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On Existence and Uniqueness of Equilibrium in a Class of Noisy Rational Expectations Models

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I study a general class of noisy rational expectations models that nests the standard Grossman and Stiglitz (1980) and Hellwig (1980) models, but relaxes the usual assumption of joint normality of asset pay-offs and supply, and allows for more general signal structures. I provide a constructive proof of existence of equilibrium, characterize the price function, and provide sufficient conditions for uniqueness within the class of equilibria with continuous price functions, which are met by both the Grossman and Stiglitz (1980) models and the Hellwig (1980) models with a continuum of investors. My solution approach does not rely on the typical "conjecture and verify" method, and I exhibit a number of non-normal examples in which asset prices can be characterized explicitly and in a closed form. The results presented here open up a broad class of models for applied work. To illustrate the usefulness of generalizing the standard model, I show that in settings with non-normally distributed pay-offs, shocks to fundamentals may be amplified purely due to learning effects, price drifts can arise naturally, and the disagreement–return relation depends in a novel way on return skewness.

Key words: Noisy Rational Expectations, Asymmetric Information, Information Aggregation, Exponential Family.

JEL Codes: D82, G12, G14

1. INTRODUCTION

In this article, I provide a constructive proof of equilibrium existence for a class of noisy rational expectations (RE) models that nests the standard Grossman and Stiglitz (1980) and Hellwig (1980) models and that does not rely on normality assumptions. The model permits many common distributions for asset pay-offs and permits general signal structures and asset supply distributions. In many natural settings, the price can be characterized explicitly in a closed form. I also provide sufficient conditions for uniqueness of equilibrium within the class of equilibria with continuous price functions, hereafter referred to as *continuous equilibria*, which are met by the Grossman and Stiglitz (1980) and the Hellwig (1980) models with a continuum of investors. In addition to being of independent theoretical interest, these results open up a broader class of models for applications; like the standard model, this generalization would also allow straightforward extensions to include multiple assets (Admati, 1985) and short-sale or borrowing constraints (Yuan, 2005).

Noisy RE models are workhorse models for studying the effects of asymmetric information in financial markets. The model of Grossman and Stiglitz (1980), and similar ones due to Hellwig (1980) and Diamond and Verrecchia (1981) are the foundation for models that guide our understanding of many economic phenomena: information acquisition in financial markets (Grossman and Stiglitz, 1980; Verrecchia, 1982), the operation of information markets (Admati and Pfleiderer, 1986, 1987, 1990), financial market crashes, crises, and contagion (Gennotte and Leland, 1990; Kodres and Pritsker, 2002; Yuan, 2005; Angeletos and Werning, 2006), cross-asset learning (Admati, 1985), insider trading (Leland, 1992), feedback effects from financial markets to firm cash flows (Hirshleifer *et al.*, 2006; Ozdenoren and Yuan, 2008), and exchange rate dynamics (Bacchetta and van Wincoop, 2006), among others.[1]

Despite their ubiquity, most noisy RE models depend on strong assumptions that all random variables are jointly normally distributed and that all agents have constant absolute risk aversion (CARA) utility functions. This set of assumptions leads to elegant solutions but fails to capture important features of asset markets. Two obvious criticisms are that normality of asset pay-offs violates limited liability and precludes any consideration of the effects of higher moments. Moreover, the standard solution method is based on conjecturing a price that is linear in signals and supply and then showing that such a conjecture is consistent with equilibrium. This method provides no guidance on how to solve more general models, and uniqueness is neither claimed nor established. Due to the complexity and apparent intractability of alternative noisy RE models, there remain open questions as to whether (i) there exist *general* variations on the standard CARA-Normal model that are analytically tractable but do not require the assumption of normally distributed pay-offs, signals, and asset supplies, and (ii) whether there exist non-linear equilibria in the standard model. The main contribution of this article is to furnish answers to both of these questions, providing a solution to a broad class of models that nests the standard model and presenting a set of sufficient conditions under which the equilibrium is unique within the class of continuous equilibria.[2]

I also show that the generalization provided here is more than a mere technical point. Indeed, even seemingly innocuous changes to pay-off or supply distributions can dramatically change standard comparative static results. For instance, I find that "small" shocks to fundamentals may lead to "large" changes in price, prices may exhibit drift-like effects, and the relation between investor disagreement and returns can depend in a novel way on return skewness. Thus, generalizing the standard model allows one to conclude that a number of "standard" results commonly accepted in the literature are not robust to plausible alternative assumptions.

The primary difficulty in solving noisy rational expectations models is that the asset price must both convey information to investors and clear the market, which presents a complicated non-linear fixed-point problem that does not fit well into any standard fixed-point theorems. When there is a hierarchical information structure with one informed and one uninformed investor, I avoid this problem by exploiting the market-clearing condition to determine *a priori* a statistic that is informationally equivalent to *any* continuous equilibrium price. The intuition for this result relies on a simple fact: for a given investor, the asset price can reveal no less than the net trade of all other investors in the economy. With this statistic pinned down, investor beliefs follow from Bayes' rule, and a simple first-order condition (FOC) characterizes demand functions. The asset

1. There is a distinct but related literature, following Kyle (1985) and Kyle (1989), that studies the consequences of asymmetric information in markets in which some traders behave strategically.

2. Following completion of an earlier draft, it was brought to my attention that Pálvölgyi and Venter (2015) demonstrate how to construct a class of discontinuous price functions in both the Grossman and Stiglitz (1980) and Hellwig (1980) models.

price is easily established by imposing market clearing. Moreover, if the statistic determined in the first step is identical for any possible price function, then uniqueness follows almost immediately.

In a dispersed information setting in which traders' information sets are not nested, it is not necessarily possible to pin down the information content of the price independently of the price function. The reason is that in this case all investors will learn from the asset price, not only the 'uninformed' investor. Nevertheless, motivated by the results in the two type case, I show that focusing on price functions that are of a particular "generalized linear" form delivers equilibrium characterization and existence results in a set of economies that nests the finite-investor case of Hellwig (1980). Moreover, in economies with a continuum of investors and an additive signal structure with normally-distributed errors, the construction used in the two-types setting can be applied directly and uniqueness is demonstrated.

This article is part of a growing literature that seeks to generalize noisy RE models beyond the CARA-Normal framework. Albagli *et al.* (2013) is the most closely related recent work. They also analyse a class of non-linear noisy RE models, but their focus is on how alternative pay-off assumptions affect information aggregation. Ausubel (1990a) and Barlevy and Veronesi (2000, 2003) are also closely related to this work. Ausubel (1990b) demonstrates existence and uniqueness of a partially revealing equilibrium in a two-good, two-agent model in which uninformed agents do not know the preferences of informed agents. Barlevy and Veronesi (2000, 2003) construct an equilibrium in a simple noisy RE model in which traders are risk-neutral and face a portfolio constraint, and they focus on a particular parametric distribution for random variables.[3] Other related work must make more substantial concessions and resort to non-standard model settings or approximation to arrive at a solution. Vanden (2008) solves a non-linear noisy RE model driven by gamma distributions but depends upon a non-standard definition of noise trading. Peress (2004) analyses the interaction between wealth effects and information acquisition by using a "small risk" log-linear approximation. Bernardo and Judd (2000) develop a computational procedure for solving noisy RE models numerically and demonstrate the non-robustness of some results from the standard Grossman and Stiglitz (1980) model. Banerjee and Green (2015) consider an economy in which the uninformed investors are uncertain about the presence of an informed investor, and Adrian (2009) studies a dynamic model in which investors are myopic and have non-Normal priors.

There are also a similar strands of literature that deal with non-noisy settings and settings in which traders behave strategically. DeMarzo and Skiadas (1998) study the properties a class of static economies that nests Grossman (1976); they demonstrate uniqueness of Grossman's (1976) fully revealing linear equilibrium and give robust examples of partially revealing equilibria when pay-offs are non-normal. Foster and Viswanathan (1993) study (linear) equilibria in the Kyle (1985) model when random variables are elliptically distributed, and Bagnoli *et al.* (2001) derive necessary and sufficient conditions on probability distributions for existence of linear equilibria in various market making models. Rochet and Vila (1994) study existence and uniqueness properties in a class of models similar to Kyle (1985) in which informed traders observe the amount of noise trade. Finally, Bhattacharya and Spiegel (1991) study non-linear equilibria in a noisy RE model with strategic informed traders, and Spiegel and Subrahmanyam (2000) consider a model of market making in which an informed trader has private information about the mean and variance of an asset's pay-off.

3. Chamley (2008) points out an error in the computation of the value of information in Barlevy and Veronesi (2000). Nevertheless, the characterization of the financial market equilibrium in their paper is correct.

2. MODEL

The model is of a simple static economy, as in Grossman and Stiglitz (1980) and Hellwig (1980). There are two dates, $t\in\{0,1\}$. At the first date, $t=0$, investors trade financial assets. At the final date, $t=1$, assets make liquidating pay-outs. There are two assets, a risky asset with pay-off $\widetilde{V}$, distributed on some set $\mathcal{V}\subseteq\mathbb{R}$,[4] and a risk-free asset, which is the numeraire, that pays off 1 and has price normalized to 1. It is trivial to extend the model to permit an exogenous, positive return on the risk-free asset.

There are $N+1$ investors, indexed by $i\in\{1,\ldots,N\}\cup\{U\}$, who have utility over wealth at $t=1$, with CARA utility functions, which I formalize in the following Assumption.

Assumption 1 (CARA utility). *Investors have CARA utility with risk tolerance* τ_i: $u_i(w)=-\exp\left\{-\frac{1}{\tau_i}w\right\}$.

Investors are endowed with shares of the risky asset and holdings of the risk-free asset that they can trade in the financial market. Without loss of generality, I normalize the endowments to zero, since a CARA investor's demand for risky assets does not depend on initial wealth. Investors $i\in\{1,\ldots,N\}$ observe signals $\widetilde{S}_i$, each distributed on $\mathcal{S}\subseteq\mathbb{R}$ before the financial market opens. They can also condition their demands on the equilibrium price. I refer to these investors as *informed investors*. I denote the collection of all signals by $\widetilde{S}\equiv(\widetilde{S}_1,\ldots,\widetilde{S}_N)\in\mathcal{S}^N$. The final investor U does not observe any signals but can condition his/her demand on the equilibrium price. This investor is referred to as the *uninformed investor*. Owing to the assumption of CARA utility, standard aggregation theorems (Rubinstein, 1974; Ingersoll, 1987, p. 217–219) imply that each investor can be thought of as a representative agent for an underlying mass of investors who observe a common signal $\widetilde{S}_i$ (or no signal, in the case of the uninformed investor) and have aggregate risk tolerance equal to τ_i. Thus, without loss of generality, I assume that no two signals are identical. That is there do not exist distinct $i,j\in\{1,\ldots,N\}$ for which $Prob(\widetilde{S}_i=\widetilde{S}_j)=1$.

The pay-off $\widetilde{V}$ and signals $\widetilde{S}$ are jointly distributed according to some cumulative distribution function (cdf) $F_{\widetilde{V},\widetilde{S}}:\mathcal{V}\times\mathcal{S}^N\to[0,1]$. The marginal cdfs and the joint cdfs of subsets of the signals use analogous notation (*e.g.* the marginal of $\widetilde{S}_i$ is denoted $F_{\widetilde{S}_i}$). Conditional cdfs are written in the form $F_{\widetilde{V}|\widetilde{S}_i}$. If a random variable has a probability density function (pdf), I use the same notational conventions but with lower case f in place of F.

To prevent fully revealing prices and provide a motive for trade, there is a random component to the supply of the asset. The supply is equal to $\bar{z}+\widetilde{Z}$, where $\bar{z}\in\mathbb{R}$ is a known constant, and the supply shock $\widetilde{Z}$ is distributed independently of all other random variables in the economy, according to cdf $F_{\widetilde{Z}}$ on some set $\mathcal{Z}\subseteq\mathbb{R}$.[5] Since $\bar{z}$ is known, one could simply absorb it into the shock $\widetilde{Z}$; however, once one moves beyond the normal distribution there is often no notationally simple way to do this, therefore, it is convenient to be able to adjust the constant separately.

All investors are price takers. All probability distributions and other parameters of the economy are common knowledge, and therefore, investors are only asymmetrically informed about the asset

4. In general, I use the notational convention that random variables are denoted by capital letters with tildes, supports of random variables and functions by calligraphic capital letters, and realizations of random variables by lower case letters without tildes. An exception to this convention is that I follow tradition and use $\widetilde{\varepsilon}$ for error terms when specifying particular functional forms for signals below.

5. It is equivalent to introduce noise or liquidity traders who submit price inelastic demand functions uncorrelated with the asset pay-off. One can also permit price elasticity in the supply by specifying supply$=\bar{z}+\widetilde{Z}+\zeta(p)$, where ζ is an increasing function.

pay-off $\widetilde{V}$. Also, signals are taken to be exogenous—they have been fixed via some unmodelled information-acquisition stage before the financial market opens.

2.1. *Equilibrium*

The definition of equilibrium in the financial market is standard and makes the typical measurability restriction on the price function suggested by Kreps (1977) to rule out prices that reveal more than the pooled information of all investors. Let $P(s,z)$ denote the equilibrium risky-asset price for given realizations of $\widetilde{S}=s$ and $\widetilde{Z}=z$. Let $X_i(s_i,p)$ denote the quantity of shares demanded by informed investor i as a function of his/her signal and price, and let $X_U(p)$ denote the quantity of shares demanded by the uninformed investor as a function of the price.

Definition 1 (Financial market equilibrium). *A noisy rational expectations equilibrium in the financial market is a measurable function $P:\mathcal{S}^N\times\mathcal{Z}\to\mathbb{R}$, and measurable demand functions for investors $X_i(\cdot)$ such that all investors maximize expected utility, conditional on their information sets*

$$X_i(s_i,p)\in\arg\max_{x\in\mathbb{R}}\mathbb{E}\left[-\exp\left\{-\frac{1}{\tau_i}x(\widetilde{V}-p)\right\}\middle|\widetilde{S}_i=s_i,P(\widetilde{S},\widetilde{Z})=p\right],\quad i\in\{1,\dots,N\},$$

$$X_U(p)\in\arg\max_{x\in\mathbb{R}}\mathbb{E}\left[-\exp\left\{-\frac{1}{\tau_U}x(\widetilde{V}-p)\right\}\middle|P(\widetilde{S},\widetilde{Z})=p\right],$$

and markets clear in all states

$$\sum_{i=1}^{N}X_i(s_i,P(s,z))+X_U(P(s,z))=\bar{z}+z,\quad\forall(s,z)\in\mathcal{S}^N\times\mathcal{Z}.$$

3. TWO-TYPES MODEL

In this section, I examine the special case in which there is a single informed investor, as in Grossman and Stiglitz (1980). This setting illustrates the main insights of the article and can be addressed under a rather general set of assumptions.

The following two assumptions are essential for the characterization of the equilibrium. Further technical assumptions will be introduced below as needed.

Assumption 2 (Single informed). *There is a single informed investor, $N=1$. All quantities associated with him/her are subscripted by I (e.g. signal $\widetilde{S}_I$, risk-tolerance τ_I).*

Assumption 3 (Exponential family). *The conditional distribution of the pay-off $\widetilde{V}$ given $\widetilde{S}_I=s_I$ has a cdf that can be written in the form*

$$dF_{\widetilde{V}|\widetilde{S}_I}(v|s_I)=\exp\{k_Is_Iv-g_I(k_Is_I)\}dH_I(v),\quad v\in\mathcal{V},s_I\in\mathcal{S}.\tag{3.1}$$

where $k_I>0$ is a constant, the function $g_I:\mathcal{G}_I\to\mathbb{R}$ has domain $\mathcal{G}_I$ which is an interval satisfying $k_I\mathcal{S}\subseteq\mathcal{G}_I$,[6] and the function $H_I:\mathbb{R}\to\mathbb{R}$ is (weakly) increasing and right-continuous.

6. I follow the notational convention that given scalars $\alpha,\beta\in\mathbb{R}$ and sets $A,B\subseteq\mathbb{R}$, the set $\alpha A+\beta B$ is defined as $\alpha A+\beta B\equiv\{\alpha a+\beta b:a\in A,b\in B\}$.

Assumption 2 is self-explanatory. Assumption 3 may appear complex at first glance. It requires that the informed investor's conditional distribution lies in a so-called *exponential family* with parameter $k_I s_I$.[7] This allows for a unified treatment of many common distributions, including the normal, binomial, exponential, and gamma, all of which are exponential families, as well as various signal structures. Indeed, the combination of Assumption 1 and 3 (and later, its generalization in Assumption 10) is the key to the article. In tandem, these assumptions will lead to informed investor demand functions that are linear in the signal s_I, though not typically in the price.

I present a number of properties of exponential families in Appendix A.1, but I will address a few important points here. Let $M_{\widetilde{V}|\widetilde{S}_I}(u|s_I) \equiv \mathbb{E}[\exp\{u\widetilde{V}\}|\widetilde{S}_I = s_I]$ denote the conditional moment generating function (mgf). The mgf of a probability distribution encapsulates information about all of its moments, which can be obtained by differentiating. Lemma A6 shows that the conditional mgf for an exponential family is $M_{\widetilde{V}|\widetilde{S}_I}(u|s_I) = \exp\{g_I(u+k_I s_I) - g_I(k_I s_I)\}$. Hence, all conditional moments are determined by the shape of g_I and the value of $k_I s_I$. The set $\mathcal{G}_I$ represents the set of admissible parameters for the distribution of $\widetilde{V}$. The requirement that $k_I \mathcal{S} \subseteq \mathcal{G}_I$ ensures that all possible realizations of the "parameter" $k_I s_I$ lie within this set. For instance, if the pay-off is conditionally exponentially distributed with rate $-k_I s_I$ this condition ensures that the distribution of $\widetilde{S}_I$ is such that the rate is always positive.

Note that the constant k_I can always be normalized to 1 after appropriately rescaling the signal $\widetilde{S}_I$; however, allowing for explicit consideration of the constant will prove convenient in the multiple-investor version of the model. Both the assumption that k_I is positive and that $k_I s_I$ is a linear function of s_I are without loss of generality. If the $k_I s_I$ terms in the distribution were instead replaced by the more general expression $k'_I b_I(s'_I)$ for some non-zero k'_I, signal s'_I, and function b_I, one could define an informationally equivalent signal $\widetilde{S}_I = \mathrm{sgn}(k'_I) b_I(\widetilde{S}'_I)$ and let $k_I = |k'_I|$ to place the distribution in the desired form. I choose to do this as part of the Assumption so as to not have to carry around extra notation.

If the standard CARA-Normal model is an idealization of a world in which investors run linear regressions to predict asset pay-offs, then Assumption 3 generalizes to a world in which investors run *generalized linear models* to predict asset pay-offs.[8] Suppose that $\widetilde{X}$ is a vector of regressors that are to be used as predictors. In a generalized linear model, the econometrician specifies that the pay-off is drawn according to an exponential family distribution with mean that is a function of $\widetilde{X}'\beta$ for some coefficient vector β. An example is logistic regression, which models the probability of a binary outcome by specifying log-odds that are linear in a set of regressors. In the general case considered here, the regressor for the informed investor is his/her signal and Lemma A8 in the Appendix shows that the conditional mean is $g'_I(k_I s_I)$. Hence, k_I is like a regression coefficient.

Assumption 3 allows for many common continuous and discrete distributions for pay-offs and for various assumptions about the joint distributions of pay-offs and signals. The following two examples illustrate some natural settings in which it is met.

Example 1 (Binomial distribution, general signal structure). *The asset pay-off follows a binomial distribution on* $\mathcal{V} = \{V_L, V_H\}$. *The informed investor receives a signal* $\widetilde{S}_0$, *jointly*

7. See Bernardo and Smith (2000, Ch. 4.5.3) or Casella and Berger (2002, Ch. 3.4) for details. The terminology may be somewhat confusing if the reader has not previously encountered exponential families. Requiring the pay-off to be distributed according to a distribution in an exponential family is not the same as requiring that the pay-off be distributed according to the exponential *distribution*. The specification in equation (3.1) is substantially more general and includes the exponential distribution as a special case.

8. See McCullagh and Nelder (1989) for a textbook treatment of generalized linear models.

distributed with $\widetilde{V}$. Suppose that this signal is non-degenerate in that it does not fully reveal $\widetilde{V}$.

I now show how this setup fits the exponential family assumption after potentially transforming the signal. Define the random variable $\widetilde{S}_I = \log\left(\frac{f_{\widetilde{V}|\widetilde{S}_0}(V_H|\widetilde{S}_0)}{f_{\widetilde{V}|\widetilde{S}_0}(V_L|\widetilde{S}_0)}\right)$ as the informed investor's log-odds. The log-odds are a sufficient statistic for $\widetilde{S}_0$: once one has observed $\widetilde{S}_I$, additionally observing $\widetilde{S}_0$ directly does not provide any additional information about $\widetilde{V}$. The conditional pdf can be written as a function of the realization s_I

$$f_{\widetilde{V}|\widetilde{S}_I}(v|s_I) = \begin{cases} \frac{\exp\{s_I\}}{1+\exp\{s_I\}} & v = V_H \\ \frac{1}{1+\exp\{s_I\}} & v = V_L. \end{cases}$$

Since $\widetilde{V}$ takes only two values, the numerators of these expressions can be captured in one term, $\exp\left\{\frac{v-V_L}{V_H-V_L}s_I\right\}$, which allows one to write the pdf as

$$\begin{aligned} f_{\widetilde{V}|\widetilde{S}_I}(v|s_I) &= \frac{\exp\{\frac{v-V_L}{V_H-V_L}s_I\}}{1+\exp\{s_I\}}\mathbb{I}\{v\in\{V_L,V_H\}\} \\ &= \exp\left\{\tfrac{1}{V_H-V_L}s_I v - \tfrac{1}{V_H-V_L}s_I V_L - \log\left(1+\exp\left\{(V_H-V_L)\tfrac{s_I}{V_H-V_L}\right\}\right)\right\}\mathbb{I}\{v\in\{V_L,V_H\}\}, \end{aligned} \tag{3.2}$$

where the second line moves the denominator into the exponential in the numerator and pulls apart the $\frac{v-V_L}{V_H-V_L}$ term.

By inspection, this distribution is in the form of equation (3.1) with

$$k_I = \frac{1}{V_H - V_L} \tag{3.3}$$

$$g_I(k_I s_I) = V_L k_I s_I + \log(1+\exp\{(V_H-V_L)k_I s_I\}) \tag{3.4}$$

$$dH_I(v) = \begin{cases} 1 & v = V_L \\ 1 & v = V_H \\ 0 & v \notin \{V_L, V_H\}, \end{cases} \quad \text{or, equivalently,} \quad H_I(v) = \begin{cases} 0 & v < V_L \\ 1 & V_L \le v < V_H \\ 2 & V_H \ge v. \end{cases} \tag{3.5}$$

To identify the sets that appear in Assumption 3, note that the support of the function g_I is $\mathcal{G}_I = \mathbb{R}$, as it is clear by inspection that g_I is defined on the entire real line. The support of the log-odds, $\mathcal{S}$, depends on the underlying signal structure, but since $\mathcal{G}_I = \mathbb{R}$ the condition $k_I\mathcal{S} \subseteq \mathcal{G}_I$ will always be satisfied.

Example 2 (General pay-off distribution, additive Normal signal). *Suppose that $\widetilde{V}$ is distributed according to an arbitrary distribution $F_{\widetilde{V}}$ on some set $\mathcal{V} \subseteq \mathbb{R}$. The informed investor receives an additive signal about the pay-off, $\widetilde{S}_I = \widetilde{V} + \widetilde{\varepsilon}_I$, where $\widetilde{\varepsilon}_I \sim N(0, \sigma_I^2)$ is an independently distributed error.*

Let $\phi(\cdot|\mu,\sigma^2)$ denote the density of a $N(\mu,\sigma^2)$ random variable, and use Bayes' rule to compute the joint distribution of $\widetilde{V}$ and $\widetilde{S}_I$

$$dF_{\widetilde{V},\widetilde{S}_I}(v,s_I) = \phi(s_I|v,\sigma_I^2)dF_{\widetilde{V}}(v).$$

Using Bayes rule again, the conditional distribution of $\widetilde{V}$ satisfies

$$dF_{\widetilde{V}|\widetilde{S}_I}(v|s_I) = \frac{\phi(s_I|v,\sigma_I^2)dF_{\widetilde{V}}(v)}{\int_{\mathcal{V}} \phi(s_I|x,\sigma_I^2)dF_{\widetilde{V}}(x)dx}.$$

To continue, plug in the normal density $\phi(s_I|v,\sigma_I^2) = \frac{\exp\{-(s_I-v)^2/2\sigma_I^2\}}{\sqrt{2\pi\sigma_I^2}}$. The key step for verifying the exponential family form is to expand the quadratic function in the exponential and notice that terms that are constant with respect to x and v and appear in both the numerator and denominator cancel

$$\begin{aligned} f_{\widetilde{V}|\widetilde{S}_I}(v|s_I) &= \frac{\exp\left\{\frac{s_I}{\sigma_I^2}v - \frac{1}{2}\frac{v^2}{\sigma_I^2}\right\}dF_{\widetilde{V}}(v)}{\int_{\mathcal{V}} \exp\left\{\frac{s_I}{\sigma_I^2}x - \frac{1}{2}\frac{x^2}{\sigma_I^2}\right\}dF_{\widetilde{V}}(x)} \\ &= \exp\left\{\frac{s_I}{\sigma_I^2}v - \log\left(\int_{\mathcal{V}} \exp\left\{\frac{s_I}{\sigma_I^2}x - \frac{1}{2}\frac{x^2}{\sigma_I^2}\right\}dF_{\widetilde{V}}(x)\right)\right\}\exp\left\{-\frac{1}{2}\frac{v^2}{\sigma_I^2}\right\}dF_{\widetilde{V}}(v), \end{aligned}$$

where the second equality pulls the expression in the denominator into the exponential function in the numerator.

By inspection, this density is in the desired form with

$$\begin{aligned} k_I &= \frac{1}{\sigma_I^2} \\ g_I(k_I s_I) &= \log\left(\int_{\mathcal{V}} \exp\left\{k_I s_I x - \frac{1}{2}\frac{x^2}{\sigma_I^2}\right\}dF_{\widetilde{V}}(x)\right) \\ dH_I(v) &= \exp\left\{-\frac{1}{2}\frac{v^2}{\sigma_I^2}\right\}dF_{\widetilde{V}}(v), \text{ or, equivalently,} \quad H_I(v) = \int_{-\infty}^{v} \exp\left\{-\frac{1}{2}\frac{x^2}{\sigma_I^2}\right\}dF_{\widetilde{V}}(x). \end{aligned}$$

The sets that appear in Assumption 3 are as follows. The support of g_I is $\mathcal{G}_I = \mathbb{R}$, since the $-x^2$ term in the exponential implies that the integral that defines g_I converges for any $k_I s_I \in \mathbb{R}$. The support of $\widetilde{S}_I$ is $\mathcal{S} = \mathbb{R}$ due to the normally distributed error. Clearly, $k_I \mathcal{S} \subseteq \mathcal{G}_I$, as required.

