into the FOC (13) yields:

$$x_n = \frac{\tau_I i_n - \left(1 + \tau_I + (N-1)\,\tau_I \varphi \left(1 - \frac{\pi_P}{\pi_I}\right)\right) p_n - s_n A \sigma_V^2}{2\frac{1+\tau_I+(N-1)\tau_I\varphi}{(N-1)\pi_P} + A\sigma_V^2}. \tag{55}$$

Writing this in terms of p rather than p_n using (6) yields the best response:

$$x_n = \frac{\tau_I i_n - A\sigma_V^2 s_n - \left(1 + \tau_I + (N-1)\,\tau_I \varphi \left(1 - \frac{\pi_P}{\pi_I}\right)\right) p}{A\sigma_V^2 + \frac{1+\tau_I+(N-1)\tau_I\varphi}{(N-1)\pi_P} + \frac{\tau_I\varphi}{\pi_I}}. \tag{56}$$

An equilibrium is found when this is matched with the conjecture (5).

First,

$$\frac{\pi_S}{\pi_I} = \frac{A\sigma_V^2}{\tau_I}, \tag{57}$$

which can be substituted into (8) to find

$$\varphi = \frac{\tau_I}{\tau_I + A^2 \sigma_V^2 \sigma_S^2}. \tag{58}$$

Second,

$$\frac{\pi_P}{\pi_I} = \frac{1 + \tau_I + (N-1)\,\tau_I \varphi \left(1 - \frac{\pi_P}{\pi_I}\right)}{\tau_I}, \tag{59}$$

which can be solved as

$$\frac{\pi_P}{\pi_I} = 1 + \frac{1}{\tau_I + (N-1)\,\tau_I \varphi}. \tag{60}$$

Third,

$$\pi_I = \frac{\tau_I}{A\sigma_V^2 + \frac{1+\tau_I+(N-1)\tau_I\varphi}{(N-1)\pi_P} + \frac{\tau_I\varphi}{\pi_I}}, \tag{61}$$

which, using (60), can be solved as

$$\pi_I = \left(\frac{N-2}{N-1} - 2\varphi\right)\frac{\tau_I}{A\sigma_V^2}. \tag{62}$$

Collecting (57), (60), and (62) yields:

$$X_n\left(p \mid i_n, s_n\right) = \frac{1}{A\sigma_V^2}\left(\frac{N-2}{N-1} - 2\varphi\right)\left(\tau_I i_n - A\sigma_V^2 s_n - \left(\tau_I + \frac{1}{1+(N-1)\varphi}\right)p\right). \tag{63}$$

The market clearing price satisfies

$$\tau_I \sum_{n=1}^{N} i_n - A\sigma_V^2 \sum_{n=1}^{N} s_n - N\left(\tau_I + \frac{1}{1+(N-1)\varphi}\right)p = 0, \tag{64}$$

which implies

$$p = \frac{1+(N-1)\varphi}{1+\tau_I+(N-1)\tau_I\varphi}\left(\tau_I \frac{\sum_{n=1}^{N} i_n}{N} - A\sigma_V^2 \frac{\sum_{n=1}^{N} s_n}{N}\right). \tag{65}$$

From (6) and (63), we have

$$\lambda = \frac{1}{(N-1)\pi_P} = \frac{A\sigma_V^2}{N-2-2(N-1)\varphi}\left(\frac{1+(N-1)\varphi}{1+\tau_I+(N-1)\tau_I\varphi}\right), \tag{66}$$

which can be substituted into the SOC ($2\lambda + A\mathrm{var}\left\{v \mid i_n, s_n, p\right\} \geq 0$) to yield:

$$\varphi \leq \frac{N-2}{2(N-1)}. \tag{67}$$

Since φ is given by (58), this is equivalent to

$$N\tau_I \leq (N-2)\left(A\sigma_V\sigma_S\right)^2. \tag{68}$$

Lastly, we need to rule out trivial no-trade equilibria by requiring $\pi_P \neq 0$. This, combined with (68), produces the necessary and sufficient condition for the equilibrium existence:

$$N\tau_I < (N-2)\,(A\sigma_V\sigma_S)^2. \tag{69}$$

□

Proof of Proposition 2. Since the error in the price signal (20) is the same function of the ratio π_S/π_I as that in the strategic equilibrium, φ is given by (9) and the conditional variance $\operatorname{var}\{v \mid i_n, s_n, p\}$ is given by (11). The conditional expectation is

$$\mathrm{E}\{v \mid i_n, s_n, p\} = \frac{1}{1+\tau_I+(N-1)\,\tau_I\varphi}\left(\tau_I i_n + (N-1)\,\tau_I\varphi\frac{N\pi_P}{(N-1)\,\pi_I}\left(p - \frac{\pi_I}{\pi_P}\frac{i_n}{N} + \frac{\pi_S}{\pi_P}\frac{s_n}{N}\right)\right), \tag{70}$$

where (20) replaces the strategic counterpart (7) in (10). Simplifying above yields

$$\mathrm{E}\{v \mid i_n, s_n, p\} = \frac{\tau_I}{1+\tau_I+(N-1)\,\tau_I\varphi}\left((1-\varphi)\,i_n + \frac{N\varphi\pi_P}{\pi_I}p + \frac{\varphi\pi_S}{\pi_I}s_n\right). \tag{71}$$

Substituting this into the FOC (21), we have

$$x_n^{PT} = \frac{\tau_I(1-\varphi)\,i_n - \left(1+\tau_I+(N-1)\,\tau_I\varphi - \frac{N\tau_I\varphi\pi_P}{\pi_I}\right)p - \left(A\sigma_V^2 - \frac{\tau_I\varphi\pi_S}{\pi_I}\right)s_n}{A\sigma_V^2}. \tag{72}$$

Then market clearing implies

$$p = \frac{\tau_I(1-\varphi)\frac{\sum_{n=1}^N i_n}{N} - \left(A\sigma_V^2 - \frac{\tau_I\varphi\pi_S}{\pi_I}\right)\frac{\sum_{n=1}^N s_n}{N}}{1+\tau_I+(N-1)\,\tau_I\varphi - \frac{N\tau_I\varphi\pi_P}{\pi_I}}. \tag{73}$$

In equilibrium, this is consistent with the conjectured price (19). First, from

$$\frac{\pi_I}{\pi_P} = \frac{\tau_I\left(1-\varphi\right)}{1+\tau_I+(N-1)\,\tau_I\varphi-\frac{N\tau_I\varphi\pi_P}{\pi_I}}, \tag{74}$$

we have

$$\frac{\pi_I}{\pi_P} = \frac{\tau_I\left(1+(N-1)\,\varphi\right)}{1+\tau_I+(N-1)\,\tau_I\varphi}. \tag{75}$$

Next, from

$$\frac{\pi_S}{\pi_I} = \frac{A\sigma_V^2-\frac{\tau_I\varphi\pi_S}{\pi_I}}{\tau_I\left(1-\varphi\right)}, \tag{76}$$

we have

$$\frac{\pi_S}{\pi_I} = \frac{A\sigma_V^2}{\tau_I}, \tag{77}$$

which can be substituted to (9) to find that φ is given by (16).

Substituting (75) and (77) into (72), we have

$$x_n^{PT} = \frac{1-\varphi}{A\sigma_V^2}\left(\tau_I i_n - A\sigma_V^2 s_n - \left(1+\frac{\tau_I}{1+(N-1)\,\varphi}\right)p\right). \tag{78}$$

The market clearing price is thus given by (17). Since the SOC $(A\text{var}\left\{v \mid i_n, s_n, p\right\} \geq 0)$ always holds, we only need to rule out a trivial no-trade equilibrium by excluding $\varphi = 1$. With φ given by (16), this means $(A\sigma_V\sigma_S)^2 > 0$.

$\square$

Proof of Proposition 3. Since $\frac{\tau_I}{(A\sigma_V\sigma_S)^2} < \frac{N-2}{N}$, both a strategic equilibrium with the demand schedule (15) and a competitive equilibrium with the demand schedule (23) exist. Dividing (15) by (23) as in (25) yields

$$\chi = \frac{N-2}{N-1} - \frac{N}{N-1}\left(\frac{\varphi}{1-\varphi}\right). \tag{79}$$

With φ given by (16), this is equivalent to (27).

$\square$

Proof of Proposition 4. With χ given by (26), we want to find the necessary and sufficient condition for $\chi \to 1$. Since $\frac{\varphi}{1-\varphi} \geq 0$, we have

$$\chi \leq \frac{N-2}{N-1}. \tag{80}$$

Thus, for χ to approach one, it is necessary that $N \to \infty$, in which case

$$\chi \to 1 - \frac{\varphi}{1-\varphi}. \tag{81}$$

Thus it is also necessary that $\varphi \to 0$. The two necessary conditions are sufficient because they imply $\chi \to 1$.

With φ given by (16), the condition $\varphi \to 0$ is equivalent to $\frac{\tau_I}{(A\sigma_V\sigma_S)^2} \to 0$.