These two examples are by no means the only ones that satisfy the exponential family assumption. I focus on them because they employ some commonly made assumptions about pay-offs and produce straightforward solutions for the asset price. The existence and uniqueness results below are presented in general terms and are not limited to these examples.

3.1. *Characterizing the equilibrium*

The goal in this section is to characterize the equilibrium price in the two-types model. The essential difficulty to overcome is that the equilibrium price must both clear the market and be consistent with investors' beliefs. If the random variables in the model were jointly normally distributed, the standard solution approach is well known. It involves conjecturing a price function that is linear (affine) in the realizations of signal s_I and supply z, solving the investors' updating and portfolio problems given the price function, and substituting their demand functions into the market-clearing condition. Solving the resulting linear equation for the price and matching the coefficients with the original conjecture produces, a system of three equations with three unknowns. Grossman and Stiglitz (1980) show that these equations can be solved in closed form for the coefficients of the price function.

With a non-normal joint distribution, the "conjecture and verify" technique is not available since the functional form of the price is not clear *a priori*. However, suppose that the informed investor has a demand function that is additively separable in the signal and some transformation of the price, $X_I(s_I,p)=as_I-g(p)$. The market-clearing condition requires that in any equilibrium,

$$as_I-g(P(\cdot))+X_U(P(\cdot))=z+\bar{z},$$

or rearranging

$$\frac{1}{a}(g(P(\cdot))-X_U(P(\cdot))+\bar{z})=s_I-\frac{1}{a}z.$$

Since the left-hand side depends on (s_I,z) only through $P(\cdot)$, this implies that *any* equilibrium price function must reveal the statistic $\widetilde{S}_I-\frac{1}{a}\widetilde{Z}$. Hence, one can determine the information content of the price function independently of its functional form.[9] With this statistic in hand, the uninformed investor's equilibrium beliefs are pinned down, and one can also characterize his/her demand independently of the price function. Finding an equilibrium price then reduces to finding a price that clears the market.

The rest of this section walks through the equilibrium derivation described above and presents heuristic proofs. All results are proven rigorously in Appendix A.2. I begin by writing down the informed investor's partial equilibrium demand function, which takes the linear form above due to Assumptions 1–3. Let $(\underline{V},\overline{V})$ denote the interior of the convex hull of $\mathcal{V}$. This set is an open interval since $\mathcal{V}\subseteq\mathbb{R}$. Demand will only be finite for prices $p\in(\underline{V},\overline{V})$ since those are the prices that preclude arbitrage. I will also require the following purely technical assumption, which guarantees that the FOC is necessary and sufficient for an optimum for the informed investor.

Assumption 4. *The interval $\mathcal{G}_I$ is open.*

Lemma 1 (Informed demand). *Suppose that Assumptions 1–4 hold and that $p\in(\underline{V},\overline{V})$. Let $G_I\equiv(g_I')^{-1}$, where $g_I(\cdot)$ is the function from the informed investor beliefs in equation* (3.1). *The demand function of the informed investor is*

$$X_I(s_I,p)=\tau_I(k_Is_I-G_I(p)). \tag{3.6}$$

I give a brief sketch of the proof here. The informed investor's optimization problem is

$$\max_{x\in\mathbb{R}}\mathbb{E}\left[-\exp\left\{-\frac{1}{\tau_I}x(\widetilde{V}-p)\right\}|\widetilde{S}_I=s_I\right]=\max_{x\in\mathbb{R}}-\exp\left\{\tfrac{1}{\tau_I}xp+g_I\left(k_Is_I-\tfrac{1}{\tau_I}x\right)-g_I(k_Is_I)\right\},$$

where I use Lemma A6 to compute the conditional expectation in a closed form. Since $\mathcal{G}_I$, the domain of g_I, is assumed to be an open interval, this problem involves maximizing a continuously

9. I use the term "information content" informally. More precisely, determining the "information content" means determining a univariate random variable that depends on both $\widetilde{S}_I$ and $\widetilde{Z}$, such that the σ-algebras generated by the price and by this random variable are identical. For parsimony and to be comparable with prior literature, I suppress the measure theoretic details here, as throughout the article.

differentiable and strictly concave function over an open set, so the FOC is necessary and sufficient for an optimum. The FOC reduces to

$$g_I'\left(k_I s_I - \frac{1}{\tau_I}x\right) = p.$$

Supposing that g_I' is invertible and $G_I \equiv (g_I')^{-1}$ is well defined at p, one can apply G_I to both sides and rearrange to deliver the demand function in the Lemma. The function $G_I(\cdot)$, which depends on the joint distribution of the signal and pay-off, has an intuitive interpretation as the investor's *price reaction function*.

As the informed demand takes the desired additively separable form, one may substitute into the market-clearing condition and rearrange to obtain

$$\frac{1}{k_I}G_I(P(s_I,z)) - \frac{X_U(P(s_I,z))}{k_I\tau_I} + \frac{\bar{z}}{k_I\tau_I} = s_I - \frac{z}{k_I\tau_I}. \tag{3.7}$$

Hence, conditioning on the equilibrium price allows the uninformed investor to infer the realized value s_U of the statistic $\widetilde{S}_U \equiv \widetilde{S}_I - \frac{1}{k_I\tau_I}\widetilde{Z}$. The next Lemma formalizes this point.

Lemma 2. *Suppose that Assumptions 1–4 hold. Let $P(\cdot)$ be any equilibrium price function, and choose any (s_I,z) and $(\hat{s}_I,\hat{z}) \in \mathcal{S}\times\mathcal{Z}$. If $P(s_I,z) = P(\hat{s}_I,\hat{z})$ then $s_I - \frac{z}{k_I\tau_I} = \hat{s}_I - \frac{\hat{z}}{k_I\tau_I}$.*

As noted earlier, it is the additively separable form of informed demand that allows one to determine the information content of price without solving for equilibrium.[10] Additive separability implies that the informed investor's trading aggressiveness $\frac{\partial X_I}{\partial s_I}(s_I,p) = \tau_I k_I$ is independent of the price. Since the information revealed by the price depends on the trading aggressiveness, but the aggressiveness does not depend on the price, one can pin down $\widetilde{S}_U$ independently of $P(\cdot)$. In Section 1 of the online Appendix available as Supplementary Material, I show that CARA utility and the exponential family assumption are also *necessary* for an investor with twice continuously differentiable utility function to have an additively separable demand. This suggests that constructing equilibrium by identifying a linear statistic independently of the price function may be difficult to generalize beyond the setting considered here, at least in situations in which investors are risk-averse and face no constraints on demand.[11]

From Lemma 2, one may be tempted to conclude that any equilibrium price function can depend on (s_I,z) only through the quantity s_U. However, without further assumptions, that need not be the case. The Lemma implies only that any equilibrium price reveals *at least* $\widetilde{S}_U = s_U$. That is, the price reveals that the realization of $(\widetilde{S}_I,\widetilde{Z})$ lies on the line segment $\left\{(s_I,z): s_I - \frac{z}{k_I\tau_I} = s_U\right\}$. It does not imply that the price reveals *only* this fact. For the purpose of constructing an equilibrium, this is not a problem—one can simply focus on price functions that depend only on $\widetilde{S}_U$. However, it will be shown in Section 3.2 below that by restricting attention to continuous functions of s_I and z one can in fact rule out the existence of other equilibria.

I turn now to the uninformed investor's problem. The conditional distribution of $\widetilde{V}$ given $\widetilde{S}_U$ follows from Bayes rule, but since the exact form is not important for the derivation that

10. Indeed, the derivation of equilibrium could have proceeded by assuming additive separability in the informed demand and only later writing down conditions on primitives that guarantee such a functional form. I abstain from this approach to avoid placing restrictions directly on equilibrium objects.

11. Albagli *et al.* (2013) show that a similar linear statistic construction can be used in a model with a continuum of risk-neutral traders who face a portfolio constraint.

follows I defer the result to Lemma A11 in Appendix A.2. With this distribution pinned down, solving the uninformed's portfolio problem is simple. One may suppose that he observes $\widetilde{S}_U$ directly, rather than explicitly updating from the price. His/her demand will then depend on the realized value s_U and the (numerical) value of the price, p, but without any updating from p itself. To capture this fact, I modify notation slightly and denote his/her demand as a function of s_U and p by $X_U(s_U,p)$.[12] Under the following assumption, which is analogous to Assumption 4 and guarantees that the objective function is defined on an open set, a standard FOC pins down $X_U(s_U,p)$.

Assumption 5. *The conditional distribution of $\widetilde{V}$ given $\widetilde{S}_U = s_U$ has a conditional mgf that converges within some (potentially infinite) non-empty open interval containing zero and diverges outside this interval: specifically, there exist $0<\eta_0,\delta_0\leq\infty$, which may depend on s_U, such that*

$$\int_{\mathcal{V}} \exp\{ut\} dF_{\widetilde{V}|\widetilde{S}_U}(v|s_U) < \infty \quad \forall u \in (-\delta_0,\eta_0)$$

$$\int_{\mathcal{V}} \exp\{ut\} dF_{\widetilde{V}|\widetilde{S}_U}(v|s_U) = \infty \quad \forall u \in (-\infty,-\delta_0] \cup [\eta_0,\infty).$$

Lemma 3 (Uninformed demand). *Suppose that Assumptions 1–5 hold, $p\in(\underline{V},\overline{V})$, and the uninformed investor's information set consists only of $\widetilde{S}_U$. His/her demand, $X_U(s_U,p)$, is characterized implicitly by the following equation*

$$\int_{\mathcal{V}} (v-p)\exp\left\{-\frac{1}{\tau_U}X_U(s_U,p)v\right\} dF_{\widetilde{V}|\widetilde{S}_U}(v|s_U) = 0, \tag{3.8}$$

where the conditional distribution $F_{\widetilde{V}|\widetilde{S}_U}$ is given in Lemma A11.

Lemma 3 characterizes the uninformed investor's demand, assuming that an equilibrium price function exists that depends only on $s_U = s_I - \frac{z}{k_I \tau_I}$. It remains to characterize this price function and demonstrate existence. From this point forward, I will abuse notation by writing $P(s_U)$ to denote such a price function, despite the fact that Definition 1 formally defined $P(\cdot)$ as a function of (s_I,z).

Fix any $s_U \in \text{support}(\widetilde{S}_U) \equiv \mathcal{S} - \frac{1}{k_I\tau_I}\mathcal{Z}$. The expression for uninformed demand in equation (3.8) must hold state-by-state in equilibrium so that when an equilibrium price $P(s_U)$ is substituted in,

$$\int_{\mathcal{V}} (v-P(s_U))\exp\left\{-\frac{1}{\tau_U}X_U(s_U,P(s_U))v\right\} dF_{\widetilde{V}|\widetilde{S}_U}(v|s_U) = 0. \tag{3.9}$$

The market-clearing condition requires that in equilibrium the demand of the uninformed investor equals the supply of the asset net of the informed demand

$$X_U(s_U,P(s_U)) = \bar{z} - \tau_I\left(k_I s_U - G_I(P(s_U))\right).$$

Substituting this expression into equation (3.9), therefore, produces an expression that characterizes the equilibrium price implicitly, supposing that it exists.

12. The one-argument demand function introduced earlier, $X_U(p)$, relates to this two-argument function as $X_U(p) = X_U(P^{-1}(p),p)$.

Proposition 1. *Suppose that Assumptions 1–5 hold, and assume that a price function that depends on* (s_I, z) *only through* $s_U = s_I - \frac{z}{k_I \tau_I}$ *exists. Then,* $P(\cdot)$ *is characterized implicitly as*

$$\int_{\mathcal{V}} (v - P(s_U)) \exp\left\{ \left[\frac{\tau_I}{\tau_U} (k_I s_U - G_I(P(s_U))) - \frac{\bar{z}}{\tau_U} \right] v \right\} dF_{\widetilde{V}|\widetilde{S}_U}(v|s_U) = 0. \tag{3.10}$$

To better understand the meaning of the integral in equation (3.10), rearrange and write out the utility function in general terms, $u_U(w) = -\exp\left\{-\frac{1}{\tau_U} w\right\}$ to obtain

$$P(s_U) = \frac{\mathbb{E}\left[\widetilde{V} u'_U \left(\widetilde{V} (\bar{z} - \tau_I (k_I s_U - G_I(P))) \right) \middle| \widetilde{S}_U = s_U \right]}{\mathbb{E}\left[u'_U \left(\widetilde{V} (\bar{z} - \tau_I (k_I s_U - G_I(P))) \right) \middle| \widetilde{S}_U = s_U \right]}.$$

This looks like a typical asset pricing Euler equation except that the "endowment" of the agent, $\bar{z} - \tau_I (k_I s_U - G_I(P))$, is the residual supply. Accordingly, one can interpret equation (3.10) as a representative agent pricing formula in which the representative uninformed investor's risky-asset holding is itself endogenously determined. This implicit characterization of the price also clarifies that the assumption of CARA utility for the uninformed investor is not necessary and can be generalized to essentially arbitrary utility functions, up to restrictions to guarantee existence of expected utility.

In particular situations, the integral in equation (3.10) can be evaluated in a closed form, which gives the possibility of solving explicitly for $P(s_U)$. One would then check that the function $P(\cdot)$ so defined is one-to-one in s_U, which is a necessary condition due to Lemma 2. Without an explicit expression for the integral, one can establish existence with an intermediate value theorem argument. It suffices to show that for fixed s_U, the aggregate excess demand function $\tau_I(k_I s_U - G_I(p) + X_U(s_U, p) - \bar{z}$ is a continuous function of the variable p and crosses zero at least once. This guarantees existence of some point $p^*(s_U) \in \mathbb{R}$ at which equation (3.10) is satisfied. Having found a $p^*(s_U)$ corresponding to each s_U, define the function $P(s_U) \equiv p^*(s_U)$. As long as this function is one-to-one, then it is an equilibrium price function. To provide this existence result, I require some additional technical assumptions.

Assumption 6. *The support of* $k_I(\widetilde{S}_U - \frac{\bar{z}}{k_I \tau_I})$ *is a subset of* $\mathcal{G}_I$*. That is* $k_I \mathcal{S} - \frac{1}{\tau_I} \mathcal{Z} - \frac{1}{\tau_I} \bar{z} \subseteq \mathcal{G}_I$.

Assumption 7. *The supply* $\widetilde{Z}$ *is distributed according to a density function* $f_{\widetilde{Z}}$ *that is log-concave. That is* $\log f_{\widetilde{Z}}$ *is a concave function.*[13]

Assumption 8. *The function* $K(v, s_U)$ *in the uninformed cdf* $F_{\widetilde{V}|\widetilde{S}_U}(v|s_U)$ *in equation* (A.7) *in Lemma A11 is continuous in* s_U *for each* v.

Assumptions 6–8 are mostly economically innocuous. Assumption 6 is a minimal condition for existence. It is necessary and sufficient for equilibrium to exist when only informed investors participate in the risky-asset market. It guarantees that for any (s_I, z) there exists a solution p^* to the market-clearing condition in that case, $\tau_I(k_I s_I - G_I(p)) = z + \bar{z}$. Assumption 7 is sufficient (though

13. This assumption implies that the support $\mathcal{Z}$ is a (potentially infinite) interval (Theorem 1.8, Ch. 4 Karlin, 1968). There is an analogue of logconcavity for discrete distributions that could be appended to Assumption 7, but at the cost of additional notational complexity. See An (1997) for details.

not necessary) to guarantee that the price function produced by equation (3.10) is monotone. Assumption 8 is a continuity condition on uninformed investor beliefs and will guarantee that the price function is continuous in s_U. It is needed for the uniqueness result, but not for existence.

The following Proposition records the fact that under the assumptions above, there is an equilibrium in the model.

Proposition 2 (Equilibrium existence). *Suppose that Assumptions 1–7 hold. Then there exists an equilibrium price function which is defined implicitly by the expression in Proposition 1. If Assumption 8 also holds, this price function is continuous in s_U.*

A drawback of the function in Proposition 1 is that without further assumptions it can only be characterized implicitly. This makes it difficult to interpret and limits its usefulness for applied work. While comparative statics can be performed using the implicit function theorem, embedding the economy here into larger models is difficult without explicit solutions. Nevertheless, if one also assumes that the conditional distribution of $\widetilde{V}$ given $\widetilde{S}_U$ is in the exponential family, an explicit solution is available.

Assumption 9 (Exponential family, conditional on $\widetilde{S}_U$). *The conditional distribution of $\widetilde{V}$ given $\widetilde{S}_U = s_U$ can be written*

$$dF_{\widetilde{V}|\widetilde{S}_U}(v|s_U) = \exp\{k_U b_U(s_U)v - g_U(k_U b_U(s_U))\}dH_U(v), \quad v \in \mathcal{V}, s_U \in Support(\widetilde{S}_U) \tag{3.11}$$

where $k_U > 0$ is a constant, the function $b_U : support(\widetilde{S}_U) \to \mathbb{R}$ is increasing, the function $g_U : \mathcal{G}_U \to \mathbb{R}$ has domain $\mathcal{G}_U$ which is an interval satisfying $k_U b_U(Support(\widetilde{S}_U)) \subseteq \mathcal{G}_U$, and the function $H_U : \mathbb{R} \to \mathbb{R}$ is (weakly) increasing and right-continuous.

As in the case of the informed investor, the assumption that k_U is positive is without loss of generality. Unlike the case of the informed investor, it is analytically convenient to allow explicitly for the possibility that s_U interacts non-linearly with k_U via the function $b_U(s_U)$.[14] I show in equation (A.17) in Appendix A.3 that Assumption 9 is always met in the binomial setting of Example 1. In that case, regardless of the distribution of the supply shock, the conditional distribution of $\widetilde{V}$ given $\widetilde{S}_U$ remains binomial, which is an exponential family. I also show in equation (A.22) that the Assumption is met in Example 2 if the supply shock follows a normal distribution. In that case, $\widetilde{S}_U$ can be written $\widetilde{S}_U = \widetilde{V} + \widetilde{\varepsilon}_I - \frac{1}{k_I \tau_I}\widetilde{Z}$, and it was shown in Section 3 that an additive signal with normally distributed error leads to a exponential family conditional distribution.

If Assumption 9 holds then Proposition 1 produces an explicit function for the price, which I record in the following Corollary.

Corollary 1. *Suppose that Assumptions 1–7 and 9 hold. Let G be the aggregate (risk-tolerance-weighted) price reaction function*

$$G(p) \equiv \tau_I G_I(p) + \tau_U G_U(p).$$

An equilibrium exists and the price function is given by

$$P(s_U) = G^{-1}(\tau_I k_I s_U + \tau_U k_U b_U(s_U) - \bar{z}).$$

14. In the case of the informed investor, this possibility was taken care of via the definition of s_I so as to not have to carry around additional notation.

Here I briefly sketch the proof. From equation (3.6), informed demand is

$$X_I(s_I,p)=\tau_I(k_I s_I - G_I(p)),$$

and under Assumption 9, the uninformed investor's FOC produces a similar demand function

$$X_U(s_U,p)=\tau_U(k_U b_U(s_U)-G_U(p)).$$

The market-clearing condition pins down the equilibrium price

$$\tau_I(k_I s_I - G_I(P(s_U)))+\tau_U(k_U b_U(s_U)-G_U(P(s_U)))=\bar{z}+z,$$

and rearranging to isolate P produces the expression in the Corollary. Since Assumption 9 required b_U to be increasing, this function is monotone in s_U as required.

3.2. *Equilibrium uniqueness*

To prove uniqueness, I restrict attention to price functions that are continuous in the signal and supply. Continuity seems to be a reasonable condition to impose in a "smooth" model in which informed beliefs and demand depend continuously on s_I. However, this assumption does exogenously restrict the equilibria under consideration. Jordan (1982) shows in a non-noisy economy that there may exist complicated discontinuous price functions that are arbitrarily close to fully revealing, and Pálvölgyi and Venter (2015) construct discontinuous equilibria in the standard CARA-Normal setting.

Under the continuity assumption, the following Lemma records the fact that any price function must reveal only the value of s_U.

Lemma 4. *Suppose that Assumptions 1–4 hold and that $\mathcal{S}$ and $\mathcal{Z}$ are (potentially infinite) intervals. Choose any $s_U \in Support(\widetilde{S}_U) \equiv \mathcal{S}-\frac{1}{k_I\tau_I}\mathcal{Z}$. Then any continuous equilibrium price function is constant on the line segment $\{(s_I,z): s_I-\frac{z}{k_I\tau_I}=s_U\}$. That is, any continuous price function depends on (s_I,z) only through $s_U=s_I-\frac{z}{k_I\tau_I}$.*

Lemma 4 implies that *any* continuous equilibrium price function is informationally equivalent to the statistic $\widetilde{S}_U$. The idea behind the Lemma is as follows. Suppose that there exists a continuous price function that is not constant along the given line segment. Then one can find a point (s_I^0,z^0) and a sufficiently small $\varepsilon>0$ such that the set of points along the segment that are close to this point, $\{(s_I,z): s_I-\frac{z}{k_I\tau_I}=s_I^0-\frac{z^0}{k_I\tau_I}\}\cap(B_\varepsilon(s_I^0,z^0)\setminus\{(s_I^0,z^0)\})$, is both non-empty (due to $\mathcal{S}$ and $\mathcal{Z}$ being intervals) and contains only points that lead to different equilibrium prices $P(s_I,z)\neq P(s_I^0,z^0)$ (by continuity and the choice of (s_I^0,z^0)). Furthermore, since $\mathcal{S}$ and $\mathcal{Z}$ are intervals, the ball $B_\varepsilon(s_I^0,z^0)$ also contains points that do not lie in $\{(s_I,z): s_I-\frac{z}{k_I\tau_I}=s_I^0-\frac{z^0}{k_I\tau_I}\}$. Because P is continuous, some of these points have prices that are equal to the prices for other nearby points that *do* lie on the line segment. This contradicts Lemma 2, which implied that identical equilibrium prices can only arise for states that lie on the same line segment $\{(s_I,z): s_I-\frac{z}{k_I\tau_I}=s_U\}$.

Given the typically *ad hoc* method of analysing noisy rational expectations models, the question of uniqueness has remained open (Veldkamp, 2011, p. 93). However, it is now straightforward to demonstrate uniqueness among continuous equilibria, which requires proving that for fixed s_U, the aggregate excess demand function $\tau_I(k_I s_U - G_I(p)+X_U(s_U,p)-\bar{z}$ crosses zero *at most* once as p increases.

Proposition 3 (Equilibrium uniqueness). *Suppose that Assumptions 1–8 hold and that the supports $\mathcal{S}$ and $\mathcal{Z}$ are intervals. Then the equilibrium price function characterized in Proposition 1 exists, is continuous, and is unique within the class of continuous price functions.*

Uniqueness is in some sense unsurprising. In models in which agents do not learn from price, multiplicity can arise if wealth effects are sufficiently strong to prevent aggregate demand from sloping downward at all prices. CARA utility rules out wealth effects, and by Lemma 4 the equilibrium is equivalent to one in which the uninformed do not condition on price, but instead observe only the statistic $\widetilde{S}_U$. It follows that aggregate demand is downward sloping and the equilibrium is unique. As a corollary, this implies that the usual linear equilibrium in Grossman and Stiglitz (1980) is unique among continuous equilibria.

4. DERIVATION OF EQUILIBRIUM IN EXAMPLE 1

For illustrative purposes, in this section, I walk through the equilibrium derivation for Example 1. I defer all technical details to Appendix A.3, along with the derivation for Example 2, and here present only the essential details. Along with the assumption that the conditional distribution of $\widetilde{V}$ follows a binomial distribution, suppose that the informed investor's log-odds $\widetilde{S}_I$ follows an arbitrary continuous distribution with density $f_{\widetilde{S}_I}$, and the supply shock $\widetilde{Z}$ follows an arbitrary continuous distribution with continuously differentiable and log-concave density $f_{\widetilde{Z}}$. These smoothness assumptions will carry over to the price function. Furthermore, to avoid tedious consideration of boundary behaviour, suppose that the log-odds $\widetilde{S}_I$ has full support on $\mathbb{R}$.[15]

To begin, one requires the informed investor's demand function, which can be determined using Lemma 1. Equations (3.3) and (3.4) show $k_I = \frac{1}{V_H - V_L}$ and $g_I(k_I s_I) = V_L k_I s_I + \log(1 + \exp\{(V_H - V_L)k_I s_I\})$, so the demand function follows after computing the price reaction function $G_I = (g_I')^{-1}$. We have

$$g_I'(k_I s_I) = V_L + (V_H - V_L)\frac{\exp\{(V_H - V_L)k_I s_I\}}{1 + \exp\{(V_H - V_L)k_I s_I\}},$$

so

$$G_I(p) = \frac{1}{V_H - V_L}\log\left(\frac{p - V_L}{V_H - p}\right).$$

Lemma 1 delivers the informed demand

$$X_I(s_I, p) = \frac{\tau_I}{V_H - V_L}\left(s_I - \log\left(\frac{p - V_L}{V_H - p}\right)\right).$$

The market-clearing condition is

$$\frac{\tau_I}{V_H - V_L}\left(s_I - \log\left(\frac{P(\cdot) - V_L}{V_H - P(\cdot)}\right)\right) + X_U(P(\cdot)) = \bar{z} + z,$$

and rearranging to isolate terms involving $P(\cdot)$ implies that the statistic inferred by the uninformed investor is

$$\widetilde{S}_U = \widetilde{S}_I - \frac{V_H - V_L}{\tau_I}\widetilde{Z}.$$

15. Another way to avoid the possibility of full revelation of the signal at the boundaries is to assume that the supply shock $\widetilde{Z}$ has full support.

Let $f_{\widetilde{V}|\widetilde{S}_U}$ denote the uninformed investor's conditional pdf, which I characterize explicitly in equation (A.15) in Appendix A.3. Owing to the binomial distribution for $\widetilde{V}$, this conditional distribution remains binomial but with log-odds

$$b_U(s_U) \equiv \log\left(\frac{f_{\widetilde{V}|\widetilde{S}_U}(V_H|s_U)}{1-f_{\widetilde{V}|\widetilde{S}_U}(V_H|s_U)}\right). \tag{4.1}$$

Since the uninformed investor also faces a binomially distributed asset, his/her demand is linear in his/her log-odds

$$X_U(s_U,p)=\frac{\tau_U}{V_H-V_L}\left(b_U(s_U)-\log\left(\frac{p-V_L}{V_H-p}\right)\right).$$

The market-clearing condition requires that in equilibrium, the price $P(s_U)$ satisfies

$$\frac{\tau_I}{V_H-V_L}\left(s_I-\log\left(\frac{P(s_U)-V_L}{V_H-P(s_U)}\right)\right)+\frac{\tau_U}{V_H-V_L}\left(b_U(s_U)-\log\left(\frac{P(s_U)-V_L}{V_H-P(s_U)}\right)\right)=z+\bar{z}.$$

Rearranging this equation produces an explicit expression for $P(s_U)$

$$P(s_U)=V_L+(V_H-V_L)\frac{\exp\left\{\frac{\tau_I}{\tau_I+\tau_U}s_U+\frac{\tau_U}{\tau_I+\tau_U}b_U(s_U)-\frac{V_H-V_L}{\tau_I+\tau_U}\bar{z}\right\}}{1+\exp\left\{\frac{\tau_I}{\tau_I+\tau_U}s_U+\frac{\tau_U}{\tau_I+\tau_U}b_U(s_U)-\frac{V_H-V_L}{\tau_I+\tau_U}\bar{z}\right\}}. \tag{4.2}$$

5. APPLICATIONS

In this section, I briefly illustrate a few novel implications of the non-normal model in the context of the binomial example developed above. In all applications, I continue to assume that $\widetilde{S}_I$ has full support and that the density of $\widetilde{Z}$ is continuously differentiable. This implies that the uninformed investor's log-odds $b_U(s_U)$ is a differentiable function of s_U and therefore that the price function (equation (4.2)) is also differentiable. Though a full consideration of any of the applications is beyond the scope of this article, they suggest some directions in which one could further develop the model.

5.1. *Price reaction to information*

Motivated by the fact that asset prices seem often to be subject to movements that cannot be easily explained by news (Roll, 1988; Cutler *et al.*, 1989), a number of researchers have attempted to generate large price movements via learning from prices (Gennotte and Leland, 1990; Romer, 1993; Barlevy and Veronesi, 2003; Yuan, 2005). While learning from endogenous variables provides a plausible explanation for price movements without public news, it turns out to be somewhat difficult to capture such effects in standard models without introducing additional frictions. As shown by Gennotte and Leland (1990) and Yuan (2005), in a standard Grossman and Stiglitz (1980) model investor demand curves are everywhere downward sloping, and hence no investors act as "feedback traders", trading in the same direction as a price movement. It follows that prices never react more strongly than in an otherwise-identical setting with only informed investors.[16]

16. Admati (1985) shows in a multiple-asset setting and Wang (1993) in a dynamic setting that demand curves for some assets may be upward-sloping when pay-offs are normally distributed. However, since linear demand curves never bend backward over themselves it is not possible to generate large price movements of the sort considered here.

It turns out that in the binomial model, uninformed investor demand may be upward sloping in some regions, even in the absence of frictions, leading to amplified price reactions.[17] In a rational expectations setting, price changes generally have three effects on asset demand. The first two effects are standard substitution and income effects. CARA utility rules out any income effect, and the substitution effect tends to make demand curves slope down. The third effect, which is novel to settings with asymmetric information, is an information effect (Admati, 1985). All else equal, if a lower price signals that the asset pay-off is likely to be lower, agents want to buy less of the asset as its price decreases. This effect tends to make demand curves slope up. Taken together, in a model in which agents have CARA utility, demand can slope upward only in situations in which the information effect is sufficiently strong to swamp the substitution effect. The following Proposition provides a characterization of such situations.

Proposition 4. *If the distribution of the informed investor's log-odds* $\widetilde{S}_I$ *satisfies* $\frac{\partial}{\partial x}\frac{f'_{\widetilde{S}_I}(x)}{f_{\widetilde{S}_I}(x)} \leq \frac{\exp\{x\}}{1+\exp\{x\}}\frac{1}{1+\exp\{x\}}$, *for all* $x \in \mathcal{S}$, *then uninformed investor demand slopes down at all prices* p. *Conversely, if there exists a non-empty open interval* (a,b) *on which* $\frac{\partial}{\partial x}\frac{f'_{\widetilde{S}_I}(x)}{f_{\widetilde{S}_I}(x)} > \frac{\exp\{x\}}{1+\exp\{x\}}\frac{1}{1+\exp\{x\}}$, *then there exists a log-concave distribution for the supply shock and a non-empty open interval* $(\underline{p},\overline{p})$ *such that for for prices* $p \in (\underline{p},\overline{p})$, *uninformed investor demand slopes up.*

This Proposition shows that uninformed investor demand may slope upward purely due to learning effects, even without any additional frictions or constraints. The condition on the elasticity of the pdf, $\frac{\partial}{\partial x}\frac{f'_{\widetilde{S}_I}(x)}{f_{\widetilde{S}_I}(x)}$, is essentially a requirement that in some region the information conveyed by a price change must be sufficiently large to overcome the substitution effect.

A simple distribution for $\widetilde{S}_I$ that captures both cases in the Proposition is a power distribution with exponent $a>0$ and support parameter $k>0$:

$$f_{\widetilde{S}_I}(x) = \frac{a}{k^a}x^{a-1}\mathbb{I}\{x \in (0,k]\}$$

It follows that for any $x \in (0,k)$,

$$\frac{\partial}{\partial x}\frac{f'_{\widetilde{S}_I}(x)}{f_{\widetilde{S}_I}(x)} = \frac{1-a}{x^2}.$$

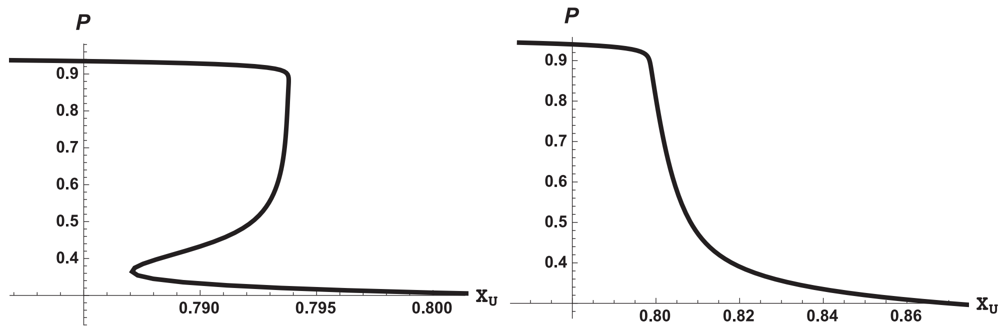
Hence, if $a \geq 1$, uninformed demand slopes down, regardless of the supply distribution, since for all $x \in (0,k)$,

$$\frac{\partial}{\partial x}\frac{f'_{\widetilde{S}_I}(x)}{f_{\widetilde{S}_I}(x)} = \frac{1-a}{x^2} \leq 0 < \frac{\exp\{x\}}{1+\exp\{x\}}\frac{1}{1+\exp\{x\}}$$

Conversely, if $a<1$, then there exist an interval of x's close to zero such that

$$\frac{\partial}{\partial x}\frac{f'_{\widetilde{S}_I}(x)}{f_{\widetilde{S}_I}(x)} = \frac{1-a}{x^2} > \frac{1}{4} \geq \frac{\exp\{x\}}{1+\exp\{x\}}\frac{1}{1+\exp\{x\}},$$

17. The models mentioned above consider frictions that cause *aggregate* demand to bend backward which generates a discontinuous price function. As shown in Section 4, the price function is continuous in the binomial model, so true "crashes" of this sort do not arise here.



(a) Uninformed demand curve for $a = 1/2$, $k = 4$. **(b)** Uninformed demand curve for $a = 2$, $k = 4$.

FIGURE 1

Panel (a): Uninformed demand curve when log-odds $\widetilde{S}_I$ follow a power distribution with $a = 1/2$, $k = 4$ and the supply shock is distributed $N(0, 0.05)$. Panel (b): Uninformed demand curve when log-odds $\widetilde{S}_I$ follow a power distribution with $a = 2$, $k = 4$ and the supply shock is distributed $N(0, 0.05)$. Other parameters: $\tau_I = 0.2, \tau_U = 0.8, V_L = 0, V_H = 1, \bar{z} = 1$.

and hence there exist supply distributions such that unformed demand is upward sloping over some region of prices. Figure 1 illustrates both possibilities in a setting with a normally-distributed supply shock.[18]

5.2. *Other applications*

In this section, I briefly consider two other applications concerning price drifts and the relation between returns and investor disagreement.

5.2.1. Price drifts. Post-announcement drift (Bernard and Thomas, 1989, 1990) and time-series momentum effects (Moskowitz *et al.*, 2012) are well documented, and have been argued by some to be evidence of irrationality among market participants (Daniel *et al.*, 1998; Hong and Stein, 1999). However, there has been recent interest in rationalizing such effects in noisy RE models (Allen *et al.*, 2006). However, Banerjee *et al.* (2009) show that in standard one-period CARA-Normal models asset prices always exhibit reversals.[19] It turns out that once one entertains the possibility of alternative pay-off distributions, this is no longer true.

To formulate a notion of drifts and reversals in the model, I use a definition that is similar to that used in the static section of Banerjee *et al.* (2009).

Definition 2. *Prices exhibit price drift (reversals) at price $\hat{p}$ if $\mathbb{E}[\widetilde{V} - P|P = p]$ is an increasing (decreasing) function of p at $p = \hat{p}$.*

This definition is meant as a reduced form for a three date model in which the final price is $P_2 = \widetilde{V}$, the time one equilibrium price is $P_1 = P(s_U)$, and the price before any information is revealed

18. Note that for this illustration the fact that the power distribution places zero probability on log-odds less than zero (success probabilities less than 1/2) is without loss of generality. One can always translate and rescale the distribution to arrange for an arbitrary support. Another remedy is to consider a symmetric power distribution with density $f_{\widetilde{S}_I}(x) = \frac{a}{2k^a}|x|^{a-1}\mathbb{I}\{x \in [-k, k]\}$, though this density is not differentiable at zero for all choices of a.

19. Strictly speaking, Banerjee *et al.* (2009) consider a Hellwig-style model (1980) with dispersed information, but it is easy to show that the absence of drifts carries over to a Grossman and Stiglitz (1980) model.

is normalized to a constant value $P_0 = \frac{V_H + V_L}{2}$ consistent with a setting, in which there is no asymmetric information nor noise shock (though the exact value of this constant is immaterial for the result). The definition then refers to whether $\mathbb{E}[P_2 - P_1 | P_1 - P_0]$ is increasing or decreasing in $P_1 - P_0$.

As shown by Banerjee *et al.* (2009), in static CARA-Normal models asset prices exhibit reversals at all p. The intuition for this is quite simple—when random variables are jointly normally distributed and noise is present traders are cautious to adjust their expectations about the pay-off less than one-for-one with changes in the price. While it is difficult to formulate conditions which guarantee either drifts or reversals in the general version of the binomial model, the following Proposition shows that both can arise naturally, even when all random variables follow symmetric distributions and uninformed log-odds $b_U(s_U)$ are "well-behaved".

Proposition 5. *Suppose that both $\widetilde{S}_I$ and $\widetilde{Z}$ are symmetrically distributed with mean zero and that the fixed component of supply is $\bar{z} = 0$. Suppose further that the derivative of the uninformed investor's log-odds, $b_U'(s_U)$, is uniformly strictly bounded between 0 and 1. Then,*

- *Prices exhibit reversals in a neighbourhood of $p = \frac{V_H + V_L}{2}$*
- *There exist $\underline{p} < \overline{p}$ such that prices exhibit drifts at $p < \underline{p}$ and $p > \overline{p}$.*

This Proposition shows that the unconditional relation between prices and future returns is generally ambiguous. More precisely, the result implies that if an econometrician conditions on "large" price movements, they should tend to find evidence of drifts, whereas if they condition on "small" price movements they should tend to find evidence of reversals.

5.2.2. The relation between disagreement and returns. A number of empirical papers document a negative relation between investor disagreement and future returns (*e.g.* Diether *et al.*, 2002; Goetzmann and Massa, 2005). The difference of opinions (DO) theory of Miller (1977) implies that when investors agree to disagree and there are short-sale constraints, stocks about which there is more disagreement will tend to have higher valuations and hence lower returns. On the contrary, in CARA-Normal models, there is a positive relation between disagreement and returns—high disagreement is associated with high risk and hence high returns. Thus, one interpretation of the existing empirical evidence is that investors agree to disagree and do not fully condition on prices.[20] However, I show below that the above predictions may be clouded in settings when uncertainty is not normally distributed. The reason is that return skewness also plays a role in the disagreement–return relation. The economic intuition for this effect is quite simple. If returns are, for example negatively skewed, then a large amount of disagreement also tends to be associated with a below average return.

Consider a slight reinterpretation of the binomial model in which there is a unit mass of agents with identical risk tolerance τ. Let λ denote the proportion of investors who are informed. This setup can be accommodated by setting the aggregate risk tolerances of each group to $\tau_I = \lambda\tau$ and $\tau_U = (1-\lambda)\tau$. In this simple setting, disagreement is given by the cross-sectional variance in expectations about $\widetilde{V}$ which, up to multiplication by the constant $\lambda(1-\lambda)$, is

$$\left(\mathbb{E}[\widetilde{V}|\widetilde{S}_I] - \mathbb{E}[\widetilde{V}|\widetilde{S}_U]\right)^2.$$

20. Banerjee (2011) points out the difficulty of distinguishing RE and DO models in a static setting. To distinguish the hypotheses, he considers how disagreement relates to the dynamic properties of returns and trading volume and finds evidence largely consistent with conditioning on prices.

The following proposition characterizes the covariance between (dollar) returns, $\widetilde{V}-P(\widetilde{S}_U)$, and investor disagreement.

Proposition 6. *Let* $\pi_I(s_I)=\frac{\exp\{s_I\}}{1+\exp\{s_I\}}$ *denote the informed investor's conditional probability of a high pay-off. Then,*

$$\begin{aligned}&\mathrm{Cov}\left(\widetilde{V}-P(\widetilde{S}_U),\left(\mathbb{E}[\widetilde{V}|\widetilde{S}_I]-\mathbb{E}[\widetilde{V}|\widetilde{S}_U]\right)^2\right)\\&=(V_H-V_L)^2\left\{\mathrm{Cov}\left(\mathbb{E}[\widetilde{V}-P(\widetilde{S}_U)|\widetilde{S}_U],\mathrm{Var}(\pi_I(\widetilde{S}_I)|\widetilde{S}_U)\right)\right.\\&\qquad\left.+(V_H-V_L)\mathbb{E}\left[\widehat{\mathrm{Skew}}(\pi_I(\widetilde{S}_I)|\widetilde{S}_U)\right]\right\},\end{aligned}$$

where

$$\widehat{\mathrm{Skew}}(\pi_I(\widetilde{S}_I)|\widetilde{S}_U)=\mathbb{E}\left[\left(\pi_I(\widetilde{S}_I)-\mathbb{E}[\pi_I(\widetilde{S}_I)|\widetilde{S}_U]\right)^3\middle|\widetilde{S}_U\right]$$

is the unnormalized conditional skewness (third central moment).

The proposition says that the relation between disagreement and returns is driven by two forces, the covariance between the conditional risk premium and the conditional variance of the probability of a high pay-off, and the (unnormalized) conditional skewness.[21] The first effect is intuitive and appears in earlier work. In a rational expectations setting, high disagreement is associated with high risk and hence high returns. The second effect is also intuitive but has not appeared before given the exclusive focus on linear normal models. Hence, Proposition 6 suggests a novel role for return skewness in determining the sign of the disagreement return relation, which may be important if the sign of this relation is to be interpreted as evidence as to whether investors rationally learn from prices.

6. INFORMATION AGGREGATION

In this section, I consider the aggregation of dispersed information. Perhaps surprisingly, one can still characterize the equilibrium price in certain cases, sometimes explicitly. My main goal in this section is to write down an equation for the price—a multidimensional analogue of the characterization in Proposition 1—rather than provide the most general possible existence result. The technical conditions that are sufficient to guarantee existence are analogous to those in Assumptions 4–7 above. Nevertheless, to demonstrate that the characterization results are not vacuous, in Appendix A.6 I provide full existence proofs for a special case of the binomial economy from Example 1 when traders receive additive signals about the log-odds of the pay-off and the general case of Example 2 in which traders receive conditionally iid signals about the pay-off itself.

The results in the two-types setting in Section 3 indicated that the key ingredient for the tractability of standard CARA-normal models is not the linearity of the price function itself, but rather the fact that the equilibrium price is informationally equivalent to a linear statistic of the signal and supply. In that vein, I make the following definition.

21. Strictly speaking, the second term involves the conditional third moment, under the uninformed investors' information set, of the informed investors' expectation of the pay-off. In a setting in which investors learn about the pay-off itself rather than a parameter governing the pay-off, this term is truly equal to the conditional third moment of the pay-off. See the earlier manuscript Breon-Drish (2012) for details.

Definition 3. *An equilibrium is a* generalized linear equilibrium *and the associated equilibrium price function is a* generalized linear price function *if the equilibrium price can be written in the form $P(L(s,z))$ for some linear function $L:\mathcal{S}^N\times\mathcal{Z}\to\mathbb{R}$ and some monotone function P: $L(\mathcal{S}^N\times\mathcal{Z})\to\mathbb{R}$.*

By restricting attention to generalized linear equilibria, one is essentially generalizing the "conjecture and verify" method from conjecturing a price function to conjecturing the information content of the price function. Thus, the results below are silent about price functions that are not of the generalized linear form. Similar conjectures about the information content of price also appear in some models with trading constraints (*e.g.* Yuan, 2005; Bai *et al.*, 2006; Marin and Olivier, 2008). However, I show in Section 2 of the online Appendix available as Supplementary Material and sketch briefly below that if one is willing to entertain an additive signal structure, as in Example 2, and a continuum of informed investors, as is often the case in applied work, then generalized linear equilibria are unique within the class of continuous equilibria.

Consider the full version of the model in which there are N informed investors, each of whom observes a signal $\widetilde{S}_i$ jointly distributed with $\widetilde{V}$, along with an uninformed investor U. In a generalized linear equilibrium, observation of the equilibrium price is equivalent to observation of some linear statistic of the form $L(\widetilde{S},\widetilde{Z})=\sum_{j=1}^{N}a_j\widetilde{S}_j-\widetilde{Z}$.[22] Supposing for a moment that a generalized linear equilibrium exists, the difficulty lies in determining the coefficients of the function L. With a single informed investor, the market-clearing condition uniquely pins down the coefficients, but introducing multiple informed investors complicates this step. Each investor would like to use the price to learn about the others' signals, hence L must be solved for as part of the equilibrium.

The following assumption is the natural multiple-investor analogue of Assumption 3 and allows one to identify the statistic L that must be revealed by any generalized linear price function.

Assumption 10 (Exponential family, conditional on $\widetilde{S}_i$ and $L(\cdot)$). *For any $i\in\{1,\ldots,N\}$ and any linear statistic of the form $L(\widetilde{S},\widetilde{Z})=\sum_{j=1}^{N}a_j\widetilde{S}_j-\widetilde{Z}$, the conditional distribution of $\widetilde{V}$ given $\widetilde{S}_i=s_i$ and $L(\widetilde{S},\widetilde{Z})=\ell$ can be written*

$$dF_{\widetilde{V}|\widetilde{S}_i,\widetilde{L}}(v|s_i,\ell)=\exp\{\hat{L}_i(s_i,\ell)v-g_i(\hat{L}_i(s_i,\ell);a)\}dH_i(v;a), \tag{6.1}$$

for some function $\hat{L}_i(s_i,\ell)=k_{i1}(a)s_i+k_{i2}(a)b_i(\ell;a)$, where $k_{i1},k_{i2}:\mathbb{R}^N\to\mathbb{R}$, $b_i(\cdot;a)$ is increasing in its first argument, $g_i(\cdot;a)$ is twice continuously differentiable with $g_i''(\cdot;a)>0$, and $H_i(\cdot;a)$ is (weakly) increasing.

Assumption 10 may appear rather dense and intimidating. However, it simply requires that the conditional distribution of the pay-off given $(\widetilde{s}_i,L(\cdot))$ lies in an exponential family. Here, the "constants" $k_{i1}(a)$ and $k_{i2}(a)$ as well as the functions b_i, g_i, and H_i that characterize the distribution may depend on the coefficients a in the linear statistic. Returning to the earlier interpretation of exponential family beliefs from Section 3, the functions $k_{i1}(a)$ and $k_{i2}(a)$ are like coefficients on the investor's private signal $\widetilde{S}_i$ and the "public" signal $L(\widetilde{S},\widetilde{Z})$ when she fits a generalized linear model to predict the asset's pay-off.

With Assumption 10 in place, one can pin down the statistic L on which the price must depend. Suppose that a generalized linear equilibrium exists and conditional beliefs are as in

22. The assumption that the coefficient on on $\widetilde{Z}$ is -1 is without loss of generality. One can always rescale L by dividing by a constant and absorb the constant into the function P.

Assumption 10. By computing expected utilities and differentiating, the demand of investor i can be written in closed form as a weighted sum of private information and public information

$$X_i(s_i,\ell,p)=\tau_i\left(k_{i1}(a)s_i+k_{i2}(a)b_i(\ell;a)-G_i(p;a)\right),$$

where $G_i(p;a)\equiv(g_i')^{-1}(\cdot;a)$ is the price reaction function.

In equilibrium, the market-clearing holds

$$\sum_{i=1}^{N}\tau_i\left(k_{i1}(a)s_i+k_{i2}(a)b_i(\ell;a)-G_i(P(\ell);a)\right)+X_U(P(\ell))-z-\bar{z}=0,$$

which can be rearranged as

$$\sum_{i=1}^{N}\tau_i G_i(P(\ell);a)-X_U(P(\ell))-\sum_{i=1}^{N}\tau_i k_{i2}(a)b_i(\ell;a)+\bar{z}=\sum_{i=1}^{N}\tau_i k_{i1}(a)s_i-z. \tag{6.2}$$

Equation (6.2) says that the left-hand side, a function of $\ell=\sum_{j=1}^{N}a_j s_j-z$, equals $\sum_{i=1}^{N}\tau_i k_{i1}(a)s_i-z$. For this to be true globally in (s,z), it must be the case that ℓ also equals $\sum_{i=1}^{N}\tau_i k_{i1}(a)s_i-z$. Thus, the coefficients must satisfy $a_i=\tau_i k_{i1}(a)$ for all i. The following Lemma records this fact.

Lemma 5 (Price-information equations). *Suppose that Assumptions 1 and 10 hold and that $P(L(\cdot))$ is a generalized linear price function. The coefficients $a^*\equiv(a_1^*,\dots,a_N^*)$ of the function L solve the following system of N "price-information" equations*

$$a_i^*=\tau_i k_{i1}(a^*)\quad i\in\{1,\dots,N\}. \tag{6.3}$$

Hence, in equilibrium the function L is given by $L^*(s,z)\equiv\sum_{i=1}^{N}a_i^* s_i-z$.

The price information equations are similar to the usual system defining the price coefficients in the Hellwig (1980) model. Examining the equations shows that prices aggregate information in an intuitive way. The coefficient a_i^* on each investor's signal is equal to his/her responsiveness to private information, weighted by his/her risk tolerance. Thus, investors whose beliefs respond more strongly to their signals (high k_{i1}) or who are more risk tolerant (high τ_i) will see their signals weighted more heavily in the price. While it is difficult to formulate general conditions on the functions k_{i1} that guarantee existence of a solution to the price information equations, it is straightforward to use a fixed-point argument based on Lemma 3.1 of Hellwig (1980) to prove existence in particular cases. I do this for the Example economies in Appendix A.6.

Having pinned down L^*, the construction of the price function proceeds as in the two-types case. However, it is worth pointing out that, depending on whether there are multiple solutions to the price information equations, there may exist multiple generalized linear equilibria. Given a particular solution to the price-information equations, the following Proposition provides an implicit characterization of the asset price.

Proposition 7. *Suppose that Assumptions 1 and 10 hold, and suppose that there exists a solution $a^*\equiv(a_1^*,\dots,a_N^*)$ to the price-information equations from Lemma 5, with corresponding function L^*.*

Define b as the aggregate (risk-tolerance-weighted) version of $k_{i2}(a^*)b_i(\cdot;a^*)$

$$b(\ell;a^*)\equiv\sum_{i=1}^{N}\tau_i k_{i2}(a^*)b_i(\ell;a^*)$$

and let G be the aggregate (risk-tolerance-weighted) price reaction function

$$G(p;a^*)\equiv\sum_{i=1}^{N}\tau_i G_i(p;a^*).$$

If the function $P(\cdot)$ *characterized implicitly by the following equation exists and is monotone in* L^*

$$\int_{\mathcal{V}}(v-P(L^*(s,z)))\exp\left\{\frac{1}{\tau_U}[L^*(s,z)+b(L^*(s,z);a^*)-G(P(L^*(s,z));a^*)-\bar{z}]v\right\}\times dF_{\widetilde{V}|\widetilde{L}^*}(v|L^*(s,z))=0,$$

then a generalized linear equilibrium exists, with price function $P(L^*(s,z))$.

Proposition 7 is the multiple-investor analogue of the implicit characterization in Proposition 1. Having determined L^*, investor demand functions are known, and to characterize the price one simply imposes market clearing by substituting the residual supply of the asset into the uninformed investor's FOC. As in the two-investor case, a drawback of Proposition 7 is that without further assumptions the price can only be characterized implicitly. However, if one also requires for the uninformed investor an exponential family condition on beliefs, or considers a setting with only informed investors, an explicit solution is available.

Assumption 11 (Exponential family, conditional on $L(\cdot)$**).** *For any linear statistic of the form* $L(\widetilde{S},\widetilde{Z})=\sum_{j=1}^{N}a_j\widetilde{S}_j-\widetilde{Z}$, *the conditional distribution of* $\widetilde{V}$ *given* $L(\widetilde{S},\widetilde{Z})=\ell$ *can be written*

$$dF_{\widetilde{V}|\widetilde{L}}(v|\ell)=\exp\{\hat{L}_U(\ell)v-g_i(\hat{L}_U(\ell);a)\}dH_U(v;a), \tag{6.4}$$

for some function $\hat{L}_U(\ell)=k_{U2}(a)b_U(\ell;a)$, *where* $k_{U2}:\mathbb{R}^N\to\mathbb{R}$, $b_U(\cdot;a)$ *is increasing in its first argument,* $g_U(\cdot;a)$ *is twice continuously differentiable with* $g_U''>0$, *and* $H_U(\cdot;a)$ *is (weakly) increasing.*

Assumption 11 holds holds in the $N>1$ cases of Examples 1 and 2 considered in Appendix A.6 if the supply shock follows a normal distribution. See equation (A.38) for Example 1, and equation (A.40) and the immediately following discussion for Example 2. Those equations characterize the beliefs of an arbitrary informed investor, of which an uninformed investor is a limiting case when the private signal becomes infinitely noisy.