$\square$

Proof of Proposition 5. Since residual uncertainty does not affect how traders learn about v, φ, defined by (8), and the conditional expectation $\mathrm{E}\left\{\hat{v} \mid i_n, s_n, p\right\}$ are still given by (9) and (10) (recall $\mathrm{E}\left\{y\right\} = 0$). The only change from the strategic equilibrium in Proposition 1 is the variance. From (38),

$$\operatorname{var}\left\{\hat{v} \mid i_n, s_n, p\right\} = \frac{1}{\tau^*}\left(\sigma_V^2 + \tau^* \sigma_Y^2\right), \tag{82}$$

where to simplify notations we define

$$\tau^* := 1 + \tau_I + (N-1)\,\tau_I \varphi. \tag{83}$$

Thus, the equilibrium demand schedule, the market clearing price, and the price impact are the same as those in the strategic equilibrium, (63), (65), and (66), when σ_V^2 is replaced with $\sigma_V^2 + \tau^* \sigma_Y^2$.

$$x_n = \frac{\frac{N-2}{N-1} - 2\varphi}{A\left(\sigma_V^2 + \tau^* \sigma_Y^2\right)} \left(\tau_I i_n - A\left(\sigma_V^2 + \tau^* \sigma_Y^2\right) s_n - \frac{\tau^*}{1 + (N-1)\varphi} p\right). \tag{84}$$

$$p = \frac{1 + (N-1)\varphi}{\tau^*} \left(\tau_I \frac{\sum_{n=1}^N i_n}{N} - A\left(\sigma_V^2 + \tau^* \sigma_Y^2\right) \frac{\sum_{n=1}^N s_n}{N}\right). \tag{85}$$

and

$$\lambda = \frac{1}{(N-1)\pi_P} = \frac{A\left(\sigma_V^2 + \tau^* \sigma_Y^2\right)}{N - 2 - 2(N-1)\varphi} \left(\frac{1 + (N-1)\varphi}{1 + \tau_I + (N-1)\tau_I \varphi}\right). \tag{86}$$

Again replacing σ_V^2 with $\sigma_V^2 + \tau^* \sigma_Y^2$ in (16) yields

$$\frac{\tau_I}{(A\sigma_V \sigma_S)^2} \left(\frac{1}{\varphi} - 1\right) = \left(1 + \left(1 + \tau_I + (N-1)\tau_I \varphi\right) \frac{\sigma_Y^2}{\sigma_V^2}\right)^2. \tag{87}$$

Since the LHS is decreasing in φ and the RHS is increasing in φ, the solution is unique. To satisfy the SOC (67) and rule out a trivial no-trade equilibrium, the solution should satisfy

$$\varphi < \frac{N-2}{2(N-1)}, \tag{88}$$

which holds if and only if the RHS is strictly greater than to the LHS for $\varphi = \frac{N-2}{2(N-1)}$.

$$\frac{\tau_I}{(A\sigma_V \sigma_S)^2} \left(\frac{1}{\frac{N-2}{2(N-1)}} - 1\right) < \left(1 + \left(1 + \tau_I + (N-1)\tau_I \frac{N-2}{2(N-1)}\right) \frac{\sigma_Y^2}{\sigma_V^2}\right)^2, \tag{89}$$

which is simplified to yield (39).

To find market competitiveness χ, we divide the quantity (84) by the quantity that the competitive traders would have traded. The competitive quantity with residual uncertainty is again the same as (23) with σ_V^2 replaced by $\sigma_V^2 + \tau^* \sigma_Y^2$.

$$x_n^{PT} = \frac{1-\varphi}{A\left(\sigma_V^2 + \tau^* \sigma_Y^2\right)} \left(\tau_I i_n - A\left(\sigma_V^2 + \tau^* \sigma_Y^2\right) s_n - \left(1 + \frac{\tau_I}{1+(N-1)\varphi}\right) p\right). \tag{90}$$

The ratio between (84) and (90) is thus unaffected by the residual uncertainty and still given by (26). The condition for perfect competition thus follows (see also the proof of Proposition 4).