If Assumption 11 holds or if there are no uninformed investors ($\tau_U=0$) then Proposition 7 produces an explicit function for the price, which I record in the following Corollary.

Corollary 2. *Suppose that Assumption 1 holds, and that either (i) Assumptions 10 and 11 hold or (ii) Assumption 10 holds and there are no uninformed investors* ($\tau_U=0$). *Suppose that there exists a solution* $a^*\equiv(a_1^*,\ldots,a_N^*)$ *to the price-information equations from Lemma 5.*

Define b as the aggregate (risk-tolerance-weighted) version of $k_{i2}(a^*)b_i(\cdot;a^*)$

$$b(\ell;a^*)\equiv\sum_{i=1}^{N}\tau_i k_{i2}(a^*)b_i(\ell;a^*)+\tau_U k_{U2}(a^*)b_U(\ell;a^*),$$

and let G be the aggregate (risk-tolerance-weighted) price reaction function

$$G(p;a^*)\equiv\sum_{i=1}^{N}\tau_i G_i(p;a^*)+\tau_U G_U(p;a^*).$$

Then if the function $G^{-1}(\cdot;a^*)$ is well defined on $\{\ell+b(\ell;a^*)-\bar{z}:\ell\in support(\widetilde{L})\}$, there exists a generalized linear equilibrium with price function

$$P(L^*(s,z))=G^{-1}\left(L^*(s,z)+b(L^*(s,z);a^*)-\bar{z};a^*\right).$$

Assumption 10 and 11 may appear to be restrictive to the point of requiring that the pay-off and signals be jointly normally distributed. That is not the case. There are other natural pay-off and signal structures that satisfy the assumptions, including a special case of Example 1 and all cases of Example 2. I compute the equilibria in these economies in Appendix A.6

6.1. *Uniqueness in a continuum case of Example 2*

While I have been unable to discover any general uniqueness results for the finite-investor model with dispersed information, in Section 2 of the online Appendix available as Supplementary Material I consider a version of the model with a continuum of investors who receive additive signals with normally distributed errors (Example 2). In this setting, uniqueness within the class of continuous equilibria re-emerges under an additional economically innocuous restriction that rules out certain singularly continuous (Cantor-like) functions. I briefly sketch the derivation here.

In this model, each investor $i\in[0,1]$ receives a signal $\widetilde{S}_i=\widetilde{V}+\widetilde{\varepsilon}_i$, where $\widetilde{\varepsilon}_i$ follow independent normal distributions $N(0,\sigma_i^2)$. The key difference with the finite-investor version of Example 2 is that with a continuum of investors no individual ε_i's enter the price. When that is true, each investor's optimal portfolio is additively separable in s_i and p for any continuous price function. This allows one to bring to bear the techniques from the two-types case in Section 3 which relied heavily on additive separability.

Here I briefly sketch the proof of additive separability and refer the interested reader to the details in the online Appendix available as Supplementary Material. Note that because $\widetilde{S}_i$ and $\widetilde{Z}$ are conditionally independent given $\widetilde{V}$ one can use Bayes' rule to write the joint distribution of $(\widetilde{V},\widetilde{S}_i,P(\widetilde{V},\widetilde{Z}))$ as

$$dF_{\widetilde{V},\widetilde{S}_i,P(\cdot)}(v,s_i,p)\propto\exp\left\{-\frac{1}{2\sigma_i^2}(s_i-v)^2\right\}ds_i\, f_{P(\cdot)|\widetilde{V}}(p|v)dp\, f_{\widetilde{V}}(v)dv.$$

This expression is proportional to the conditional density that the investor uses when forming his/her portfolio, where the constant of proportionality depends only on (s_i,p), and therefore divides out of the FOC.

Plugging into investor i's FOC and dividing out terms that are constant with respect to v yields

$$\int_{\mathcal{V}}(v-p)\exp\left\{-\left(\frac{1}{\tau_i}X_i(s_i,p)-\frac{s_i}{\sigma_i^2}\right)v\right\}f_{P(\cdot)|\widetilde{V}}(p|v)\exp\left\{-\frac{1}{2\sigma_i^2}v^2\right\}f_{\widetilde{V}}(v)dv=0,$$

which implies that $\frac{1}{\tau_i}X_i(s_i,p)-\frac{s_i}{\sigma_i^2}$ can be written purely as a function, $h_{i,P(\cdot)}(p)$, of the realized price. Rearranging yields

$$X_i(s_i,p)=\tau_i\left(\frac{s_i}{\sigma_i^2}+h_{i,P(\cdot)}(p)\right).$$

Plugging into the market-clearing condition and using the same argument as in Lemma 4 in the hierarchical information case implies that any continuous price function must reveal only a linear combination of signals and supply shock. Uniqueness of equilibrium then follows directly.

7. CONCLUSION

In this article, I have presented a class of noisy RE economies that nests the standard CARA-Normal setting of Grossman and Stiglitz (1980) and Hellwig (1980). I provided a constructive proof of existence of equilibrium and in the two-types and continuum-of-investors settings have given sufficient conditions for uniqueness of this equilibrium within the class of continuous equilibria. I have also exhibited some examples in which explicit solutions are available. The results presented here open up a broad class of models for applications, a few of which I have given a brief treatment, including price reaction to information, price drifts and reversals, and the disagreement–return relation. The model could be generalized in a straightforward way to incorporate multiple assets (thus extending Admati (1985)) by appealing to multivariate exponential families for pay-off distributions and/or to include trading constraints by following the method of Yuan (2005). I leave these problems for future work.

APPENDIX

A. PROOFS

A.1. *Properties of the exponential family conditional distribution*

In this section, I collect a few properties of the exponential family conditional distribution from Assumption 3,

$$dF_{\widetilde{V}|\widetilde{S}_I}(v|s_I)=\exp\{k_I s_I v-g_I(k_I s_I)\}dH_I(v),\quad v\in\mathcal{V}, s_I\in\mathcal{S},$$

where $\mathcal{G}_I$ denotes the support of g_I and satisfies $k_I\mathcal{S}\subseteq\mathcal{G}_I$.

Lemma A6 (mgf). *Fix $s_I\in\mathcal{S}$ and let $M_{\widetilde{V}|\widetilde{S}_I}(u|s_I)\equiv\mathbb{E}\left[\exp\{u\widetilde{V}\}|\widetilde{S}_I=s_I\right]$ denote the conditional mgf. Then, $\{u\in\mathbb{R}: M_{\widetilde{V}|\widetilde{S}_I}(u|s_I)<\infty\}=\{u\in\mathbb{R}:u+k_I s_I\in\mathcal{G}_I\}$, and*

$$M_{\widetilde{V}|\widetilde{S}_I}(u|s_I)=\exp\{g_I(u+k_I s_I)-g_I(k_I s_I)\}.$$

Proof The density must integrate to 1

$$\int_{\mathcal{V}}\exp\{k_I s_I v-g_I(k_I s_I)\}dH_I(v)=1$$

$$\Rightarrow\quad\int_{\mathcal{V}}\exp\{k_I s_I v\}dH_I(v)=\exp\{g_I(k_I s_I)\},$$

Therefore,

$$\int_{\mathcal{V}}\exp\{uv\}\exp\{k_I s_I v-g_I(k_I s_I)\}dH_I(v)=\exp\{g_I(u+k_I s_I)-g_I(k_I s_I)\},$$

which is well defined and finite as long as $u+k_I s_I$ lies in $\mathcal{G}_I$, the domain of g_I. ||

Lemma A7 (Differentiability of g_I). *The function g_I is infinitely continuously differentiable in the interior of $\mathcal{G}_I$ and is strictly convex.*

Proof From Lemma A6, the conditional mgf of $\widetilde{V}$ is defined for any u such that $u+k_I s_I \in \mathcal{G}_I$ and one can write

$$g_I(u+k_I s_I) = \log M_{\widetilde{V}|\widetilde{S}_I}(u|s_I) + g_I(k_I s_I) \tag{A.1}$$

It is well known that mgfs are analytic in the interior of their domain of existence (Billingsley, 1995, p. 278). Since $u+k_I s_I$ ranges over the interior of $\mathcal{G}_I$ as u ranges over the interior of the interval of convergence of the mgf, it follows from equation (A.1) that the function g_I is analytic in the interior of $\mathcal{G}_I$, and therefore is infinitely continuously differentiable. To show convexity, use the previous expression for $g_I(u+k_I s_I)$ and differentiate both sides twice with respect to u

$$\begin{aligned} g_I''(u+k_I s_I) &= \frac{M_{\widetilde{V}|\widetilde{S}_I}(u|s_I) M''_{\widetilde{V}|\widetilde{S}_I}(u|s_I) - (M'_{\widetilde{V}|\widetilde{S}_I}(u|s_I))^2}{(M_{\widetilde{V}|\widetilde{S}_I}(u|s_I))^2} \\ &= \frac{\mathbb{E}\big[\exp\{u\widetilde{V}\}|\widetilde{S}_I = s_I\big]\mathbb{E}\big[\widetilde{V}^2 \exp\{u\widetilde{V}\}|\widetilde{S}_I = s_I\big] - \big(\mathbb{E}\big[\widetilde{V}\exp\{u\widetilde{V}\}|\widetilde{S}_I = s_I\big]\big)^2}{(M_{\widetilde{V}|\widetilde{S}_I}(u|s_I))^2}. \end{aligned}$$

Positivity of this expression follows from the Cauchy–Schwarz inequality, which states $\mathbb{E}[|\widetilde{X}\widetilde{Y}|] \le \sqrt{\mathbb{E}[\widetilde{X}^2]}\sqrt{\mathbb{E}[\widetilde{Y}^2]}$ for any random variables $\widetilde{X}$ and $\widetilde{Y}$, with equality if and only if $\widetilde{X}$ and $\widetilde{Y}$ are linearly dependent. Taking $\widetilde{X} = \exp\{u\frac{\widetilde{V}}{2}\}$ and $\widetilde{Y} = \widetilde{V}\exp\{u\frac{\widetilde{V}}{2}\}$, and squaring both sides of the inequality shows that $g_I''(u+ks_I) > 0$. ‖

Lemma A8 (Moments). *The conditional moments of $\widetilde{V}$ are given by the derivatives of the mgf from Lemma A6, evaluated at $u=0$*

$$\mathbb{E}\big[\widetilde{V}^n|\widetilde{S}_I = s_I\big] = \frac{d^n}{du^n}\exp\{g_I(u+k_I s_I) - g_I(k_I s_I)\}\Big|_{u=0}.$$

The first four moments are

$$\begin{aligned} \mathbb{E}\big[\widetilde{V}|\widetilde{S}_I = s_I\big] &= g_I'(k_I s_I) \\ \mathbb{E}\Big[\widetilde{V}^2|\widetilde{S}_I = s_I\Big] &= g_I''(k_I s_I) + (g_I'(k_I s_I))^2 \\ \mathbb{E}\Big[\widetilde{V}^3|\widetilde{S}_I = s_I\Big] &= g_I'''(k_I s_I) + 3g_I''(k_I s_I)g_I'(k_I s_I) + (g_I'(k_I s_I))^3 \\ \mathbb{E}\Big[\widetilde{V}^4|\widetilde{S}_I = s_I\Big] &= g_I''''(k_I s_I) + 4g_I'''(k_I s_I)g_I'(k_I s_I) + 3(g_I''(k_I s_I))^2 \\ &\quad + 6g_I''(k_I s_I)(g_I'(k_I s_I))^2 + (g_I'(k_I s_I))^4. \end{aligned}$$

Proof Recall that the n-th derivative of the mgf, evaluated at $u=0$, delivers the n-th raw moment of the distribution (Theorem 2.3.7, Casella and Berger, 2002). Hence, the general expression follows immediately from the expression for the mgf from Lemma A6.

For the first moment, one has

$$\begin{aligned} \mathbb{E}\big[\widetilde{V}|\widetilde{S}_I = s_I\big] &= \frac{d}{du} M_{\widetilde{V}|\widetilde{S}_I}(u|s_I)\Big|_{u=0} \\ &= \frac{d}{du}\exp\{g_I(u+k_I s_I) - g_I(k_I s_I)\}\Big|_{u=0} \\ &= g_I'(u+k_I s_I)\exp\{g_I(u+k_I s_I) - g_I(k_I s_I)\}\Big|_{u=0} \\ &= g_I'(k_I s_I). \end{aligned}$$

Repeating this for higher derivatives of the mgf gives the explicit results for the remaining raw moments. ‖

A.2. *Proofs of results in Section 3*

This Appendix presents proofs of all results found in Section 3. For clarity, it is divided into three parts. The first part derives some preliminary results that relate to the portfolio optimization problem faced by a CARA investor. These results will be used in later proofs. The second part provides proofs for the results from the text that relate to the partial equilibrium demand functions of informed and uninformed investors. Finally, the last part provides proofs of the existence and uniqueness results.

A.2.1. Some preliminaries. Before beginning the proofs of the main results in the article, I present two preliminary Lemmas on investors' partial equilibrium portfolio problem. The following Lemma establishes that for a CARA investor facing an asset whose pay-off has an mgf that converges on an open set, the investor's FOC is necessary and sufficient for an optimum, and there is a finite solution to the FOC if and only if there are no arbitrage opportunities in the market.

Lemma A9. *Consider an investor with CARA utility with risk tolerance τ. Suppose that conditional on his/her information set $\mathcal{F}$, the asset pay-off $\widetilde{V}\in\mathcal{V}$ is distributed according to a non-degenerate distribution with mgf that is finite only on a (potentially infinite) non-empty open interval $(-\delta,\eta)$, $\delta,\eta>0$.*[23] *Then*

- *The investor's FOC is necessary and sufficient for the optimal portfolio, which is unique when it exists.*
- *An optimal portfolio exists if and only if the asset price p lies in $(\underline{V},\overline{V})$, the interior of the convex hull of $\mathcal{V}$.*

Proof To begin, I establish that the investor's problem has a finite maximum if and only if there exists a portfolio x that satisfies the FOC. Consider the investor's problem

$$\max_x \mathbb{E}\left[-\exp\left\{-\frac{1}{\tau}x(\widetilde{V}-p)\right\}|\mathcal{F}\right]=\max_x\int_{\mathcal{V}}-\exp\left\{-\frac{1}{\tau}x(v-p)\right\}dF_{\widetilde{V}|\mathcal{F}}(v)$$

This objective function is an mgf evaluated at $-\frac{1}{\tau}x$. Assuming that the mgf of $\widetilde{V}$ is finite on $(-\delta,\eta)$, the investor's objective function is finite for portfolios x in the interval $\{x:-\frac{1}{\tau}x\in(-\delta,\eta)\}=(-\tau\eta,\tau\delta)$, and is equal to $-\infty$ outside of this interval. Since mgf are analytic (Billingsley, 1995, p.278), his/her objective function is (infinitely) continuously differentiable, and since the pay-off distribution is non-degenerate it is also strictly concave. It is well known that for a differentiable and strictly concave objective function defined on an open set, the optimization has a well-defined maximizer x^* if and only if there exists an x^* satisfying the FOC[24]

$$\int_{\mathcal{V}}(v-p)\exp\left\{-\frac{1}{\tau}x^*(v-p)\right\}dF_{\widetilde{V}|\mathcal{F}}(v)=0.$$

Moreover, due to the strict concavity of the objective function, the optimum is unique when it exists.

It remains to demonstrate that the given restrictions on p are necessary and sufficient for the existence of a solution to the FOC. To show that the existence of a finite optimal portfolio implies $p\in(\underline{V},\overline{V})$, suppose that the investor has a finite optimal portfolio but that $p\notin(\underline{V},\overline{V})$. Without loss of generality, suppose that p lies below $(\underline{V},\overline{V})$. Then, with probability 1, the asset pay-off is greater than or equal to the price

$$\mathbb{P}\left(\widetilde{V}\geq p|\mathcal{F}\right)=\int_{\mathcal{V}}\mathbb{I}_{\{v\geq p\}}dF_{\widetilde{V}|\mathcal{F}}(v)=1,$$

and with strictly positive probability the pay-off is strictly greater than the price

$$\mathbb{P}\left(\widetilde{V}>p|\mathcal{F}\right)=\int_{\mathcal{V}}\mathbb{I}_{\{v>p\}}dF_{\widetilde{V}|\mathcal{F}}(v)\geq\int_{\mathcal{V}}\mathbb{I}_{\{v>\underline{V}\}}dF_{\widetilde{V}|\mathcal{F}}(v)>0.$$

Due to these facts, it follows that for all candidate portfolios $x\in(-\tau\eta,\tau\delta)$, we have

$$\int_{\mathcal{V}}(v-p)\exp\left\{-\frac{1}{\tau}x(v-p)\right\}dF_{\widetilde{V}|\mathcal{F}}(v)=\int_{\mathcal{V}}(v-p)\exp\left\{-\frac{1}{\tau}x(v-p)\right\}\mathbb{I}_{\{v\geq p\}}dF_{\widetilde{V}|\mathcal{F}}(v)>0,$$

from which it follows that the FOC cannot be satisfied. This contradicts the assumption that the investor has a finite optimal portfolio. Hence, if the investor has an optimal portfolio, it must be the case that $p\in(\underline{V},\overline{V})$.

Conversely, suppose that $p\in(\underline{V},\overline{V})$. To show that the investor has a finite optimal portfolio, it suffices to show that there exists an $x^*\in(-\tau\eta,\tau\delta)$ that satisfies the FOC

$$\int_{\mathcal{V}}(v-p)\exp\left\{-\frac{1}{\tau}x^*(v-p)\right\}dF_{\widetilde{V}|\mathcal{F}}(v)=0.$$

To prove that such an x^* exists, I employ an intermediate value theorem argument. Because the integral that appears in the FOC is the derivative of the mgf of $\widetilde{V}-p$ and mgfs are infinitely continuously differentiable in their domain of existence, the integral is a continuous function of x.

23. At this level of generality, both the support $\mathcal{V}$ and the interval $(-\delta,\eta)$ are allowed to depend on the information set $\mathcal{F}$.

24. See any text on convex analysis (*e.g.* Boyd and Vandenberghe, 2004, Ch. 4)

Parameterize $x(\omega)=-\tau\omega$, for $\omega\in(-\delta,\eta)$. This parameterization captures all portfolios $x\in(-\tau\eta,\tau\delta)$ for which the objective function is finite. With this expression for the portfolio, one can write the FOC as

$$\int_{\mathcal{V}}(v-p)\exp\left\{-\frac{1}{\tau}x(\omega)(v-p)\right\}dF_{\widetilde{V}|\mathcal{F}}(v)=\int_{\mathcal{V}}(v-p)\exp\{\omega(v-p)\}dF_{\widetilde{V}|\mathcal{F}}(v).$$

Since $p\in(\underline{V},\overline{V})$, this integral can be divided into positive and negative parts, both of which are non-zero.

$$\begin{aligned}\int_{\mathcal{V}}(v-p)\exp\{\omega(v-p)\}dF_{\widetilde{V}|\mathcal{F}}(v)=&\int_{\mathcal{V}}\mathbb{I}_{\{\underline{V}\le v<p\}}(v-p)\exp\{\omega(v-p)\}dF_{\widetilde{V}|\mathcal{F}}(v)\\&+\int_{\mathcal{V}}\mathbb{I}_{\{p<v\le\overline{V}\}}(v-p)\exp\{\omega(v-p)\}dF_{\widetilde{V}|\mathcal{F}}(v).\end{aligned}$$

Note that the set $\{v=p\}$ contributes zero to the integral and so can be omitted.

I now show that as $\omega\uparrow\eta$, the negative integral over $\{v:\underline{V}\le v<p\}$ remains bounded, whereas the integral over $\{v:p<v\le\overline{V}\}$ becomes unboundedly large. Consider first the negative integral. Note that for $\omega\in(0,\eta)$,

$$\begin{aligned}\left|\int_{\mathcal{V}}\mathbb{I}_{\{\underline{V}\le v<p\}}(v-p)\exp\{\omega(v-p)\}dF_{\widetilde{V}|\mathcal{F}}(v)\right|&\le\int_{\mathcal{V}}\mathbb{I}_{\{\underline{V}\le v<p\}}|v-p|\exp\{\omega(v-p)\}dF_{\widetilde{V}|\mathcal{F}}(v)\\&\le\int_{\mathcal{V}}\mathbb{I}_{\{\underline{V}\le v<p\}}|v-p|dF_{\widetilde{V}|\mathcal{F}}(v)\\&<\infty,\end{aligned}$$

where the first inequality follows from the triangle inequality, the second inequality follows because $\exp\{\omega(v-p)\}<1$ on $\{v<p\}\cap\{\omega>0\}$, and the final inequality follows because $\widetilde{V}$ has a finite mgf, and therefore $\widetilde{V}$ has finite moments of all orders. This shows that the integral over $\{v:\underline{V}\le v<p\}$ remains bounded as ω tends to its upper boundary.

Now, consider the positive integral over $\{v:p<v\le\overline{V}\}$. Note that because $p\in(\underline{V},\overline{V})$, there exists some $k>p$ such that there is a strictly positive probability that the pay-off exceeds k

$$\mathbb{P}(\widetilde{V}\ge k|\mathcal{F})=\int_{\mathcal{V}}\mathbb{I}_{\{k<v\le\overline{V}\}}dF_{\widetilde{V}|\mathcal{F}}(v)>0. \tag{A.2}$$

The integral over $\{v:p<v\le\overline{V}\}$ can be bounded below using:

$$\begin{aligned}\int_{\mathcal{V}}\mathbb{I}_{\{p<v\le\overline{V}\}}(v-p)\exp\{\omega(v-p)\}dF_{\widetilde{V}|\mathcal{F}}(v)=&\int_{\mathcal{V}}\mathbb{I}_{\{p<v<k\}}(v-p)\exp\{\omega(v-p)\}dF_{\widetilde{V}|\mathcal{F}}(v)\\&+\int_{\mathcal{V}}\mathbb{I}_{\{k\le v\le\overline{V}\}}(v-p)\exp\{\omega(v-p)\}dF_{\widetilde{V}|\mathcal{F}}(v)\\\ge&\int_{\mathcal{V}}\mathbb{I}_{\{k\le v\le\overline{V}\}}(v-p)\exp\{\omega(v-p)\}dF_{\widetilde{V}|\mathcal{F}}(v)\\\ge&(k-p)\int_{\mathcal{V}}\mathbb{I}_{\{k\le v\le\overline{V}\}}\exp\{\omega(v-p)\}dF_{\widetilde{V}|\mathcal{F}}(v)\end{aligned}\tag{A.3}$$

where the first equality splits up the integral, the first inequality disregards one of the integrals, which is positive, and the second inequality uses the fact that $v-p\ge k-p>0$ for $v>k$. Now, consider the limiting value of the integral in equation (A.3) as ω tends to η. There are two cases to consider, $\eta=\infty$ and $\eta<\infty$. If $\eta=\infty$, then it follows from Fatou's lemma that

$$\liminf_{\omega\uparrow\eta}\int_{\mathcal{V}}\mathbb{I}_{\{k\le v\le\overline{V}\}}\exp\{\omega(v-p)\}dF_{\widetilde{V}|\mathcal{F}}(v)\ge\int_{\mathcal{V}}\mathbb{I}_{\{k\le v\le\overline{V}\}}\liminf_{\omega\uparrow\eta}\exp\{\omega(v-p)\}dF_{\widetilde{V}|\mathcal{F}}(v)=\infty,$$

and, therefore, integral over $\{v:p<v\le\overline{V}\}$ becomes unboundedly large as ω tends to the upper boundary. If $\eta<\infty$, then since the mgf converges only on an open set, it follows that the mgf must diverge as ω approaches η

$$\lim_{\omega\uparrow\eta}\int_{\mathcal{V}}\exp\{\omega(v-p)\}dF_{\widetilde{V}|\mathcal{F}}(v)=\infty. \tag{A.4}$$

To apply this fact to demonstrate the limiting behaviour of the integral equation (A.3), split up the integral in equation (A.4) to write it as

$$\int_{\mathcal{V}} \exp\{\omega(v-p)\} dF_{\widetilde{V}|\mathcal{F}}(v) \tag{A.5}$$

$$= \int_{\mathcal{V}} \mathbb{I}_{\{v\leq p\}} \exp\{\omega(v-p)\} dF_{\widetilde{V}|\mathcal{F}}(v) + \int_{\mathcal{V}} \mathbb{I}_{\{p<v\}} \exp\{\omega(v-p)\} dF_{\widetilde{V}|\mathcal{F}}(v)$$

$$= \int_{\mathcal{V}} \mathbb{I}_{\{v\leq p\}} \exp\{\omega(v-p)\} dF_{\widetilde{V}|\mathcal{F}}(v) + \int_{\mathcal{V}} \mathbb{I}_{\{p<v<k\}} \exp\{\omega(v-p)\} dF_{\widetilde{V}|\mathcal{F}}(v)$$

$$+ \int_{\mathcal{V}} \mathbb{I}_{\{v\geq k\}} \exp\{\omega(v-p)\} dF_{\widetilde{V}|\mathcal{F}}(v). \tag{A.6}$$

The first two integrals in equation (A.6) remain bounded as $\omega \to \eta$, since for $\omega > 0$,

$$\int_{\mathcal{V}} \mathbb{I}_{\{v\leq p\}} \exp\{\omega(v-p)\} dF_{\widetilde{V}|\mathcal{F}}(v) \leq \int_{\mathcal{V}} \mathbb{I}_{\{v\leq p\}} dF_{\widetilde{V}|\mathcal{F}}(v) < \infty,$$

$$\int_{\mathcal{V}} \mathbb{I}_{\{p<v<k\}} \exp\{\omega(v-p)\} dF_{\widetilde{V}|\mathcal{F}}(v) \leq \exp\{\eta(k-p)\} \int_{\mathcal{V}} \mathbb{I}_{\{p<v<k\}} dF_{\widetilde{V}|\mathcal{F}}(v) < \infty.$$

Given that equation (A.4) is true, it must, therefore, be the case that

$$\lim_{\omega\uparrow\eta} \int_{\mathcal{V}} \mathbb{I}_{\{v\geq k\}} \exp\{\omega(v-p)\} dF_{\widetilde{V}|\mathcal{F}}(v) = \infty.$$

Returning to equation (A.3), one concludes that the integral over $\{v : p < v \leq \overline{V}\}$ becomes unboundedly large as $\omega \uparrow \eta$ since

$$\lim_{\omega\uparrow\eta} \int_{\mathcal{V}} \mathbb{I}_{\{p<v\leq\overline{V}\}} (v-p) \exp\{\omega(v-p)\} dF_{\widetilde{V}|\mathcal{F}}(v) \geq (k-p) \lim_{\omega\uparrow\eta} \int_{\mathcal{V}} \mathbb{I}_{\{k\leq v\leq\overline{V}\}} \exp\{\omega(v-p)\} dF_{\widetilde{V}|\mathcal{F}}(v)$$

$$= \infty.$$

Summarizing the above, it has been shown that for sufficiently large $\omega \in (0,\eta)$, the portfolio $x(\omega) = -\tau\omega$ satisfies

$$\int_{\mathcal{V}} (v-p) \exp\left\{-\frac{1}{\tau} x(\omega)(v-p)\right\} dF_{\widetilde{V}|\mathcal{F}}(v) > 0.$$

Running through the same sequence of steps above for $\omega \downarrow -\delta$, it follows that for sufficiently small $\omega \in (-\delta, 0)$, the portfolio $x(\omega)$ satisfies

$$\int_{\mathcal{V}} (v-p) \exp\left\{-\frac{1}{\tau} x(\omega)(v-p)\right\} dF_{\widetilde{V}|\mathcal{F}}(v) < 0.$$

Combining these results, the intermediate value theorem implies that there exists some finite portfolio x^* that satisfies the FOC

$$\int_{\mathcal{V}} (v-p) \exp\left\{-\frac{1}{\tau} x^*(v-p)\right\} dF_{\widetilde{V}|\mathcal{F}}(v) = 0.$$

‖

The next Lemma applies Lemma A9 to the partial equilibrium portfolio problems of the informed and uninformed investors and establishes an equivalence between $(\underline{V}, \overline{V})$, and the range of the function g_I' that appears in the informed investor's beliefs.

Lemma A10. *Suppose that Assumptions 1–5 hold and that the uninformed investor observes $\widetilde{S}_U$. Fix any realization of $(\widetilde{S}_I, \widetilde{Z}) = (s_I, z)$ and the associated realization $\widetilde{S}_U = s_U = s_I - \frac{1}{k_I \tau_I} z$. The following statements are equivalent.*

- *(a) The informed investor's portfolio problem has a finite optimum and a well defined, unique optimal portfolio given by the solution to his/her FOC.*
- *(b) The uninformed investor's portfolio problem has a finite optimum and a well defined, unique optimal portfolio given by the solution to his/her FOC.*
- *(c) The price p lies in the interior of the convex hull of $\mathcal{V}$, $p \in (\underline{V}, \overline{V})$.*
- *(d) The price p lies in the range of the function $g_I'(\cdot)$, $p \in g_I'(\mathcal{G}_I)$.*

Proof I begin by proving $(a) \Longleftrightarrow (c)$. Under Assumptions 2–3, there is a single informed investor, whose conditional beliefs are of the exponential family form. Lemma A6 shows that his/her conditional mgf of $\widetilde{V}$ is finite only on the set $\{\omega : \omega + k_I s_I \in \mathcal{G}_I\}$, which under Assumption 4 is an open interval. From Lemma A9, it follows that statements (a) and (c) are equivalent.

To show $(b) \Longleftrightarrow (c)$, I again will invoke Lemma A9. Under Assumption 5, the mgf of $\widetilde{V}$, conditional on $\widetilde{S}_U$ is finite only in a non-empty open interval. From Lemma A9 it follows that statements (b) and (c) are equivalent.

To conclude, I show $(a) \Longleftrightarrow (d)$. I showed in the first part of the proof that all of the assumptions of Lemma A9 are satisfied for the informed investor. Hence, he/she has a finite optimal portfolio if and only if there is a portfolio x^* that satisfies his/her FOC. His/her objective function is

$$\max_{x\in\mathbb{R}} \mathbb{E}\left[-\exp\left\{-\frac{1}{\tau_I}x(\widetilde{V}-p)\right\}|\widetilde{S}_I = s_I\right] = \max_{x\in\mathbb{R}} -\exp\left\{\frac{1}{\tau_I}xp + g_I\left(k_I s_I - \frac{1}{\tau_I}x\right) - g_I(k_I s_I)\right\},$$

where the equality uses Lemma A6 to write out the expectation explicitly.

There is a unique, finite x^* that achieves this maximum if and only if there exists a unique, finite x^* that solves the FOC

$$g_I'\left(k_I s_I - \frac{1}{\tau_I}x^*\right) = p.$$

An x^* that satisfies the FOC exists if and only if $p \in g_I'(\mathcal{G}_I)$ — that is, if and only if $(g_I')^{-1}(p)$ exists. Lemma A7 guarantees that g_I' is strictly increasing and therefore that this x^* is unique when it exists. ‖

A.2.2. Proofs of results on investors' partial equilibrium demand functions.

Proof (Lemma 1). It was shown in the proof of Lemma A10 that as long as $p \in (\underline{V}, \overline{V})$ the informed investor's optimal portfolio is characterized by the FOC

$$g_I'\left(k_I s_I - \frac{1}{\tau_I}X_I(s_I,p)\right) = p$$

$$\Rightarrow \quad X_I(s_I,p) = \tau_I(k_I s_I - (g_I')^{-1}(p)),$$

Defining the price reaction function, $G_I = (g_I')^{-1}$, produces the expression in the Lemma. ‖

Proof (Lemma 2). It suffices to show that $P(s_I,z) = P(\hat{s}_I,\hat{z})$ implies that $s_I - \frac{z}{k_I\tau_I} = \hat{s}_I - \frac{\hat{z}}{k_I\tau_I}$. Suppose that (s_I,z) and $(\hat{s}_I,\hat{z})$ satisfy $P(s_I,z) = P(\hat{s}_I,\hat{z})$. Lemma 1 demonstrated that under Assumptions 1–4, the informed investor's demand function is $\tau_I(k_I s_I - G_I(p))$. Hence, the market-clearing condition requires

$$\begin{aligned}
\bar{z} + z - \tau_I(k_I s_I - G_I(P(s_I,z))) &= X_U(P(s_I,z)) \\
&= X_U(P(\hat{s}_I,\hat{z})) \\
&= \bar{z} + \hat{z} - \tau_I(k_I \hat{s}_I - G_I(P(\hat{s}_I,\hat{z}))) \\
&= \bar{z} + \hat{z} - \tau_I(k_I \hat{s}_I - G_I(P(s_I,z))),
\end{aligned}$$

where the first equality follows from the market-clearing condition in the state (s_I,z), the second equality uses the fact that $P(s_I,z) = P(\hat{s}_I,\hat{z})$, the third equality follows from market clearing in the state $(\hat{s}_I,\hat{z})$, and the last equality again uses $P(s_I,z) = P(\hat{s}_I,\hat{z})$.

The above chain of equalities implies that

$$s_I - \frac{z}{k_I\tau_I} = \hat{s}_I - \frac{\hat{z}}{k_I\tau_I}.$$

‖

The following Lemma presents an expression for the uninformed investor's conditional beliefs.

Lemma A11. *The conditional distribution of $\widetilde{V}$ given $\widetilde{S}_U = s_U$, $F_{\widetilde{V}|\widetilde{S}_U}(v|s_U)$, is given by*

$$dF_{\widetilde{V}|\widetilde{S}_U}(v|s_U) = K(v,s_U)\,dH_I(v), \tag{A.7}$$

where

$$K(v,s_U) = \frac{\int_{\mathcal{S}} \mathbb{I}\left\{x \in s_U + \frac{1}{k_I\tau_I}\mathcal{Z}\right\} f_Z(k_I\tau_I(x-s_U))\exp\{k_I x v - g_I(k_I x)\}\,dF_{\widetilde{S}_I}(x)}{\int_{\mathcal{S}} \mathbb{I}\left\{x \in s_U + \frac{1}{k_I\tau_I}\mathcal{Z}\right\} f_Z(k_I\tau_I(x-s_U))\,dF_{\widetilde{S}_I}(x)}.$$

Proof (Lemma A11). Since $\widetilde{S}_U$ is a convolution of $\widetilde{S}_I$ and $-\frac{1}{k_I\tau_I}\widetilde{Z}$, the conditional distribution of $\widetilde{S}_I$ given $\widetilde{S}_U$ is

$$dF_{\widetilde{S}_I|\widetilde{S}_U}(s_I|s_U)=\frac{\mathbb{I}\left\{s_I\in s_U+\frac{1}{k_I\tau_I}\mathcal{Z}\right\}f_Z\left(k_I\tau_I(s_I-s_U)\right)dF_{\widetilde{S}_I}(s_I)}{\int_{\mathcal{S}}\mathbb{I}\left\{x\in s_U+\frac{1}{k_I\tau_I}\mathcal{Z}\right\}f_Z\left(k_I\tau_I(x-s_U)\right)dF_{\widetilde{S}_I}(x)},$$

where the indicator function makes explicit the fact that depending on the support $\mathcal{Z}$, observation of $\widetilde{S}_U$ may reveal that $\widetilde{S}_I$ lies in some proper subset of $\mathcal{S}$.

Integrating the conditional cdf of $\widetilde{V}$ given $\widetilde{S}_I$ against this distribution produces the conditional cdf of $\widetilde{V}$ given $\widetilde{S}_U$

$$\begin{aligned}
&F_{\widetilde{V}|\widetilde{S}_U}(v|s_U)\\
&=\int_{\mathcal{S}}F_{\widetilde{V}|\widetilde{S}_I}(v|x)dF_{\widetilde{S}_I|\widetilde{S}_U}(x|s_U)\\
&=\frac{\int_{\mathcal{S}}\left[\int_{\mathcal{V}}\mathbb{I}\{y\le v\}\exp\{k_Ixy-g_I(k_Ix)\}dH_I(y)\right]\mathbb{I}\left\{x\in s_U+\frac{1}{k_I\tau_I}\mathcal{Z}\right\}f_Z\left(k_I\tau_I(x-s_U)\right)dF_{\widetilde{S}_I}(x)}{\int_{\mathcal{S}}\mathbb{I}\left\{x\in s_U+\frac{1}{k_I\tau_I}\mathcal{Z}\right\}f_Z\left(k_I\tau_I(x-s_U)\right)dF_{\widetilde{S}_I}(x)}\\
&=\frac{\int_{\mathcal{V}}\mathbb{I}\{y\le v\}\int_{\mathcal{S}}\mathbb{I}\left\{x\in s_U+\frac{1}{k_I\tau_I}\mathcal{Z}\right\}f_Z\left(k_I\tau_I(x-s_U)\right)\exp\{k_Ixy-g_I(k_Ix)\}dF_{\widetilde{S}_I}(x)dH_I(y)}{\int_{\mathcal{S}}\mathbb{I}\left\{x\in s_U+\frac{1}{k_I\tau_I}\mathcal{Z}\right\}f_Z\left(k_I\tau_I(x-s_U)\right)dF_{\widetilde{S}_I}(x)},
\end{aligned}$$

where the third equality holds because the integrand in the numerator is positive, and hence Fubini's Theorem allows for changing the order of integration.

This completes the proof since the expression in the Lemma is seen to be the Lebesgue–Stieltjes differential corresponding to this conditional cdf. ‖

Proof (Lemma 3). The uninformed investor's optimization problem is

$$\max_x\int_{\mathcal{V}}-\exp\left\{-\frac{1}{\tau_U}x(v-p)\right\}dF_{\widetilde{V}|\widetilde{S}_U}(v|s_U),$$

where $F_{\widetilde{V}|\widetilde{S}_U}$ is characterized in Lemma A11.

Lemma A10 implies that for $p\in(\underline{V},\overline{V})$ the optimal demand exists, is unique, and is characterized by the FOC

$$\int_{\mathcal{V}}(v-p)\exp\left\{-\frac{1}{\tau_U}X_U(s_U,p)(v-p)\right\}dF_{\widetilde{V}|\widetilde{S}_U}(v|s_U)=0.$$

Dividing out $\exp\left\{\frac{1}{\tau_U}X_U(s_U,p)p\right\}$, which is constant with respect to v, produces equation (3.8). ‖

At this point it will be useful to provide two results which characterize how informed and uninformed investor demand depends on the signals s_I and s_U, and the price p.

Lemma A12. *Suppose that Assumptions 1–5 hold and that the uninformed investor's information set consists only of $\widetilde{S}_U$. Fix s_I and s_U. Both the informed and uninformed investors' optimal portfolios are continuous and strictly decreasing in p.*

Proof (Lemma A12). Since investors have CARA utility, Proposition 1 of Cheng *et al.* (1987) implies that demand functions are strictly decreasing in price. The intuition for the result is simple—holding information constant, a change in price has both income and substitution effects. The assumption of CARA utility precludes income effects, and with only a substitution effect, demand must decrease with price.

Continuity for the demand of the informed investor is immediate upon inspection of his/her optimal demand in equation (3.6). For the uninformed investor, recall that the FOC uniquely defines his/her demand function according to

$$\int_{\mathcal{V}}(v-p)\exp\left\{-\frac{1}{\tau_U}X_U(s_U,p)(v-p)\right\}dF_{\widetilde{V}|\widetilde{S}_U}(v|s_U)=0.$$

The FOC can be written in terms of conditional mgfs as

$$M'_{\widetilde{V}|\widetilde{S}_U}\left(-\frac{1}{\tau_U}X_U(s_U,p)|s_U\right)-pM_{\widetilde{V}|\widetilde{S}_U}\left(-\frac{1}{\tau_U}X_U(s_U,p)|s_U\right)=0.$$

The conditional mgf and its derivative are continuous functions of their argument for fixed s_U and the FOC is continuous in p. Furthermore, demand is uniquely defined for fixed p. It follows from the implicit function theorem that the function $X_U(s_U,p)$ defined implicitly by this expression is itself continuous in p. ‖

Lemma A13. *Suppose that Assumptions 1–5 and 7 hold and that the uninformed investor's information set consists only of $\widetilde{S}_U$. Fix $p \in (\underline{V}, \overline{V})$. The informed investor's optimal portfolio $X_I(s_I, p)$ is strictly increasing in s_I, and the uninformed investor's optimal portfolio $X_U(s_U, p)$ is increasing in s_U.*

To prove Lemma A13, I introduce a new definition and establish a property of uninformed investor beliefs that is needed for the result. The following definition follows Definition 1 of Jewitt (1987) (or Definition 1.1 in Chap. 2 of Karlin (1968)).

Definition 4 (Total positivity). *A function $K : X \times Y \subseteq \mathbb{R}^2 \to \mathbb{R}$ is* totally positive of order 2 (TP_2) *if $K(x,y) \geq 0$ and for any choice of $x_1 < x_2$ and $y_1 < y_2$,*

$$\begin{vmatrix} K(x_1,y_1) & K(x_1,y_2) \\ K(x_2,y_1) & K(x_2,y_2) \end{vmatrix} \geq 0,$$

where $|\cdot|$ represents the determinant.[25]

Total positivity of order 2 goes by several different names in the literature, including: the monotone likelihood ratio property (Milgrom, 1981), affiliation (Milgrom and Weber, 1982), and log-supermodularity (Athey, 2002). The condition captures a notion of good news (Milgrom, 1981). If the conditional pdf of $\widetilde{V}$ given $\widetilde{S}_U$ is TP_2 then increases in the realization $\widetilde{S}_U = s_U$ produce upward shifts in the conditional distribution of $\widetilde{V}$ under the likelihood ratio stochastic ordering, which is essentially a stronger version of the first-degree stochastic dominance order.

The following Proposition records the fact that the TP_2 property is preserved under composition.

Proposition 8 (Composition formula for totally positive functions). *Let $M : X \times W \to \mathbb{R}$ and $L : W \times Y \to \mathbb{R}$ be TP_2 and suppose that $h : W \to \mathbb{R}$ is non-decreasing. Then the function*

$$K(x,y) = \int M(x,w)L(w,y)dh(w)$$

is TP_2.

The proof is provided in Karlin (1968), so I omit it here. The intuition can be easily seen by considering the case in which M and L are conditional densities. Suppose that the conditional distributions of $\widetilde{X}$ given $\widetilde{W}$ and $\widetilde{W}$ given $\widetilde{Y}$ are TP_2—increases in $\widetilde{W}$ are good news about $\widetilde{X}$ and increases in $\widetilde{Y}$ are good news about $\widetilde{W}$. The composition formula implies that increases in $\widetilde{Y}$ are also good news about $\widetilde{X}$. In the context of the model, increases in the informed investor's signal $\widetilde{S}_I$ are good news about the pay-off $\widetilde{V}$ and increases in the uninformed investor's signal $\widetilde{S}_U$ are good news about $\widetilde{S}_I$. Hence, increases in $\widetilde{S}_U$ are good news about $\widetilde{V}$.

The following Corollary records the fact that the uninformed investor's conditional distribution is TP_2.

Corollary 3 (Corollary to Lemma A11). *Under Assumption 7, the function $K(v, s_U)$ that appears in the conditional distribution $F_{\widetilde{V}|\widetilde{S}_U}(v|s_U)$ in Lemma A11 is TP_2.*

Proof (Corollary 3). Let

$$K(v,s_U) = \frac{\int_{\mathcal{S}} \mathbb{I}\left\{x \in s_U + \frac{1}{k_I \tau_I}\mathcal{Z}\right\} f_Z(k_I \tau_I (x - s_U)) \exp\{k_I x v - g_I(k_I x)\} dF_{\widetilde{S}_I}(x)}{\int_{\mathcal{S}} \mathbb{I}\left\{x \in s_U + \frac{1}{k_I \tau_I}\mathcal{Z}\right\} f_Z(k_I \tau_I (x - s_U)) dF_{\widetilde{S}_I}(x)}$$

be the function in the integral in the uninformed investors' beliefs.

Notice first that the denominator of K is positive and is a function only of s_U and not v, so its behaviour does not affect whether the function K is totally positive. Hence, it suffices to show that the numerator $\int_{\mathcal{S}} \mathbb{I}\left\{x \in s_U + \frac{1}{k_I \tau_I}\mathcal{Z}\right\} f_Z(k_I \tau_I (x - s_U)) \exp\{k_I x v - g(k_I x)\} dF_{\widetilde{S}_I}(x)$ is TP_2 in (v, s_U). The indicator function is TP_2 in (x, s_U) (Athey (2002), Lemma 3), and under Assumption 7, the function $f_Z(k_I \tau_I (x - s_U))$ is TP_2 in (x, s_U). Hence, $\mathbb{I}\left\{x \in s_U + \frac{1}{k_I \tau_I}\mathcal{Z}\right\} f_Z(k_I \tau_I (x - s_U))$ is TP_2 in (x, s_U). Furthermore, $\exp\{k_I x v - g(k_I x)\}$ is TP_2 in (v, x) because $\frac{\partial^2}{\partial v \partial x} \log \exp\{k_I x v - g(k_I x)\} > 0$. Hence, it follows from the composition formula that the numerator is TP_2 in (v, s_U) and therefore that $K(v, s_U)$ is TP_2. ||

25. In the case that K is strictly positive and twice differentiable, then K is TP_2 if and only if $\frac{\partial^2}{\partial x \partial y} \log K(x,y) \geq 0$. See, *e.g.* (Jewitt, 1987 or Karlin, 1968).

I now finally return to the proof of Lemma A13.

Proof (Lemma A13). The result for the informed investor is immediate upon inspection of his/her portfolio in equation (3.6). For the uninformed investor, Corollary 3 shows that the function $K(v,s_U)$ that appears in his/her beliefs in Lemma A11 is TP_2. It, therefore, follows from Proposition 2 of Landsberger and Meilijson (1990) that uninformed demand is increasing in s_U. ‖

A.2.3. Proofs of existence and uniqueness results.

Proof (Proposition 2). Fix realizations $(\widetilde{S}_I,\widetilde{Z})=(s_I,z)$ and the associated realization $s_U=s_I-\frac{1}{k_I\tau_I}z$. It suffices to show that there exists some price $p\in(\underline{V},\overline{V})$ for which the market-clearing condition holds

$$X_I(s_I,p)+X_U(s_U,p)-(z+\bar{z})=0.$$

I will use an intermediate value theorem argument to demonstrate existence of such a p. Define $p_0=\mathbb{E}[\widetilde{V}|\widetilde{S}_U=s_U]$. I claim that $p_0\in(\underline{V},\overline{V})$. When the price is p_0, the uninformed investor perceives the risky asset as having zero risk premium and his/her optimal demand is equal to zero. Hence, Lemma A10, which showed the equivalence of finite demand and prices lying in $(\underline{V},\overline{V})$, establishes $p_0\in(\underline{V},\overline{V})$. Consider also $p_I=g_I'(k(s_U-\frac{1}{k_I\tau_I}\bar{z}))$, which would be the equilibrium price if there were only informed investors in the economy.[26] Assumption 6 guarantees $k_I(s_U-\frac{1}{k_I\tau_I}\bar{z})\in\mathcal{G}_I$, and hence $p_I\in g_I'(\mathcal{G}_I)$. Lemma A10, therefore, implies $p_I\in(\underline{V},\overline{V})$.

Let $\underline{p}=\min\{p_0,p_I\}$ and $\overline{p}=\max\{p_0,p_I\}$. The interval $[\underline{p},\overline{p}]$ lies in $(\underline{V},\overline{V})$ due to the fact that both p_0 and p_I do. I will show that there exists a market-clearing price in this interval. If $p_0=p_I$, then the proof is done since this price clears the market. For $p_0\neq p_I$, note that

$$X_I(s_I,\underline{p})+X_U(s_U,\underline{p})-(z+\bar{z})\geq X_I(s_I,p_I)+X_U(s_U,p_0)-(z+\bar{z})=0$$

$$X_I(s_I,\overline{p})+X_U(s_U,\overline{p})-(z+\bar{z})\leq X_I(s_I,p_I)+X_U(s_U,p_0)-(z+\bar{z})=0,$$

where the first line follows because $\underline{p}\leq p_0,p_I$ and demand functions are strictly decreasing in price by Lemma A12. The second line follows similarly because $\overline{p}\geq p_0,p_I$.

Since Lemma A12 established that both investors' demand functions are continuous in p, the existence of a p at which the market-clearing condition holds now follows from the intermediate value theorem. Moreover, because aggregate demand is *strictly* decreasing in p, this market-clearing price is in fact unique for each fixed s_U.

To complete the proof of existence, it must be shown that the price function characterized here is one-to-one. Consider the function on the left-hand side of the market-clearing condition

$$\tau_I(ks_U-G_I(p))+X_U(s_U,p)-\bar{z}=0.$$

Under Assumption 7, X_U is increasing in s_U, therefore, this function is strictly increasing in s_U. This function is also strictly decreasing in p owing to Lemma A12. Hence, the equilibrium price function must be strictly increasing in s_U and therefore one-to-one. This completes the proof of existence.

It remains to demonstrate continuity of the price function. The first part of the proof showed that there exists a price function defined implicitly by

$$\tau_I(k_Is_U-G_I(P(s_U)))+X_U(s_U,P(s_U))-\bar{z}=0.$$

Lemma A12 established that the function in this expression is continuous and strictly decreasing in price, so that there is a unique market-clearing price corresponding to each s_U. The desired conclusion will, therefore, follow from the implicit function theorem if it can be shown that X_U is jointly continuous in s_U,p. Consider the FOC that defines the uninformed investor's optimal demand implicitly

$$\int_{\mathcal{V}}(v-p)\exp\left\{-\frac{1}{\tau_U}X_U(s_U,p)v\right\}K(v,s_U)dH_I(v)=0,$$

where I substitute in for $dF_{\widetilde{V}|\widetilde{S}_U}$ from Lemma A11. Assumption 8 guarantees that $K(v,s_U)$ is continuous in s_U for each v, so the integrand is continuous in (s_U,p). Since the log concavity of $\widetilde{Z}$ implies that the conditional distribution of $\widetilde{V}$

26. In that case, the market clearing condition is

$$\tau_I(k_Is_I-G_I(p))=z+\bar{z}\Rightarrow p=G_I^{-1}\left(k_Is_I-\frac{1}{\tau_I}(z+\bar{z})\right)$$
$$\Rightarrow p=g_I'\left(k_I\left(s_U-\frac{1}{k_I\tau_I}\bar{z}\right)\right),$$

where the second line follows because $G_I^{-1}=g_I'$ and $s_U=s_I-\frac{1}{k_I\tau_I}z$.

has tails that are at most exponential, an application of the dominated convergence theorem implies that passing limits through the integral sign is allowed. It follows that the integral is a continuous function of (s_U, p). Therefore, from the implicit function theorem applied to the FOC, the uninformed investor's demand function is continuous in (s_U, p). Again applying the implicit function theorem, this time to the market-clearing condition, implies that the equilibrium price is continuous in s_U. ‖

Proof (Corollary 1). Under Assumptions 1–7 existence is guaranteed, so it remains to verify the expression for the price. The proof proceeds similarly to that of Proposition 1 and could alternately be derived as a special case, so I omit some detail.

From Lemma 1 the informed investor's demand function is

$$X_I(s_I, p) = \tau_I (k_I s_I - G_I(p)),$$

and since Assumption 9 requires uninformed investor beliefs to have the exponential family form, his/her demand takes a similar form

$$X_U(s_U, p) = \tau_U (k_U b_U(s_U) - G_U(p)).$$

Imposing market clearing gives

$$\tau_I (k_I s_I - G_I(p)) + \tau_U (k_U b_U(s_U) - G_U(p)) = \bar{z} + z$$

$$\Rightarrow \tau_I (k_I s_U - G_I(p)) + \tau_U (k_U b_U(s_U) - G_U(p)) = \bar{z}$$

$$\Rightarrow \tau_I k_I s_U + \tau_U k_U b_U(s_U) - \bar{z} = G(p),$$

where the second line follows from grouping the terms involving s_I and z into a single one involving s_U and the third line rearranges and uses the definition of the aggregate price reaction function, $G = \tau_I G_I + \tau_U G_U$. Applying G^{-1} to both sides of the final line produces the expression in the Corollary. ‖

The following lemma is required for the proof of Lemma 4.

Lemma A14. *Suppose that Assumptions 1–4 hold and that $\mathcal{S}$ and $\mathcal{Z}$ are intervals. Then any continuous equilibrium price function is strictly monotone in s_I for any fixed z and is strictly monotone in z for any fixed s_I.*

Proof (Lemma A14). I prove only the first statement in the proof. The proof of the second is analogous.

Fix any $z \in \mathcal{Z}$. Given that $P(\cdot, z)$, considered as a function of its first argument, is assumed to be continuous and defined on an interval, it suffices to show that $P(\cdot, z)$ is one-to-one in the first argument. Assume that $P(\cdot, z)$ is not one-to-one. Then, there exist distinct possible realizations $s \neq \hat{s}$ of $\widetilde{S}_I$ with $P(s, z) = P(\hat{s}, z)$.