□

Proof of Proposition 6. To find the best response demand schedule, substitute the conditional expectation (46) into the FOC (13) to obtain

$$x_n = \frac{\left(\tau_I i_n + (N-1)\tau_I \varphi \left(\frac{\pi_P}{\pi_I} p_n + \frac{\pi_S}{\pi_I} \rho_S s_n\right)\right) - \tau^* p_n - A\sigma_V^2 s_n}{\frac{2\tau^*}{(N-1)\pi_P} + A\sigma_V^2}, \tag{91}$$

where τ^*, given by (83), is used to simplify the expression. Rewriting this in terms of the market clearing price, we have

$$x_n = \frac{\tau_I i_n - \left(\tau^* - (N-1)\tau_I \varphi \frac{\pi_P}{\pi_I}\right) p - \left(A\sigma_V^2 - (N-1)\tau_I \varphi \frac{\pi_S}{\pi_I} \rho_S\right) s_n}{\frac{\tau^*}{(N-1)\pi_P} + A\sigma_V^2 + \frac{\tau_I \varphi}{\pi_I}}. \tag{92}$$

In equilibrium, this is consistent with the conjectured strategy (5). From

$$\frac{\pi_S}{\pi_I} = \frac{A\sigma_V^2 - (N-1)\tau_I \varphi \frac{\pi_S}{\pi_I} \rho_S}{\tau_I}, \tag{93}$$

we have

$$\frac{\pi_S}{\pi_I} = \frac{A\sigma_V^2}{\tau_I} \frac{1}{1+(N-1)\varphi \rho_S}, \tag{94}$$

which can be substituted into (45) to yield (48).

Next, note that the best response (92) is the same as that in the strategic equilibrium with independent endowments (56) except for the coefficient for s_n. The equilibrium π_I and π_P must be the same as in (63). Thus, we have

$$X_n\left(p \mid i_n, s_n\right) = \frac{1}{A\sigma_V^2}\left(\frac{N-2}{N-1} - 2\varphi\right)\left(\tau_I i_n - \frac{A\sigma_V^2}{1+(N-1)\rho_S\varphi} s_n - \left(\tau_I + \frac{1}{1+(N-1)\varphi}\right)p\right). \tag{95}$$

The market clearing price is

$$p = \frac{1+(N-1)\varphi}{1+\tau_I+(N-1)\varphi\tau_I}\left(\tau_I \frac{\sum_{n=1}^N i_n}{N} - \frac{A\sigma_V^2}{1+(N-1)\rho_S\varphi}\frac{\sum_{n=1}^N s_n}{N}\right). \tag{96}$$

Since π_S does not affect the price impact, λ is still given by (66). The SOC and ruling out trivial no-trivial equilibrium imply (67).

To find market competitiveness, we divide (95) by the quantity that competitive traders would have traded. Conjecturing that the price is given by (19), the new information from the price is summarized by

$$\frac{N\pi_P}{(N-1)\pi_I}\left(p - \frac{\pi_I}{\pi_P}\frac{i_n}{N} + \frac{\pi_S}{\pi_P}\frac{1+(N-1)\rho_S}{N}s_n\right) = v + \frac{\sum_{n'\neq n} e_{n'}}{N-1} - \frac{\pi_S}{\pi_I}\left(\frac{\sum_{n'\neq n} s_{n'}}{N-1} - \rho_S s_n\right). \tag{97}$$

Then the condition expectation is

$$\mathrm{E}\left\{v \mid i_n, s_n, p\right\} = \frac{1}{1+\tau_I+(N-1)\varphi\tau_I}\left((1-\varphi)\tau_I i_n + \frac{N\varphi\tau_I\pi_P}{\pi_I}p + \frac{\varphi\tau_I\left(1+(N-1)\rho_S\right)\pi_S}{\pi_I}s_n\right). \tag{98}$$

Substitute this into the FOC (21) to yield

$$x_n = \frac{\tau_I(1-\varphi)i_n - \left(1+\tau_I+(N-1)\varphi\tau_I - \frac{N\varphi\tau_I\pi_P}{\pi_I}\right)p - \left(A\sigma_V^2 - \frac{\varphi\tau_I(1+(N-1)\rho_S)\pi_S}{\pi_I}\right)s_n}{A\sigma_V^2}. \tag{99}$$

In equilibrium, the best response is consistent with the market clearing price.

$$\frac{\pi_S}{\pi_I} = \frac{A\sigma_V^2}{\tau_I}\frac{1}{1+(N-1)\,\rho_S\varphi}, \tag{100}$$

which implies that the relative informational efficiency φ is still given by (48). Since (99) is basically same as that with independent endowments (72) except for π_S, the equilibrium demand schedule is given by

$$x_n^{PT} = \frac{1-\varphi}{A\sigma_V^2}\left(\tau_I i_n - \frac{A\sigma_V^2}{1+(N-1)\,\rho_S\varphi} s_n - \left(1+\frac{\tau_I}{1+(N-1)\,\varphi}\right)p\right). \tag{101}$$

Thus, market competitiveness χ is the same as (26). Then the condition for perfect competition follows.

□