Under Assumptions 1–4, the informed investor's demand function takes the form from Lemma 1. Plugging into the market-clearing condition implies that the existence of such $s \neq \hat{s}$ would imply

$$\begin{aligned}
0 &= X_U(P(s,z)) + \tau_I (k_I s - G_I(P(s,z))) - z - \bar{z} \\
&= X_U(P(\hat{s},z)) + \tau_I \left(k_I s - G_I(P(\hat{s},z))\right) - z - \bar{z} \\
&\neq X_U(P(\hat{s},z)) + \tau_I \left(k_I \hat{s} - G_I(P(\hat{s},z))\right) - z - \bar{z} \\
&= 0,
\end{aligned}$$

where the first equality follows from market clearing in state (s,z), the second equality holds because $P(s,z) = P(\hat{s},z)$, the inequality follows from $s \neq \hat{s}$, and the final equality is the market-clearing condition in state $(\hat{s},z)$. ‖

Proof (Lemma 4). Let $P(\cdot)$ be an equilibrium price function. Assume to the contrary that there exists some $s_U^0 \in$ Support$(\widetilde{S}_U)$ along which P is not constant. Let $Q = \{P(s_I, z) : s_I - \frac{z}{k_I \tau_I} = s_U^0\}$ be the set of values that P takes for states along s_U^0. Take any $p \in Q$. Then there exists $(s_I^0, z^0) \in Q$ with $P(s_I^0, z^0) = p$ and by continuity of P there exists a sequence $(s_I^n, z^n) \in Q$ converging to (s_I^0, z^0) such that $P(s_I^n, z^n) \to P(s_I, z)$. By the assumption that P is not constant along s_U^0 and is continuous, the point (s_I^0, z^0) and this sequence can be chosen so that $P(s_I^n, z^n) \neq P(s_I^0, z^0)$ for all n and $P(s_I^n, z^n)$ converges monotonically to $P(s_I^0, z^0)$. Without loss of generality, assume $P(s_I^n, z^n) \downarrow P(s_I, z)$. Since the Assumptions guarantee that the informed investor's demand takes the linear form in Lemma 1, it follows from the market-clearing condition and the fact that $s_I^n - \frac{z^n}{k_I \tau_I} = s_U^0$ for all n that

$$\tau_I (k_I s_U^0 - G_I(P(s_I^n, z^n))) + X_U(P(s_I^n, z^n)) = \bar{z} \quad \forall n, \text{ and} \tag{A.8}$$

$$\tau_I (k_I s_U^0 - G_I(P(s_I^0, z^0))) + X_U(P(s_I^0, z^0)) = \bar{z}. \tag{A.9}$$

Subtracting line (A.8) from line (A.9) and rearranging yields

$$X_U(P(s_I^n,z^n))-X_U(P(s_I^0,z^0))=\tau_I\left[G_I(P(s_I^n,z^n))-G_I(P(s_I^0,z^0))\right] \quad \forall n. \tag{A.10}$$

Now, keep z^0 fixed and let $\hat{s}_I^n$ be any sequence such that $\hat{s}_I^n$ converges (strictly) monotonically to s_I^0 and $P(\hat{s}_I^n,z^0)=P(s_I^n,z^n)$ for all n sufficiently large, $n\geq N$. Such a sequence exists because Lemma A14 implies that P must be strictly monotone and continuous in s_I for fixed z^0, and $P(s_I^n,z^n)$ is decreasing in n by construction. For $\hat{s}_I^n$ the market-clearing condition implies that

$$\tau_I\left(k_I\hat{s}_I^n-\frac{z^0}{\tau_I}-G_I(P(\hat{s}_I^n,z^0))\right)+X_U(P(\hat{s}_I^n,z^0))=\bar{z} \quad \forall n \tag{A.11}$$

Subtract line (A.9) from line (A.11) and use the fact that $s_U^0=s_I^0-\frac{z^0}{k_I\tau_I}$, to yield

$$X_U(P(\hat{s}_I^n,z^0))-X_U(P(s_I^0,z^0))-\tau_I\left[G_I(P(\hat{s}_I^n,z^0))-G_I(P(s_I^0,z^0))\right]=-\tau_I(\hat{s}_I^n-s_I^0) \quad \forall n. \tag{A.12}$$

Now, use the fact that $P(\hat{s}_I^n,z^0)=P(s_I^n,z^n)$ for $n\geq N$ and plug into (A.12) to produce

$$X_U(P(s_I^n,z^n))-X_U(P(s_I^0,z^0))-\tau_I\left[G_I(P(s_I^n,z^n))-G_I(P(s_I^0,z^0))\right]=-\tau_I(\hat{s}_I^n-s_I^0) \quad \forall n\geq N. \tag{A.13}$$

Using equation (A.10) to substitute for the left-hand side of equation (A.13) gives

$$0=-\tau_I(\hat{s}_I^n-s_I^0) \quad \forall n\geq N,$$

which is a contradiction because $\hat{s}_I^n\neq s_I^0$ for all n. ‖

Proof (Proposition 3). It was established in Proposition 2 that under Assumptions 1–8 the function $P(s_U)$ defined by equation (3.10) exists and is continuous in s_U. It was also shown in the proof of that result that for fixed s_U, there exists a unique market-clearing price. This establishes that any price function that depends on (s_I,z) only through s_U is uniquely defined by equation (3.10). It remains to establish that any continuous price function can depend only on s_U. However, since $\mathcal{S}$ and $\mathcal{Z}$ are intervals Lemma 4 guarantees this. Hence, the price function in equation (3.10) is the unique continuous price function. ‖

A.3. *Equilibrium derivation in $N=1$ case of Examples 1 and 2*

Proof (Example 1). Before deriving the equilibrium, I will verify that Assumptions 4–9 are met. Assumption 3 was verified in Section 3 in the text.

First, consider Assumption 4, which requires that $\mathcal{G}_I$, the support of g_I, be an open set. From the derivation of the informed investor beliefs in equation (3.2) in the text, the support of g_I is $\mathcal{G}_I=\mathbb{R}$, which is open.

Next, consider Assumption 5, which requires that the uninformed investor's conditional mgf converge on an open set. This assumption is also met since the uninformed investor's conditional distribution is binomial, and any binomial distribution has an mgf that converges on the entire real line (Casella and Berger, 2002). To derive explicitly the uninformed investor's conditional beliefs about $\widetilde{V}$, note that because $\widetilde{S}_U=\widetilde{S}_I-\frac{V_H-V_L}{\tau_I}\widetilde{Z}$ is a convolution the conditional distribution of $\widetilde{S}_I$ given $\widetilde{S}_U=s_U$ follows easily from Bayes' rule

$$f_{\widetilde{S}_I|\widetilde{S}_U}(s_I|s_U)=\frac{f_{\widetilde{S}_U|\widetilde{S}_I}(s_U|s_I)f_{\widetilde{S}_I}(s_I)}{f_{\widetilde{S}_U}(s_U)}=\frac{\frac{\tau_I}{V_H-V_L}f_{\widetilde{Z}}\left(\frac{\tau_I}{V_H-V_L}(s_U-s_I)\right)f_{\widetilde{S}_I}(s_I)}{f_{\widetilde{S}_U}(s_U)}, \tag{A.14}$$

where the marginal of $\widetilde{S}_U$ is

$$f_{\widetilde{S}_U}(s_U)=\int_{-\infty}^{\infty}\frac{\tau_I}{V_H-V_L}f_{\widetilde{Z}}\left(\frac{\tau_I}{V_H-V_L}(s_U-x)\right)f_{\widetilde{S}_I}(x)dx.$$

It is helpful to keep track of the support of this conditional distribution explicitly. This support will typically vary with the realization s_U if the supply shock is distributed on any set other than the entire real line. Let $\underline{Z}$ and $\overline{Z}$ denote the endpoints of the (potentially infinite) interval $\mathcal{Z}$ on which $\widetilde{Z}$ is supported, and let $\underline{s}(s_U)=s_U+\frac{V_H-V_L}{\tau_I}\underline{Z}$ and $\bar{s}(s_U)=s_U+\frac{V_H-V_L}{\tau_I}\overline{Z}$ be the corresponding endpoints of the support of the conditional distribution of $\widetilde{S}_I$.

Integrating over the density for $\widetilde{S}_I$ derived above, the uninformed investor's conditional pdf for the pay-off $\widetilde{V}$ is thus

$$\begin{aligned}f_{\widetilde{V}|\widetilde{S}_U}(v|s_U)&=\int_{\underline{s}(s_U)}^{\bar{s}(s_U)}f_{\widetilde{V}|\widetilde{S}_I}(v|x)f_{\widetilde{S}_I|\widetilde{S}_U}(x|s_U)dx\\&=\int_{\underline{s}(s_U)}^{\bar{s}(s_U)}\frac{\exp\left\{x\frac{v-V_L}{V_H-V_L}\right\}}{1+\exp\{x\}}\frac{\frac{\tau_I}{V_H-V_L}f_{\widetilde{Z}}\left(\frac{\tau_I}{V_H-V_L}(s_U-x)\right)f_{\widetilde{S}_I}(x)}{f_{\widetilde{S}_U}(s_U)}\mathbb{I}\{v\in\{V_L,V_H\}\}dx\end{aligned} \tag{A.15}$$

which is a binomial distribution with log-odds

$$b_U(s_U) \equiv \log\left(\frac{\int_{\underline{s}(s_U)}^{\overline{s}(s_U)} \frac{\exp\{x\}}{1+\exp\{x\}} f_{\widetilde{Z}}\left(\frac{\tau_I}{V_H-V_L}(s_U-x)\right) f_{\widetilde{S}_I}(x)dx}{\int_{\underline{s}(s_U)}^{\overline{s}(s_U)} \frac{1}{1+\exp\{x\}} f_{\widetilde{Z}}\left(\frac{\tau_I}{V_H-V_L}(s_U-x)\right) f_{\widetilde{S}_I}(x)dx}\right). \tag{A.16}$$

Therefore, as in equation (3.2) for the informed investor, the pdf can be condensed into the form

$$f_{\widetilde{V}|\widetilde{S}_U}(v|s_U) = \exp\left\{\frac{1}{V_H-V_L} b_U(s_U)v - \frac{1}{V_H-V_L} b_U(s_U)V_L - \log\left(1+\exp\left\{(V_H-V_L)\frac{b_U(s_U)}{V_H-V_L}\right\}\right)\right\}$$
$$\times \mathbb{I}\{v \in \{V_L, V_H\}\}. \tag{A.17}$$

To verify the remaining assumptions, first consider Assumption 6, which restricted the supports of the distributions. This assumption is met because $\mathcal{G}_I = \mathbb{R}$ implies that $k_I\mathcal{S} - \frac{1}{\tau_I}\mathcal{Z} - \frac{1}{\tau_I}\bar{z} \subseteq \mathcal{G}_I$, regardless of the values of $\mathcal{S}, \mathcal{Z}$, and $\bar{z}$. Assumption 7, which restricted the distribution of the supply shock $\widetilde{Z}$ to have a log-concave density is satisfied by assumption. Finally, Assumption 8 on the continuity of uninformed investor beliefs is met. From the expression in equation (A.17), $f_{\widetilde{V}|\widetilde{S}_U}(v|s_U)$ will be continuous in s_U as long as the log-odds $b_U(s_U)$ are. Since the supply shock $\widetilde{Z}$ has a continuously differentiable density function, the integrands that appear in the log-odds are continuous, and because f_Z is log-concave, and therefore has tails that are at most exponential, the dominated convergence theorem allows us to pass limits through the integrals in equation (A.16) and conclude that $f_{\widetilde{V}|\widetilde{S}_U}(v|s_U)$ is in fact continuous in s_U.

Since Assumptions 1–8 are met, Proposition 2 guarantees that an equilibrium exists and that the price function is continuous. Furthermore, since $\widetilde{S}_I$ and $\widetilde{Z}$ are continuously distributed on intervals, Proposition 3 guarantees that this equilibrium is unique.

There are two ways to derive the price function. The first is to use the implicit characterization technique used in Proposition 1. The second is to verify that Assumption 9 on uninformed investor beliefs is met and then use the explicit characterization from Corollary 1. I use the second method here.

By inspection of equation (A.17), the uninformed pdf is in the form in Assumption 9 with

$$k_U = \frac{1}{V_H - V_L} \tag{A.18}$$

$$b_U(s_U) = \log\left(\frac{\int_{\underline{s}(s_U)}^{\overline{s}(s_U)} \frac{\exp\{x\}}{1+\exp\{x\}} f_{\widetilde{Z}}\left(\frac{\tau_I}{V_H-V_L}(s_U-x)\right) f_{\widetilde{S}_I}(x)dx}{\int_{\underline{s}(s_U)}^{\overline{s}(s_U)} \frac{1}{1+\exp\{x\}} f_{\widetilde{Z}}\left(\frac{\tau_I}{V_H-V_L}(s_U-x)\right) f_{\widetilde{S}_I}(x)dx}\right) \tag{A.19}$$

$$g_U(k_U b_U(s_U)) = V_L k_U b_U(s_U) + \log(1+\exp\{(V_H-V_L)k_U b_U(s_U)\}) \tag{A.20}$$

$$dH_U(v) = \begin{cases} 1 & v = V_L \\ 1 & v = V_H \\ 0 & v \notin \{V_L, V_H\}, \end{cases} \quad \text{or, equivalently,} \quad H_U(v) = \begin{cases} 0 & v < V_L \\ 1 & V_L \le v < V_H \\ 2 & V_H \ge v. \end{cases} \tag{A.21}$$

The support of the function g_U is $\mathcal{G}_U = \mathbb{R}$, as it is clear by inspection that the function is defined on the entire real line. Hence, $k_U b_U(\text{Support}(\widetilde{S}_U)) \subseteq \mathcal{G}_U = \mathbb{R}$, as required.

The price can now be derived by writing down explicit expressions for each investor's demand function and then clearing the market. Lemma 1 delivers the informed investor's demand function once his/her price reaction function is characterized. Recalling the expression for the function g_I from equation (3.4), the derivative follows immediately

$$g_I'(k_I s_I) = V_L + (V_H - V_L)\frac{\exp\{(V_H-V_L)k_I s_I\}}{1+\exp\{(V_H-V_L)k_I s_I\}}.$$

Hence, his/her price reaction function, which is the inverse of g_I', is

$$G_I(p) = (g_I')^{-1}(p) = \frac{1}{V_H-V_L}\log\left(\frac{p-V_L}{V_H-p}\right),$$

and according to Lemma 1 his/her demand function is

$$X_I(s_I, p) = \frac{\tau_I}{V_H-V_L}\left(s_I - \log\left(\frac{p-V_L}{V_H-p}\right)\right).$$

Similarly, since the uninformed investor's conditional beliefs are binomial but with log-odds $b_U(s_U)$ as given in equation (A.16), his/her optimal demand is linear in $b_U(s_U)$

$$X_U(s_U, p) = \frac{\tau_U}{V_H-V_L}\left(b_U(s_U) - \log\left(\frac{p-V_L}{V_H-p}\right)\right).$$

The market-clearing condition requires that in equilibrium

$$X_I(s_I,P(s_U))+X_U(s_U,P(s_U))=z+\bar{z}$$
$$\Rightarrow \frac{\tau_I}{V_H-V_L}\left(s_I-\log\left(\frac{P(s_U)-V_L}{V_H-P(s_U)}\right)\right)$$
$$+\frac{\tau_U}{V_H-V_L}\left(b_U(s_U)-\log\left(\frac{P(s_U)-V_L}{V_H-P(s_U)}\right)\right)=z+\bar{z},$$

and some simple algebra delivers the explicit expression for the price

$$P(s_U)=V_L+(V_H-V_L)\frac{\exp\left\{\frac{\tau_I}{\tau_I+\tau_U}s_U+\frac{\tau_U}{\tau_I+\tau_U}b_U(s_U)-\frac{V_H-V_L}{\tau_I+\tau_U}\bar{z}\right\}}{1+\exp\left\{\frac{\tau_I}{\tau_I+\tau_U}s_U+\frac{\tau_U}{\tau_I+\tau_U}b_U(s_U)-\frac{V_H-V_L}{\tau_I+\tau_U}\bar{z}\right\}}.$$

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Proof (Example 2). Here I derive the equilibrium in the case of general pay-offs and additive signals with normally distributed errors, $\widetilde{S}_I=\widetilde{V}+\widetilde{\varepsilon}_I$. Suppose further that the supply shock is normally distributed, $\widetilde{Z}\sim N(0,\sigma_Z^2)$. Before deriving the equilibrium, I will verify that Assumptions 3—8 are met and therefore from Propositions 2 and 3 an equilibrium exists and is unique. I will also show that Assumption 9 is met and derive the explicit price function from Corollary 1.

Assumption 3 was verified in Section 3 in the text. Assumption 4 on the openness of $\mathcal{G}_I$ holds since it was shown in the text that $\mathcal{G}_I=\mathbb{R}$. Checking Assumption 5, which requires that the uninformed investor's conditional mgf converge on an open set, necessitates computing his/her conditional beliefs. Note that because the supply shock is distributed $N(0,\sigma_Z^2)$, the variable $\widetilde{S}_U$ is equal to the sum of the true pay-off plus a normally distributed error with variance $\sigma_U^2\equiv\sigma_I^2+\left(\frac{\sigma_I^2}{\tau_I}\right)^2\sigma_Z^2$. Hence, the conditional density of $\widetilde{V}$ given $\widetilde{S}_U=s_U$ can be derived using steps that are analogous to those used to derive the informed investor beliefs in the text:

$$dF_{\widetilde{V}|\widetilde{S}_U}(v|s_U)=\frac{\exp\left\{\frac{s_U}{\sigma_U^2}v-\frac{1}{2}\frac{v^2}{\sigma_U^2}\right\}dF_{\widetilde{V}}(v)}{\int_{\mathcal{V}}\exp\left\{\frac{s_U}{\sigma_U^2}x-\frac{1}{2}\frac{x^2}{\sigma_U^2}\right\}dF_{\widetilde{V}}(x)}$$
$$=\exp\left\{\frac{s_U}{\sigma_U^2}v-\log\left(\int_{\mathcal{V}}\exp\left\{\frac{s_U}{\sigma_U^2}x-\frac{1}{2}\frac{x^2}{\sigma_U^2}\right\}dF_{\widetilde{V}}(x)\right)\right\}\exp\left\{-\frac{1}{2}\frac{v^2}{\sigma_U^2}\right\}dF_{\widetilde{V}}(v). \quad \text{(A.22)}$$

This density is in the exponential family form with

$$k_U=\frac{1}{\sigma_U^2}$$
$$b_U(s_U)=s_U$$
$$g_U(k_Us_U)=\log\left(\int_{\mathcal{V}}\exp\left\{\frac{s_U}{\sigma_U^2}x-\frac{1}{2}\frac{x^2}{\sigma_U^2}\right\}dF_{\widetilde{V}}(x)\right)$$
$$dH_U(v)=\exp\left\{-\frac{1}{2}\frac{v^2}{\sigma_U^2}\right\}dF_{\widetilde{V}}(v),\qquad \text{or, equivalently,}\qquad H_U(v)=\int_0^v\exp\left\{-\frac{1}{2}\frac{x^2}{\sigma_U^2}\right\}dF_{\widetilde{V}}(x).$$

The support of g_U is $\mathcal{G}_U=\mathbb{R}$. As in the case of the function g_I above, note that the $-x^2$ term in the exponential in the integral implies that the integral that defines g_U converges for any $k_Us_U\in\mathbb{R}$. The support of $\widetilde{S}_U$ is $\mathbb{R}$ due to the normally distributed error. Hence, $k_U\times\text{Support}(\widetilde{S}_U)\subseteq\mathcal{G}_U$, as required. Using steps analogous to those in the proof of Lemma A6, it can be shown that

$$M_{\widetilde{V}|\widetilde{S}_U}(u|s_U)=\exp\{g_U(u+k_Us_U)-g_U(k_Us_U)\},$$

and, therefore, Assumption 5 is met because g_U is defined on the entire real line.

Assumption 6, which restricts the supports of random variables, is met since $\mathcal{G}_I=\mathbb{R}$ means that $k_I\mathcal{S}-\frac{1}{\tau_I}\mathcal{Z}-\frac{1}{\tau_I}\bar{z}\subseteq\mathcal{G}_I$, for all choices of supports or parameter values. Assumption 7 on the log concavity of $f_{\widetilde{Z}}$ is met because the supply shock is normally distributed. Finally, to verify Assumption 8 on the continuity of the uninformed's conditional distribution consider the expression in equation (A.22). Clearly the numerator is continuous in s_U, and since the integral in the denominator is proportional to the mgf of the distribution $\frac{\exp\left\{-\frac{1}{2}\frac{v^2}{\sigma_U^2}\right\}dF_{\tilde{V}}(v)}{\int_{\mathcal{V}}\exp\left\{-\frac{1}{2}\frac{x^2}{\sigma_U^2}\right\}dF_{\tilde{V}}(x)}$, evaluated at $\frac{s_U}{\sigma_U^2}$, it is also continuous in s_U.

Given that Assumptions 1–8 are met, Proposition 2 implies that an equilibrium price function exists and is continuous in s_U. Since the supports $\mathcal{S}=\mathbb{R}$ and $\mathcal{Z}=\mathbb{R}$ are intervals, Proposition 3 also implies that this price function is unique. It remains to characterize the price function. Since the uninformed investor's beliefs are in the exponential family form from Assumption 9, Corollary 1 guarantees an explicit price function, which I now derive.

The informed investor demand function follows from Lemma 1 after determining his/her price reaction function. G_I is the inverse of the derivative of g_I:

$$g_I'(k_I s_I)=\frac{\int_{\mathcal{V}} x\exp\left\{k_I s_I x-\frac{1}{2}\frac{x^2}{\sigma_I^2}\right\}dF_{\tilde{V}}(x)}{\int_{\mathcal{V}}\exp\left\{k_I s_I x-\frac{1}{2}\frac{x^2}{\sigma_I^2}\right\}dF_{\tilde{V}}(x)},$$

where $k_I=1/\sigma_I^2$, and hence his/her demand function is

$$X_I(s_I,p)=\tau_I\left(k_I s_I-G_I(p)\right)=\tau_I\left(\frac{s_I}{\sigma_I^2}-G_I(p)\right).$$

Given his/her exponential family beliefs, the uninformed investor's demand is similar to that of the informed investor. His/Her price reaction function $G_U=(g_U')^{-1}$ is the inverse of

$$g_U'(k_U s_U)=\frac{\int_{\mathcal{V}} x\exp\left\{k_U s_U x-\frac{1}{2}\frac{x^2}{\sigma_U^2}\right\}dF_{\tilde{V}}(x)}{\int_{\mathcal{V}}\exp\left\{k_U s_U x-\frac{1}{2}\frac{x^2}{\sigma_U^2}\right\}dF_{\tilde{V}}(x)},$$

so that his/her demand function is

$$X_U(s_U,p)=\tau_U\left(\frac{s_U}{\sigma_U^2}-G_U(p)\right).$$

The market-clearing condition requires that in equilibrium

$$\begin{aligned}
&X_I(s_I,P(s_U))+X_U(s_U,P(s_U))=z+\bar{z}\\
&\Rightarrow \tau_I\left(\frac{s_I}{\sigma_I^2}-G_I(P(s_U))\right)+\tau_U\left(\frac{s_U}{\sigma_U^2}-G_U(P(s_U))\right)=z+\bar{z}\\
&\Rightarrow \tau_I\left(\frac{1}{\sigma_I^2}\left(s_I-\frac{\sigma_I^2}{\tau_I}z\right)-G_I(P(s_U))\right)+\tau_U\left(\frac{s_U}{\sigma_U^2}-G_U(P(s_U))\right)-\bar{z}=0\\
&\Rightarrow \left(\frac{\tau_I}{\sigma_I^2}+\frac{\tau_U}{\sigma_U^2}\right)s_U-\bar{z}=\tau_I G_I(P(s_U))+\tau_U G_U(P(s_U)),
\end{aligned}$$

where the final line uses $s_U=s_I-\frac{\sigma_I^2}{\tau_I}z$. Finally, let $G(p)\equiv\tau_I G_I(p)+\tau_U G_U(p)$ be the aggregate price reaction function and apply to both sides to deliver

$$P(s_U)=G^{-1}\left[\left(\frac{\tau_I}{\sigma_I^2}+\frac{\tau_U}{\sigma_U^2}\right)s_U-\bar{z}\right].$$

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A.4. *Proofs of results in Section 5*

Proof (Proposition 4). Given that $P(s_U)$ is increasing in s_U, it follows that uninformed demand is increasing in the equilibrium price if and only if the total derivative of $X_U(s_U,P(s_U))$ with respect to s_U is positive. To see this, recall that the uninformed investor demand function expressed purely as a function of p relates to the demand function expressed as a function of s_U and p as $X_U(p)=X_U(P^{-1}(p),p)$. Differentiating the equilibrium demand $X_U(s_U,P(s_U))$ totally with respect to s_U gives

$$\frac{d}{ds_U}X_U(s_U,P(s_U))=\frac{\partial X_U}{\partial s_U}+\frac{\partial X_U}{\partial p}\frac{\partial P}{\partial s_U}.$$

Recall that the demand function is linear in the log-odds

$$X_U(s_U,p)=\tau_U\left(\frac{1}{V_H-V_L}\log\left(\frac{f_{\widetilde{V}|\widetilde{S}_U}(V_H|s_U)}{f_{\widetilde{V}|\widetilde{S}_U}(V_L|s_U)}\right)-\frac{1}{V_H-V_L}\log\left(\frac{p-V_L}{V_H-p}\right)\right).$$

Hence, the derivative of the equilibrium demand is

$$\begin{aligned}\frac{d}{ds_U}X_U(s_U,P(s_U))&=\frac{\tau_U}{V_H-V_L}\frac{\partial}{\partial s_U}\log\left(\frac{f_{\widetilde{V}|\widetilde{S}_U}(V_H|s_U)}{f_{\widetilde{V}|\widetilde{S}_U}(V_L|s_U)}\right)-\frac{\tau_U}{(V_H-P(s_U))(P(s_U)-V_L)}\frac{\partial P}{\partial s_U}\\&=\frac{\tau_U}{V_H-V_L}\frac{\partial}{\partial s_U}\log\left(\frac{f_{\widetilde{V}|\widetilde{S}_U}(V_H|s_U)}{f_{\widetilde{V}|\widetilde{S}_U}(V_L|s_U)}\right)-\frac{\tau_U}{(V_H-V_L)^2(1-\hat{\pi}(s_U))\hat{\pi}(s_U)}\frac{\partial P}{\partial s_U},\end{aligned}$$

where $\hat{\pi}(s_U)=\frac{\exp\left\{\frac{\tau_I}{\tau_I+\tau_U}s_U+\frac{\tau_U}{\tau_I+\tau_U}b_U(s_U)-\frac{V_H-V_L}{\tau_I+\tau_U}\bar{z}\right\}}{1+\exp\left\{\frac{\tau_I}{\tau_I+\tau_U}s_U+\frac{\tau_U}{\tau_I+\tau_U}b_U(s_U)-\frac{V_H-V_L}{\tau_I+\tau_U}\bar{z}\right\}}$ is the 'risk-neutral probability' that appears in the price function in equation (4.2). Differentiating the price function from equation (4.2) gives the partial derivative

$$\frac{\partial P}{\partial s_U}=(V_H-V_L)\hat{\pi}(s_U)(1-\hat{\pi}(s_U))\left(\frac{\tau_I}{\tau_I+\tau_U}+\frac{\tau_U}{\tau_I+\tau_U}\frac{\partial}{\partial s_U}\log\left(\frac{f_{\widetilde{V}|\widetilde{S}_U}(V_H|s_U)}{f_{\widetilde{V}|\widetilde{S}_U}(V_L|s_U)}\right)\right),$$

so that the previous expression can be simplified to

$$\frac{d}{ds_U}X_U(s_U,P(s_U))=\frac{\tau_U}{V_H-V_L}\frac{\tau_I}{\tau_I+\tau_U}\left(\frac{\partial}{\partial s_U}\log\left(\frac{f_{\widetilde{V}|\widetilde{S}_U}(V_H|s_U)}{f_{\widetilde{V}|\widetilde{S}_U}(V_L|s_U)}\right)-1\right).$$

The sign of this expression depends on whether the derivative of the uninformed investor's log-odds is greater or less than 1.

The derivative of the log-odds is

$$\frac{\partial}{\partial s_U}\log\left(\frac{f_{\widetilde{V}|\widetilde{S}_U}(V_H|s_U)}{f_{\widetilde{V}|\widetilde{S}_U}(V_L|s_U)}\right)=\frac{\frac{\partial}{\partial s_U}f_{\widetilde{V}|\widetilde{S}_U}(V_H|s_U)}{f_{\widetilde{V}|\widetilde{S}_U}(V_H|s_U)(1-f_{\widetilde{V}|\widetilde{S}_U}(V_H|s_U))},$$

and therefore signing $\frac{\partial}{\partial s_U}\log\left(\frac{f_{\widetilde{V}|\widetilde{S}_U}(V_H|s_U)}{f_{\widetilde{V}|\widetilde{S}_U}(V_L|s_U)}\right)-1$ is equivalent to signing

$$\frac{\partial}{\partial s_U}f_{\widetilde{V}|\widetilde{S}_U}(V_H|s_U)-f_{\widetilde{V}|\widetilde{S}_U}(V_H|s_U)(1-f_{\widetilde{V}|\widetilde{S}_U}(V_H|s_U)).$$

From the derivation for Example 1 in Appendix A.3 the uninformed investor's conditional pdf is

$$f_{\widetilde{V}|\widetilde{S}_U}(v|s_U)=\int_{\underline{s}(s_U)}^{\bar{s}(s_U)}\frac{\exp\{x\frac{v-V_L}{V_H-V_L}\}}{1+\exp\{x\}}\frac{\frac{\tau_I}{V_H-V_L}f_{\widetilde{Z}}\left(\frac{\tau_I}{V_H-V_L}(s_U-x)\right)f_{\widetilde{S}_I}(x)}{f_{\widetilde{S}_U}(s_U)}dx, \tag{A.23}$$

and, therefore, the derivative is

$$\begin{aligned}&\frac{\partial}{\partial s_U}f_{\widetilde{V}|\widetilde{S}_U}(V_H|s_U)\\&=\int_{\underline{s}(s_U)}^{\bar{s}(s_U)}\frac{\exp\{x\}}{1+\exp\{x\}}\frac{f_{\widetilde{S}_I}(x)\frac{\tau_I}{V_H-V_L}\frac{\partial}{\partial s_U}f_{\widetilde{Z}}\left(\frac{\tau_I}{V_H-V_L}(s_U-x)\right)}{f_{\widetilde{S}_U}(s_U)}dx\\&\quad-\int_{\underline{s}(s_U)}^{\bar{s}(s_U)}\frac{\exp\{x\}}{1+\exp\{x\}}\frac{f_{\widetilde{S}_I}(x)\frac{\tau_I}{V_H-V_L}f_{\widetilde{Z}}\left(\frac{\tau_I}{V_H-V_L}(s_U-x)\right)}{f_{\widetilde{S}_U}(s_U)}dx\,\frac{\frac{\partial}{\partial s_U}f_{\widetilde{S}_U}(s_U)}{f_{\widetilde{S}_U}(s_U)}\\&\quad+\frac{\exp\{x\}}{1+\exp\{x\}}\frac{\frac{\tau_I}{V_H-V_L}f_{\widetilde{Z}}\left(\frac{\tau_I}{V_H-V_L}(s_U-x)\right)f_{\widetilde{S}_I}(x)}{f_{\widetilde{S}_U}(s_U)}\Bigg|_{x=\underline{s}(s_U)}^{\bar{s}(s_U)}\\&=\int_{\underline{s}(s_U)}^{\bar{s}(s_U)}\frac{\exp\{x\}}{1+\exp\{x\}}\frac{f_{\widetilde{S}_I}(x)\frac{\tau_I}{V_H-V_L}\left[-\frac{\partial}{\partial x}f_{\widetilde{Z}}\left(\frac{\tau_I}{V_H-V_L}(s_U-x)\right)\right]}{f_{\widetilde{S}_U}(s_U)}dx\\&\quad-\int_{\underline{s}(s_U)}^{\bar{s}(s_U)}\frac{\exp\{x\}}{1+\exp\{x\}}f_{\widetilde{S}_I|\widetilde{S}_U}(x|s_U)dx\,\frac{\frac{\partial}{\partial s_U}f_{\widetilde{S}_U}(s_U)}{f_{\widetilde{S}_U}(s_U)}+\frac{\exp\{x\}}{1+\exp\{x\}}f_{\widetilde{S}_I|\widetilde{S}_U}(x|s_U)\Bigg|_{x=\underline{s}(s_U)}^{\bar{s}(s_U)},\end{aligned} \tag{A.24}$$

where the first equality pulls the derivative inside the integral from equation (A.23), and the second equality uses the fact that $\frac{\partial}{\partial s_U} f_{\widetilde{Z}}\left(\frac{\tau_I}{V_H - V_L}(s_U - x)\right) = -\frac{\partial}{\partial x} f_{\widetilde{Z}}\left(\frac{\tau_I}{V_H - V_L}(s_U - x)\right)$ to rewrite the expression in the first integral and the fact that $f_{\widetilde{S}_I|\widetilde{S}_U}(x|s_U) = \frac{\frac{\tau_I}{V_H - V_L} f_{\widetilde{Z}}\left(\frac{\tau_I}{V_H - V_L}(s_U - x)\right) f_{\widetilde{S}_I}(x)}{f_{\widetilde{S}_U}(s_U)}$ to simplify the other two terms.

Perform integration by parts on the first integral in equation (A.24)

$$
\begin{aligned}
&\int_{\underline{s}(s_U)}^{\bar{s}(s_U)} \frac{\exp\{x\}}{1+\exp\{x\}} \frac{f_{\widetilde{S}_I}(x) \frac{\tau_I}{V_H - V_L}\left[-\frac{\partial}{\partial x} f_{\widetilde{Z}}\left(\frac{\tau_I}{V_H - V_L}(s_U - x)\right)\right]}{f_{\widetilde{S}_U}(s_U)} dx \\
&= \int_{\underline{s}(s_U)}^{\bar{s}(s_U)} \frac{\partial}{\partial x}\left[\frac{\exp\{x\}}{1+\exp\{x\}} f_{\widetilde{S}_I}(x)\right] \frac{\frac{\tau_I}{V_H - V_L} f_{\widetilde{Z}}\left(\frac{\tau_I}{V_H - V_L}(s_U - x)\right)}{f_{\widetilde{S}_U}(s_U)} dx \\
&\qquad - \frac{\exp\{x\}}{1+\exp\{x\}} \frac{f_{\widetilde{S}_I}(x) \frac{\tau_I}{V_H - V_L} f_{\widetilde{Z}}\left(\frac{\tau_I}{V_H - V_L}(s_U - x)\right)}{f_{\widetilde{S}_U}(s_U)}\Bigg|_{x=\underline{s}(s_U)}^{\bar{s}(s_U)} \\
&= \int_{\underline{s}(s_U)}^{\bar{s}(s_U)} \frac{\exp\{x\}}{1+\exp\{x\}} \frac{1}{1+\exp\{x\}} f_{\widetilde{S}_I|\widetilde{S}_U}(x|s_U) dx + \int_{\underline{s}_I}^{\bar{s}_I} \frac{\exp\{x\}}{1+\exp\{x\}} \frac{f'_{\widetilde{S}_I}(x)}{f_{\widetilde{S}_I}(x)} f_{\widetilde{S}_I|\widetilde{S}_U}(x|s_U) dx \\
&\qquad - \frac{\exp\{x\}}{1+\exp\{x\}} f_{\widetilde{S}_I|\widetilde{S}_U}(x|s_U)\Bigg|_{x=\underline{s}(s_U)}^{\bar{s}(s_U)}.
\end{aligned}
\tag{A.25}
$$

Now, differentiate $\frac{\frac{\partial}{\partial s_U} f_{\widetilde{S}_U}(s_U)}{f_{\widetilde{S}_U}(s_U)}$

$$
\begin{aligned}
&\frac{\frac{\partial}{\partial s_U} f_{\widetilde{S}_U}(s_U)}{f_{\widetilde{S}_U}(s_U)} \\
&= \frac{\frac{\partial}{\partial s_U} \int_{\underline{s}(s_U)}^{\bar{s}(s_U)} f_{\widetilde{S}_I}(x) \frac{\tau_I}{V_H - V_L} f_{\widetilde{Z}}\left(\frac{\tau_I}{V_H - V_L}(s_U - x)\right) dx}{f_{\widetilde{S}_U}(s_U)} \\
&= \int_{\underline{s}(s_U)}^{\bar{s}(s_U)} \frac{f_{\widetilde{S}_I}(x) \frac{\tau_I}{V_H - V_L} \frac{\partial}{\partial s_U} f_{\widetilde{Z}}\left(\frac{\tau_I}{V_H - V_L}(s_U - x)\right)}{f_{\widetilde{S}_U}(s_U)} dx + \frac{f_{\widetilde{S}_I}(x) \frac{\tau_I}{V_H - V_L} f_{\widetilde{Z}}\left(\frac{\tau_I}{V_H - V_L}(s_U - x)\right)}{f_{\widetilde{S}_U}(s_U)}\Bigg|_{x=\underline{s}(s_U)}^{\bar{s}(s_U)} \\
&= \int_{\underline{s}(s_U)}^{\bar{s}(s_U)} \frac{f_{\widetilde{S}_I}(x) \frac{\tau_I}{V_H - V_L}\left[-\frac{\partial}{\partial x} f_{\widetilde{Z}}\left(\frac{\tau_I}{V_H - V_L}(s_U - x)\right)\right]}{f_{\widetilde{S}_U}(s_U)} dx + \frac{f_{\widetilde{S}_I}(x) \frac{\tau_I}{V_H - V_L} f_{\widetilde{Z}}\left(\frac{\tau_I}{V_H - V_L}(s_U - x)\right)}{f_{\widetilde{S}_U}(s_U)}\Bigg|_{x=\underline{s}(s_U)}^{\bar{s}(s_U)} \\
&= \int_{\underline{s}(s_U)}^{\bar{s}(s_U)} \frac{f'_{\widetilde{S}_I}(x) \frac{\tau_I}{V_H - V_L} f_{\widetilde{Z}}\left(\frac{\tau_I}{V_H - V_L}(s_U - x)\right)}{f_{\widetilde{S}_U}(s_U)} dx \\
&= \int_{\underline{s}(s_U)}^{\bar{s}(s_U)} \frac{f'_{\widetilde{S}_I}(x)}{f_{\widetilde{S}_I}(x)} \frac{f_{\widetilde{S}_I}(x) \frac{\tau_I}{V_H - V_L} f_{\widetilde{Z}}\left(\frac{\tau_I}{V_H - V_L}(s_U - x)\right)}{f_{\widetilde{S}_U}(s_U)} dx \\
&= \int_{\underline{s}(s_U)}^{\bar{s}(s_U)} \frac{f'_{\widetilde{S}_I}(x)}{f_{\widetilde{S}_I}(x)} f_{\widetilde{S}_I|\widetilde{S}_U}(x|s_U) dx,
\end{aligned}
\tag{A.26}
$$

where the third equality again uses $\frac{\partial}{\partial s_U} f_{\widetilde{Z}}\left(\frac{\tau_I}{V_H - V_L}(s_U - x)\right) = -\frac{\partial}{\partial x} f_{\widetilde{Z}}\left(\frac{\tau_I}{V_H - V_L}(s_U - x)\right)$, the fourth equality performs integration by parts on the integral which produces a boundary term that cancels the existing boundary term, the next-to-last equality multiplies and divides by $f_{\widetilde{S}_I}(x)$ in the integral, and the final line simplifies using the expression for $f_{\widetilde{S}_I|\widetilde{S}_U}(x|s_U)$.

Substituting equation (A.25) and (A.26) into equation (A.24) gives

$$
\begin{aligned}
&\frac{\partial}{\partial s_U} f_{\widetilde{V}|\widetilde{S}_U}(V_H|s_U) \\
&= \int_{\underline{s}(s_U)}^{\bar{s}(s_U)} \frac{\exp\{x\}}{1+\exp\{x\}} \frac{1}{1+\exp\{x\}} f_{\widetilde{S}_I|\widetilde{S}_U}(x|s_U) dx + \int_{\underline{s}(s_U)}^{\bar{s}(s_U)} \frac{\exp\{x\}}{1+\exp\{x\}} \frac{f'_{\widetilde{S}_I}(x)}{f_{\widetilde{S}_I}(x)} f_{\widetilde{S}_I|\widetilde{S}_U}(x|s_U) dx \\
&\qquad - \int_{\underline{s}(s_U)}^{\bar{s}(s_U)} \frac{\exp\{x\}}{1+\exp\{x\}} f_{\widetilde{S}_I|\widetilde{S}_U}(x|s_U) dx \int_{\underline{s}(s_U)}^{\bar{s}(s_U)} \frac{f'_{\widetilde{S}_I}(x)}{f_{\widetilde{S}_I}(x)} f_{\widetilde{S}_I|\widetilde{S}_U}(x|s_U) dx
\end{aligned}
$$

Putting everything together, the goal is to sign

$$
\begin{aligned}
&\frac{\partial}{\partial s_U} f_{\widetilde{V}|\widetilde{S}_U}(V_H|s_U) - f_{\widetilde{V}|\widetilde{S}_U}(V_H|s_U)(1-f_{\widetilde{V}|\widetilde{S}_U}(V_H|s_U)) \\
&= \int_{\underline{s}(s_U)}^{\overline{s}(s_U)} \tfrac{\exp\{x\}}{1+\exp\{x\}} \tfrac{1}{1+\exp\{x\}} f_{\widetilde{S}_I|\widetilde{S}_U}(x|s_U)dx + \int_{\underline{s}(s_U)}^{\overline{s}(s_U)} \tfrac{\exp\{x\}}{1+\exp\{x\}} \tfrac{f'_{\widetilde{S}_I}(x)}{f_{\widetilde{S}_I}(x)} f_{\widetilde{S}_I|\widetilde{S}_U}(x|s_U)dx \\
&\quad - \int_{\underline{s}(s_U)}^{\overline{s}(s_U)} \tfrac{\exp\{x\}}{1+\exp\{x\}} f_{\widetilde{S}_I|\widetilde{S}_U}(x|s_U)dx \int_{\underline{s}(s_U)}^{\overline{s}(s_U)} \tfrac{f'_{\widetilde{S}_I}(x)}{f_{\widetilde{S}_I}(x)} f_{\widetilde{S}_I|\widetilde{S}_U}(x|s_U)dx \\
&\quad - \int_{\underline{s}(s_U)}^{\overline{s}(s_U)} \tfrac{\exp\{x\}}{1+\exp\{x\}} f_{\widetilde{S}_I|\widetilde{S}_U}(x|s_U)dx \int_{\underline{s}(s_U)}^{\overline{s}(s_U)} \tfrac{1}{1+\exp\{x\}} f_{\widetilde{S}_I|\widetilde{S}_U}(x|s_U)dx \\
&= \mathrm{Cov}\left(\tfrac{\exp\{\widetilde{S}_I\}}{1+\exp\{\widetilde{S}_I\}}, \tfrac{1}{1+\exp\{\widetilde{S}_I\}} \middle| \widetilde{S}_U = s_U \right) + \mathrm{Cov}\left(\tfrac{\exp\{\widetilde{S}_I\}}{1+\exp\{\widetilde{S}_I\}}, \tfrac{f'_{\widetilde{S}_I}(\widetilde{S}_I)}{f_{\widetilde{S}_I}(\widetilde{S}_I)} \middle| \widetilde{S}_U = s_U \right),
\end{aligned} \tag{A.27}
$$

where the first equality plugs in for $\frac{\partial}{\partial s_U} f_{\widetilde{V}|\widetilde{S}_U}(V_H|s_U)$ and $f_{\widetilde{V}|\widetilde{S}_U}(V_H|s_U)(1-f_{\widetilde{V}|\widetilde{S}_U}(V_H|s_U))$, and the second equality rearranges terms and uses the definition of covariance.

To derive a sufficient condition for downward-sloping demand, note that the first covariance term can be written

$$
\begin{aligned}
\mathrm{Cov}\left(\tfrac{\exp\{\widetilde{S}_I\}}{1+\exp\{\widetilde{S}_I\}}, \tfrac{1}{1+\exp\{\widetilde{S}_I\}} \middle| \widetilde{S}_U = s_U \right) &= \mathrm{Cov}\left(\tfrac{\exp\{\widetilde{S}_I\}}{1+\exp\{\widetilde{S}_I\}}, 1 - \tfrac{\exp\{\widetilde{S}_I\}}{1+\exp\{\widetilde{S}_I\}} \middle| \widetilde{S}_U = s_U \right) \\
&= -\mathrm{Cov}\left(\tfrac{\exp\{\widetilde{S}_I\}}{1+\exp\{\widetilde{S}_I\}}, \tfrac{\exp\{\widetilde{S}_I\}}{1+\exp\{\widetilde{S}_I\}} \middle| \widetilde{S}_U = s_U \right),
\end{aligned}
$$

so that equation (A.27) is

$$
\begin{aligned}
&\frac{\partial}{\partial s_U} f_{\widetilde{V}|\widetilde{S}_U}(V_H|s_U) - f_{\widetilde{V}|\widetilde{S}_U}(V_H|s_U)(1-f_{\widetilde{V}|\widetilde{S}_U}(V_H|s_U)) \\
&= \mathrm{Cov}\left(\tfrac{\exp\{\widetilde{S}_I\}}{1+\exp\{\widetilde{S}_I\}}, \tfrac{f'_{\widetilde{S}_I}(\widetilde{S}_I)}{f_{\widetilde{S}_I}(\widetilde{S}_I)} - \tfrac{\exp\{\widetilde{S}_I\}}{1+\exp\{\widetilde{S}_I\}} \middle| \widetilde{S}_U = s_U \right).
\end{aligned}
$$

If $\frac{f'_{\widetilde{S}_I}(\widetilde{S}_I)}{f_{\widetilde{S}_I}(\widetilde{S}_I)} - \frac{\exp\{\widetilde{S}_I\}}{1+\exp\{\widetilde{S}_I\}}$ is a decreasing function of $\widetilde{S}_I$, then this covariance will be unambiguously negative, regardless of the value of s_U. Under the assumption that $f_{\widetilde{S}_I}(x)$ is twice continuously differentiable, this is true if and only if the derivative is non-positive

$$
\frac{\partial}{\partial x} \frac{f'_{\widetilde{S}_I}(x)}{f_{\widetilde{S}_I}(x)} - \frac{\exp\{x\}}{1+\exp\{x\}} \frac{1}{1+\exp\{x\}} \leq 0, \tag{A.28}
$$

as stated in the Proposition.

I will now prove the converse result. Suppose that there exists a non-empty, bounded open interval (a,b) on which[27]

$$
\frac{\partial}{\partial x} \frac{f'_{\widetilde{S}_I}(x)}{f_{\widetilde{S}_I}(x)} - \frac{\exp\{x\}}{1+\exp\{x\}} \frac{1}{1+\exp\{x\}} > 0.
$$

Choose any supply shock distribution $f_{\widetilde{Z}}$ with support $[\underline{Z},\overline{Z}]$ satisfying $\frac{V_H-V_L}{\tau_I}(\overline{Z}-\underline{Z}) < b-a$. Without loss of generality, suppose that the support is centered at zero and can thus be written $[-\overline{Z},\overline{Z}]$ for some $\overline{Z}>0$. If it were not, one could always shift it by some constant and absorb the constant into the fixed component of supply. Hence, the previous condition on the support can be written $2\frac{V_H-V_L}{\tau_I}\overline{Z} < b-a$.

27. The assumption that the interval is bounded is without loss of generality. If the inequality holds on an unbounded interval, then one can simply restrict attention to some bounded subinterval for the construction that follows.

Let $\delta = \frac{b-a}{2} - \frac{V_H - V_L}{\tau_I}\overline{Z} > 0$. Conditional on $\widetilde{S}_U = s_U$, the support of $\widetilde{S}_I$ is $[s_U - \frac{V_H-V_L}{\tau_I}\underline{Z}, s_U + \frac{V_H-V_L}{\tau_I}\overline{Z}]$. Hence, for $s_U \in \left(\frac{a+b}{2} - \delta, \frac{a+b}{2} + \delta\right)$, the support of $\widetilde{S}_I$ is entirely contained in (a,b) because for such s_U,

$$\left[s_U - \frac{V_H - V_L}{\tau_I}\underline{Z}, s_U + \frac{V_H - V_L}{\tau_I}\overline{Z}\right] \subseteq \left(\frac{a+b}{2} - \delta - \frac{V_H - V_L}{\tau_I}\overline{Z}, \frac{a+b}{2} + \delta + \frac{V_H - V_L}{\tau_I}\overline{Z}\right) = (a,b),$$

where the set inclusion follows from taking the union of all of the supports as s_U ranges over $\left(\frac{a+b}{2} - \delta, \frac{a+b}{2} + \delta\right)$, and the equality follows after substituting in for δ.

This implies that for $s_U \in \left(\frac{a+b}{2} - \delta, \frac{a+b}{2} + \delta\right)$ the support of $\widetilde{S}_I$ is contained in the set in which

$$\frac{\partial}{\partial x}\frac{f'_{\widetilde{S}_I}(x)}{f_{\widetilde{S}_I}(x)} - \frac{\exp\{x\}}{1+\exp\{x\}}\frac{1}{1+\exp\{x\}} > 0.$$

It follows that for such s_U,

$$\mathrm{Cov}\left(\frac{\exp\{\widetilde{S}_I\}}{1+\exp\{\widetilde{S}_I\}}, \frac{f'_{\widetilde{S}_I}(\widetilde{S}_I)}{f_{\widetilde{S}_I}(\widetilde{S}_I)} - \frac{\exp\{\widetilde{S}_I\}}{1+\exp\{\widetilde{S}_I\}}\middle|\widetilde{S}_U = s_U\right) > 0,$$

and, therefore, that uninformed demand is upward sloping at prices $p \in (P(\frac{a+b}{2} - \delta), P(\frac{a+b}{2} + \delta))$. ‖

Proof (Proposition 5). To save space, define the following function

$$B(s_U) = \frac{\tau_I}{\tau_I + \tau_U}s_U + \frac{\tau_U}{\tau_I + \tau_U}b_U(s_U),$$

which is a risk-tolerance-weighted average of s_U and the uninformed investor's log-odds $b_U(s_U)$.

Under the uniform boundedness assumption there exist $0 < \underline{\kappa} \le \overline{\kappa} < 1$ such that

$$\underline{\kappa} \le b'_U(s_U) \le \underline{\kappa}. \tag{A.29}$$

Furthermore, this condition implies that b_U and B are strictly increasing functions and hence have strictly increasing inverses, both of which will be used below.

Since the distributions are assumed symmetric, the functions s_U and $b_U(s_U) = \log\left(\frac{f_{\widetilde{V}|\widetilde{S}_U}(V_H|s_U)}{1-f_{\widetilde{V}|\widetilde{S}_U}(V_H|s_U)}\right)$ cross once, and this crossing is at $s_U = 0$. A crossing point must satisfy

$$\begin{aligned} s_U &= \log\left(\frac{f_{\widetilde{V}|\widetilde{S}_U}(V_H|s_U)}{1-f_{\widetilde{V}|\widetilde{S}_U}(V_H|s_U)}\right) \\ \iff & f_{\widetilde{V}|\widetilde{S}_U}(V_H|s_U) = \exp\{s_U\}\left(1 - f_{\widetilde{V}|\widetilde{S}_U}(V_H|s_U)\right) \\ \iff & \int_{\underline{s}(s_U)}^{\overline{s}(s_U)} \frac{\exp\{x\} - \exp\{s_U\}}{1+\exp\{x\}} f_{\widetilde{Z}}\left(\frac{\tau_I}{V_H - V_L}(s_U - x)\right) f_{\widetilde{S}_I}(x)dx = 0, \end{aligned}$$

where the second line rearranges the first, and the third line plugs in for the conditional pdf and rearranges. At $s_U = 0$, the left-hand side of the most recent line is

$$\int_{\frac{V_H - V_L}{\tau_I}\underline{Z}}^{\frac{V_H - V_L}{\tau_I}\overline{Z}} \frac{\exp\{x\} - 1}{1+\exp\{x\}} f_{\widetilde{Z}}\left(\frac{\tau_I}{V_H - V_L}(-x)\right) f_{\widetilde{S}_I}(x)dx,$$

which is equal to zero because $\frac{\exp\{x\}-1}{1+\exp\{x\}}$ is an odd function, the product $f_{\widetilde{Z}}\left(\frac{\tau_I}{V_H - V_L}(-x)\right)f_{\widetilde{S}_I}(x)$ is symmetric about zero by assumption, and the support $[\frac{V_H-V_L}{\tau_I}\underline{Z}, \frac{V_H-V_L}{\tau_I}\overline{Z}]$ is also centered around zero owing to symmetry. Combining the fact that b_U is strictly increasing, the equality of s_U and $b_U(s_U)$ at $s_U = 0$, and the fact that $B'(s_U) = \frac{\tau_I}{\tau_I + \tau_U} + \frac{\tau_U}{\tau_I + \tau_U}b'(s_U) > b'(s_U)$ it follows that

$$\begin{aligned} B(s_U) < b_U(s_U) < 0 \qquad & \text{for } s_U < 0 \\ 0 < b_U(s_U) < B(s_U) \qquad & \text{for } s_U > 0. \end{aligned}$$

I will now use the above results to show that in a neighbourhood of $p=0$, $\frac{\partial}{\partial p}\mathbb{E}[\widetilde{V}|P=p]<1$ and for sufficiently extreme p, $\frac{\partial}{\partial p}\mathbb{E}[\widetilde{V}|P=p]>1$, which will establish the claim in the Proposition. Note that

$$\mathbb{E}[\widetilde{V}|P=p]=\mathbb{E}[\widetilde{V}|\widetilde{S}_U=P^{-1}(p)],$$

because $P(s_U)$ is simply a monotone transformation of s_U.[28]

Hence,

$$\mathbb{E}[\widetilde{V}|P=p]=V_L+(V_H-V_L)\frac{\exp\{b_U(P^{-1}(p))\}}{1+\exp\{b_U(P^{-1}(p))\}},$$

and differentiating with respect to p gives

$$\begin{aligned}\frac{\partial}{\partial p}\mathbb{E}[\widetilde{V}|P=p]&=(V_H-V_L)\frac{\exp\{b_U(P^{-1}(p))\}}{\left[1+\exp\{b_U(P^{-1}(p))\}\right]^2}b_U'(P^{-1}(p))\frac{\partial P^{-1}}{\partial p}\\&=\frac{\exp\left\{b_U\left(B^{-1}\left(\log\left(\frac{p-V_L}{V_H-p}\right)\right)\right)\right\}}{\left[1+\exp\left\{b_U\left(B^{-1}\left(\log\left(\frac{p-V_L}{V_H-p}\right)\right)\right)\right\}\right]^2}b_U'\left[B^{-1}\left(\log\left(\frac{p-V_L}{V_H-p}\right)\right)\right]\\&\quad\times(B^{-1})'\left[\log\left(\frac{p-V_L}{V_H-p}\right)\right]\frac{(V_H-V_L)^2}{(V_H-p)(p-V_L)},\end{aligned}$$

where the second line inverts the price function from equation (4.2), $P^{-1}(p)=B^{-1}\left(\log\left(\frac{p-V_L}{V_H-p}\right)\right)$ and differentiates.

The inverse function theorem implies that

$$(B^{-1})'\left[\log\left(\frac{p-V_L}{V_H-p}\right)\right]=\frac{1}{B'\left[B^{-1}\left(\log\left(\frac{p-V_L}{V_H-p}\right)\right)\right]},$$

so that the previous line can be written

$$\frac{\partial}{\partial p}\mathbb{E}[\widetilde{V}|P=p]=\frac{\exp\left\{b_U\left(B^{-1}\left(\log\left(\frac{p-V_L}{V_H-p}\right)\right)\right)\right\}}{\left[1+\exp\left\{b_U\left(B^{-1}\left(\log\left(\frac{p-V_L}{V_H-p}\right)\right)\right)\right\}\right]^2}\frac{b_U'\left[B^{-1}\left(\log\left(\frac{p-V_L}{V_H-p}\right)\right)\right]}{B'\left[B^{-1}\left(\log\left(\frac{p-V_L}{V_H-p}\right)\right)\right]}\frac{(V_H-V_L)^2}{(V_H-p)(p-V_L)}.$$

28. Using the transformation of random variables formula, the conditional density of $\widetilde{S}_I$ given $P=p$ is

$$\begin{aligned}f_{\widetilde{S}_I|P}(s_I|p)&=\frac{f_{\widetilde{S}_I}(s_I)f_{\widetilde{Z}}\left(\frac{\tau_I}{V_H-V_L}(P^{-1}(p)-s_I)\right)\frac{\partial P^{-1}(p)}{\partial p}}{\int_{-\infty}^{\infty}f_{\widetilde{S}_I}(x)f_{\widetilde{Z}}\left(\frac{\tau_I}{V_H-V_L}(P^{-1}(p)-x)\right)\frac{\partial P^{-1}(p)}{\partial p}dx}\\&=\frac{f_{\widetilde{S}_I}(s_I)f_{\widetilde{Z}}\left(\frac{\tau_I}{V_H-V_L}(P^{-1}(p)-s_I)\right)}{\int_{-\infty}^{\infty}f_{\widetilde{S}_I}(x)f_{\widetilde{Z}}\left(\frac{\tau_I}{V_H-V_L}(P^{-1}(p)-x)\right)dx}\\&=f_{\widetilde{S}_I|\widetilde{S}_U}(s_I|P^{-1}(p)),\end{aligned}$$

where the second equality cancels the Jacobian term $\frac{\partial P^{-1}(p)}{\partial p}$ from the fraction and the final equality follows from equation (A.14). Multiplying by $f_{\widetilde{V}|\widetilde{S}_I}(v|s_I)$ and integrating over s_I delivers $f_{\widetilde{V}|P}(v|p)=f_{\widetilde{V}|\widetilde{S}_U}(v|P^{-1}(p))$.

Prices exhibit reversals at $p=\frac{V_H+V_L}{2}$ because

$$\begin{aligned}\frac{\partial}{\partial p}\mathbb{E}\left[\widetilde{V}|P=p\right]\Big|_{p=\frac{V_H+V_L}{2}} &= \frac{\exp\left\{b_U(B^{-1}(0))\right\}}{\left[1+\exp\left\{b_U(B^{-1}(0))\right\}\right]^2}\frac{b_U'\left[B^{-1}(0)\right]}{B'\left(B^{-1}(0)\right)}\frac{(V_H-V_L)^2}{(V_H-\frac{V_H+V_L}{2})(\frac{V_H+V_L}{2}-V_L)}\\ &= \frac{1}{4}\frac{b_U'\left[B^{-1}(0)\right]}{B'\left(B^{-1}(0)\right)}\frac{(V_H-V_L)^2}{\frac{1}{4}(V_H-V_L)^2}\\ &= \frac{b_U'\left[B^{-1}(0)\right]}{B'\left(B^{-1}(0)\right)}\\ &< 1, \end{aligned} \tag{A.30}$$

where the second equality uses the fact that $b_U(B^{-1}(0))=b_U(0)=0$, and the inequality follows from the fact that the bounds in equation (A.29) imply that for any $x\in\mathbb{R}$,

$$\frac{b_U'(x)}{B'(x)}=\frac{b_U'(x)}{\frac{\tau_I}{\tau_I+\tau_U}+\frac{\tau_U}{\tau_I+\tau_U}b_U'(x)}\in\left[\frac{\underline{\kappa}}{\frac{\tau_I}{\tau_I+\tau_U}+\frac{\tau_U}{\tau_I+\tau_U}\underline{\kappa}},\frac{\overline{\kappa}}{\frac{\tau_I}{\tau_I+\tau_U}+\frac{\tau_U}{\tau_I+\tau_U}\overline{\kappa}}\right]\subset(0,1). \tag{A.31}$$

By continuity, the inequality in equation (A.30) also holds for p near $\frac{V_H+V_L}{2}$, which establishes the first claim in the Proposition.

Now, without loss of generality, consider $p>\frac{V_H+V_L}{2}$ (the case in which $p<\frac{V_H+V_L}{2}$ is symmetric). For $p>\frac{V_H+V_L}{2}$ one has $\frac{p-V_L}{V_H-p}>1$ so that $\log\left(\frac{p-V_L}{V_H-p}\right)>0$. Hence,

$$\begin{aligned}\frac{\partial}{\partial p}\mathbb{E}[\widetilde{V}|P=p] &= \frac{\exp\left\{b_U\left(B^{-1}\left(\log\left(\frac{p-V_L}{V_H-p}\right)\right)\right)\right\}}{\left[1+\exp\left\{b_U\left(B^{-1}\left(\log\left(\frac{p-V_L}{V_H-p}\right)\right)\right)\right\}\right]^2}\frac{b_U'\left[B^{-1}\left(\log\left(\frac{p-V_L}{V_H-p}\right)\right)\right]}{B'\left[B^{-1}\left(\log\left(\frac{p-V_L}{V_H-p}\right)\right)\right]}\frac{(V_H-V_L)^2}{(V_H-p)(p-V_L)}\\ &> \frac{\exp\left\{b_U\left(B^{-1}\left(\log\left(\frac{p-V_L}{V_H-p}\right)\right)\right)\right\}}{\left[1+\exp\left\{b_U\left(B^{-1}\left(\log\left(\frac{p-V_L}{V_H-p}\right)\right)\right)\right\}\right]^2}\frac{\underline{\kappa}}{\frac{\tau_I}{\tau_I+\tau_U}+\frac{\tau_U}{\tau_I+\tau_U}\underline{\kappa}}\frac{(V_H-V_L)^2}{(V_H-p)(p-V_L)}\\ &= \frac{\exp\left\{b_U\left(B^{-1}\left(\log\left(\frac{p-V_L}{V_H-p}\right)\right)\right)\right\}}{1+\exp\left\{b_U\left(B^{-1}\left(\log\left(\frac{p-V_L}{V_H-p}\right)\right)\right)\right\}}\frac{1}{1+\exp\left\{b_U\left(B^{-1}\left(\log\left(\frac{p-V_L}{V_H-p}\right)\right)\right)\right\}}\\ &\quad\times\frac{\underline{\kappa}}{\frac{\tau_I}{\tau_I+\tau_U}+\frac{\tau_U}{\tau_I+\tau_U}\underline{\kappa}}\frac{(V_H-V_L)^2}{(V_H-p)(p-V_L)}\\ &> \frac{1}{2}\frac{1}{1+\exp\left\{b_U\left(B^{-1}\left(\log\left(\frac{p-V_L}{V_H-p}\right)\right)\right)\right\}}\frac{\underline{\kappa}}{\frac{\tau_I}{\tau_I+\tau_U}+\frac{\tau_U}{\tau_I+\tau_U}\underline{\kappa}}\frac{(V_H-V_L)^2}{(V_H-p)(p-V_L)}, \end{aligned} \tag{A.32}$$

where the first line simply writes out the expression for $\frac{\partial}{\partial p}\mathbb{E}[\widetilde{V}|P=p]$, the second line uses equation (A.31) to bound $\frac{b_U'(x)}{B'(x)}$ from below, the third line splits the first fraction in the expression into a product, and the final line uses the fact that $b_U(B^{-1}\left(\log\left(\frac{p-V_L}{V_H-p}\right)\right))>b_U(0)>0$ for $p>\frac{V_H+V_L}{2}$ to bound the first term by $1/2$.

To continue, I will bound $b_U\left(B^{-1}\left(\log\left(\frac{p-V_L}{V_H-p}\right)\right)\right)$, which appears in the denominator in equation (A.32), from above. Recall that the inverse function theorem implies that

$$\frac{\partial}{\partial x}b_U(B^{-1}(x))=\frac{b_U'\left[B^{-1}(x)\right]}{B'\left(B^{-1}(x)\right)},$$

Hence, since $B^{-1}(0)=b_U(0)=0$, one can write $b_U(B^{-1}(x))$ as the following integral, and use the bounds on b_U' to bound it

$$\begin{aligned}b_U(B^{-1}(x)) &= \int_0^x\frac{\partial}{\partial y}b_U(B^{-1}(y))dy\\ &= \int_0^x\frac{b_U'\left[B^{-1}(y)\right]}{B'\left(B^{-1}(y)\right)}dy\\ &< \int_0^x\frac{\overline{\kappa}}{\frac{\tau_I}{\tau_I+\tau_U}+\frac{\tau_U}{\tau_I+\tau_U}\overline{\kappa}}dy\\ &= \frac{\overline{\kappa}}{\frac{\tau_I}{\tau_I+\tau_U}+\frac{\tau_U}{\tau_I+\tau_U}\overline{\kappa}}x, \end{aligned}$$

where the inequality follows from equation (A.31).

Continuing from equation (A.32), this inequality implies

$$\frac{\partial}{\partial p}\mathbb{E}[\widetilde{V}|P=p] > \frac{1}{2}\frac{\kappa}{\frac{\tau_I}{\tau_I+\tau_U}+\frac{\tau_U}{\tau_I+\tau_U}\kappa}\frac{1}{1+\exp\left\{\frac{\bar{\kappa}}{\frac{\tau_I}{\tau_I+\tau_U}+\frac{\tau_U}{\tau_I+\tau_U}\bar{\kappa}}\log\left(\frac{p-V_L}{V_H-p}\right)\right\}}\frac{(V_H-V_L)^2}{(V_H-p)(p-V_L)}$$

$$=\frac{1}{2}\frac{\kappa}{\frac{\tau_I}{\tau_I+\tau_U}+\frac{\tau_U}{\tau_I+\tau_U}\kappa}\frac{1}{(V_H-p)^{\frac{\bar{\kappa}}{\frac{\tau_I}{\tau_I+\tau_U}+\frac{\tau_U}{\tau_I+\tau_U}\bar{\kappa}}}+(p-V_L)^{\frac{\bar{\kappa}}{\frac{\tau_I}{\tau_I+\tau_U}+\frac{\tau_U}{\tau_I+\tau_U}\bar{\kappa}}}}\frac{(V_H-V_L)^2}{p-V_L}$$

$$\times(V_H-p)^{\frac{\bar{\kappa}}{\frac{\tau_I}{\tau_I+\tau_U}+\frac{\tau_U}{\tau_I+\tau_U}\bar{\kappa}}-1}$$

As $p\to V_H$, the $(V_H-p)^{\frac{\bar{\kappa}}{\frac{\tau_I}{\tau_I+\tau_U}+\frac{\tau_U}{\tau_I+\tau_U}\bar{\kappa}}-1}$ term becomes arbitrarily large because $\frac{\bar{\kappa}}{\frac{\tau_I}{\tau_I+\tau_U}+\frac{\tau_U}{\tau_I+\tau_U}\bar{\kappa}}-1<0$, while the remaining terms remain bounded away from zero. Hence, for all p sufficiently large, $\frac{\partial}{\partial p}\mathbb{E}[\widetilde{V}|P=p]>1$, which establishes the second claim in the Proposition. ‖

Proof (Proposition 6). The fact that the covariance is a bilinear operator allows one to pull constants out of the covariance

$$\mathrm{Cov}\left(\widetilde{V}-P(\widetilde{S}_U),\left(\mathbb{E}[\widetilde{V}|\widetilde{S}_I]-\mathbb{E}[\widetilde{V}|\widetilde{S}_U]\right)^2\right)$$

$$=\mathrm{Cov}\left(\widetilde{V}-P(\widetilde{S}_U),\left(\mathbb{E}[\widetilde{V}|\widetilde{S}_I]-\mathbb{E}[\widetilde{V}|\widetilde{S}_U]\right)^2\right)$$

Using the law of total covariance the covariance in the previous line can be written

$$\mathrm{Cov}\left(\widetilde{V}-P(\widetilde{S}_U),\left(\mathbb{E}[\widetilde{V}|\widetilde{S}_I]-\mathbb{E}[\widetilde{V}|\widetilde{S}_U]\right)^2\right)$$

$$=\mathrm{Cov}\left(\mathbb{E}[\widetilde{V}-P(\widetilde{S}_U)|\widetilde{S}_U],\mathbb{E}[\left(\mathbb{E}[\widetilde{V}|\widetilde{S}_I]-\mathbb{E}[\widetilde{V}|\widetilde{S}_U]\right)^2\Big|\widetilde{S}_U]\right)$$

$$+\mathbb{E}\left[\mathrm{Cov}\left(\widetilde{V}-P(\widetilde{S}_U),\left(\mathbb{E}[\widetilde{V}|\widetilde{S}_I]-\mathbb{E}[\widetilde{V}|\widetilde{S}_U]\right)^2|\widetilde{S}_U\right)\right]. \tag{A.33}$$

Recall that $\widetilde{S}_I$ is the log-odds of the informed investor, hence his/her conditional probability of the 'up' state is $\pi_I(\widetilde{S}_I)=\frac{\exp\{\widetilde{S}_I\}}{1+\exp\{\widetilde{S}_I\}}$. Hence, the conditional expected values are

$$\mathbb{E}[\widetilde{V}|\widetilde{S}_I]=V_L+(V_H-V_L)\pi_I(\widetilde{S}_I)$$

$$\mathbb{E}[\widetilde{V}|\widetilde{S}_U]=V_L+(V_H-V_L)\mathbb{E}[\pi_I(\widetilde{S}_I)|\widetilde{S}_U]$$

so that the second term in equation (A.33) is

$$\mathbb{E}\left[\left(\mathbb{E}[\widetilde{V}|\widetilde{S}_I]-\mathbb{E}[\widetilde{V}|\widetilde{S}_U]\right)^2\Big|\widetilde{S}_U\right]=(V_H-V_L)^2\mathbb{E}\left[\left(\pi_I(\widetilde{S}_I)-\mathbb{E}[\pi_I(\widetilde{S}_I)|\widetilde{S}_U]\right)^2\Big|\widetilde{S}_U\right]$$

$$=(V_H-V_L)^2\,\mathrm{Var}(\pi_I(\widetilde{S}_I)|\widetilde{S}_U). \tag{A.34}$$

Furthermore, the first term in equation (A.33) is

$$\mathrm{Cov}\left(\widetilde{V}-P(\widetilde{S}_U),\left(\mathbb{E}[\widetilde{V}|\widetilde{S}_I]-\mathbb{E}[\widetilde{V}|\widetilde{S}_U]\right)^2\Big|\widetilde{S}_U\right)$$

$$=\mathrm{Cov}\left(\widetilde{V}-\mathbb{E}[\widetilde{V}|\widetilde{S}_U],\left(\mathbb{E}[\widetilde{V}|\widetilde{S}_I]-\mathbb{E}[\widetilde{V}|\widetilde{S}_U]\right)^2\Big|\widetilde{S}_U\right)$$

$$=\mathbb{E}\left[\left(\widetilde{V}-\mathbb{E}[\widetilde{V}|\widetilde{S}_U]\right)\left(\mathbb{E}[\widetilde{V}|\widetilde{S}_I]-\mathbb{E}[\widetilde{V}|\widetilde{S}_U]\right)^2\Big|\widetilde{S}_U\right]$$

$$=\mathbb{E}\left[\left(\mathbb{E}[\widetilde{V}|\widetilde{S}_I]-\mathbb{E}[\widetilde{V}|\widetilde{S}_U]\right)^3\Big|\widetilde{S}_U\right]$$

$$=(V_H-V_L)^3\mathbb{E}\left[\left(\pi_I(\widetilde{S}_I)-\mathbb{E}[\pi_I(\widetilde{S}_I)|\widetilde{S}_U]\right)^3\Big|\widetilde{S}_U\right]. \tag{A.35}$$

where the first equality follows because $P(\widetilde{S}_U)$ and $\mathbb{E}[\widetilde{V}|\widetilde{S}_U]$ are both $\widetilde{S}_U$-measurable, and hence do not affect the value of the conditional covariance, the second equality follows from the definition of covariance, the third line from the law of iterated expectations, and the final line from the expressions above for the conditional expectation of $\widetilde{V}$.

Substituting eqs. (A.34) and (A.35) into equation (A.33) delivers the expression in the Proposition. ‖

A.5. *Proofs of results in Section 6*

Proof (Lemma 5). In a generalized linear equilibrium, $(\widetilde{S}_i, P(L(\widetilde{S},\widetilde{Z})))$ is informationally equivalent to $(\widetilde{S}_i, L(\widetilde{S},\widetilde{Z}))$. Hence, informed investor i's conditional beliefs are given by equation (6.1). Maximizing utility using the expression for the mgf of an exponential family distribution as in Lemma A6, it is straightforward to show that given $(\widetilde{S}_I, L(\widetilde{S},\widetilde{Z}))=(s_i,\ell)$ investor i's demand function is given by

$$\begin{aligned} X_i(s_i,\ell,p) &= \tau_i\left(\hat{L}_i(s_i,\ell)-G_i(p;a)\right) \\ &= \tau_i\left(k_{i1}(a)s_i+k_{i2}(a)b_i(\ell;a)-G_i(p;a)\right), \end{aligned}$$

where his/her price reaction function, $G_i(p;a)=(g_i')^{-1}(\cdot;a)$, exists because $g_i''>0$.

In equilibrium, the market-clearing condition must hold

$$\sum_{i=1}^{N}\tau_i\left(k_{i1}(a)s_i+k_{i2}(a)b_i(\ell;a)-G_i(P;a)\right)+X_U(\ell,P)-z-\bar{z}=0,$$

which can be rearranged to obtain

$$\sum_{i=1}^{N}\tau_i k_{i1}(a)s_i-z-\sum_{i=1}^{N}\tau_i G_i(P;a)-\bar{z}=-X_U(\ell,P)-\sum_{i=1}^{N}\tau_i k_{i2}(a)b_i(\ell;a).$$

Define $v(\ell;a)=\sum_{i=1}^{N}\tau_i k_{i2}(a)b_i(\ell;a)$, and the weighted price reaction function $G(P;a)=\sum_{i=1}^{N}\tau_i G_i(P;a)$. Using the fact that $\ell=L(s,z)=\sum_{i=1}^{N}a_i s_i-z$ one obtains

$$\sum_{i=1}^{N}\tau_i k_{i1}(a)s_i-z=G(P;a)-X_U(L(s,z),P)-v(L(s,z);a)+\bar{z}.$$

For this to be consistent with the generalized linear equilibrium, it must be the case that the coefficients a_i in the statistic $L(\cdot)$ equal $\tau_i k_{i1}(a)$ for all i. ∥

Proof (Proposition 7). From the proof of Lemma 5, the informed investor demand in a generalized linear equilibrium is

$$X_i(s_i,\ell,p)=\tau_i\left(k_{i1}(a^*)s_i+k_{i2}(a^*)b_i(\ell;a^*)-G_i(p;a^*)\right)$$

where $a^*=(a_1^*,\ldots,a_N^*)$ are the equilibrium coefficients pinned down in Lemma 5.

Similarly, given $L(\widetilde{S},\widetilde{Z}))=\ell$, the uninformed investor's demand, as a function of ℓ and the numerical value of the price, p, is characterized by

$$\int_{\mathcal{V}}(v-p)\exp\left\{-\frac{1}{\tau_U}X_U(\ell,p)v\right\}dF_{\widetilde{V}|\widetilde{L}}(v|\ell).$$

In equilibrium, the market-clearing condition must hold

$$\sum_{i=1}^{N}\tau_i\left(k_{i1}(a^*)s_i+k_{i2}(a^*)b_i(\ell;a^*)-G_i(P;a^*)\right)+X_U(\ell,P)-z-\bar{z}=0,$$

and under the assumption that the price information equations hold we can substitute $\tau_i k_{i1}(a^*)=a_i^*$

$$\sum_{i=1}^{N}a_i^*s_i-z+v(L(s,z);a^*)-G(P;a^*)-\bar{z}=-X_U(L(s,z),P).$$

Substituting this expression for $X_U(\cdot)$ into the integral that characterizes uninformed investor demand produces an equation that defines the price function implicitly

$$\int_{\mathcal{V}}(v-P)\exp\left\{\frac{1}{\tau_U}\left[\sum_{i=1}^{N}a_i^*s_i-z+v(L(s,z);a^*)-G(P;a)-\bar{z}\right]v\right\}dF_{\widetilde{V}|\widetilde{L}}(v|L(s,z)). \tag{A.36}$$

Hence, as long as a solution a^* to the price information exists and the function defined in equation (A.36) exists and is monotone (and hence one-to-one) it defines an equilibrium price P. ∥

Proof (Corollary 2). The proof proceeds similarly to that of Proposition 7 and could alternately be derived as a special case, therefore, I omit most details.

In a generalized linear equilibrium, informed investor i's demand functions are given by the expressions in the proof of Lemma 5. Similarly, the uninformed investor's demand is

$$\begin{aligned} X_U(\ell,p) &= \tau_U\left(\hat{L}_U(\ell) - G_U(p;a^*)\right) \\ &= \tau_U\left(k_{U2}(a^*)b_U(\ell;a^*) - G_U(p;a^*)\right), \end{aligned}$$

where $G_U(p;a^*) = (g_U')^{-1}(\cdot;a^*)$ is his/her price reaction function.

Imposing market clearing and rearranging the resulting expression produces the expressions in the text. Since the price reaction functions are strictly increasing in p, so are their inverses, and therefore G^{-1} is also strictly increasing. Hence, as long as it exists the candidate price function will be monotone in L^* as required. ‖

A.6. *Equilibrium derivation in $N > 1$ cases of Examples 1 and 2*

Proof (Example 1). Here I maintain the binomial distribution for the pay-off $\widetilde{V}$, but for tractability, I place more structure on the informed investor's signal. Suppose that agents have a prior distribution over the log-odds $\widetilde{W} \equiv \log \frac{\widetilde{\pi}}{1-\widetilde{\pi}}$ given by a so-called "tilted normal" distribution[29]

$$f_{\widetilde{W}}(w) = \frac{1+\exp\{w\}}{1+\exp\{\mu_W\}}\phi\left(w \middle| \mu_W - \frac{1}{2}\sigma_W^2, \sigma_W^2\right).$$

Each informed investor receives a signal $\widetilde{S}_i = \widetilde{W} + \widetilde{\varepsilon}_i$, where $\widetilde{\varepsilon}_i \sim N(0,\sigma_i^2)$ is independently distributed. For simplicity, $\tau_U = 0$, so that there is no uninformed investor, though there is no essential difficulty in including him/her by allowing some of the σ_i^2 to tend to ∞. The supply shock $\widetilde{Z}$ is distributed $N(0,\sigma_Z^2)$.

As there is no uninformed investor, if one can place the informed investor beliefs in the exponential family form of Assumption 10, Corollary 2 will deliver an explicit expression for the price. The derivation of the conditional pdf of $\widetilde{V}$ given $\widetilde{S}_i$ and $\widetilde{L}$ is straightforward but computationally intensive. Let $L(s,z) \equiv \sum_{j=1}^N a_j s_j - z$ and compute the joint distribution of $(\widetilde{W}, \widetilde{S}_i, L(\widetilde{S}, \widetilde{Z}))$:

$$\begin{aligned} & f_{\widetilde{W},\widetilde{S}_i,\widetilde{L}}(w,s_i,\ell) \\ &= f_{\widetilde{W}}(w) f_{\widetilde{S}_i,\widetilde{L}|\widetilde{W}}(s_i,\ell|w) \\ &= \frac{1+\exp\{w\}}{1+\exp\{\mu_W\}}\phi\left(w \middle| \mu_W - \frac{\sigma_W^2}{2}, \sigma_W^2\right)\phi\left(s_i,\ell \middle| \left(\begin{pmatrix} w \\ \left(\sum_{j=1}^N a_j\right)v \end{pmatrix}, \begin{pmatrix} \sigma_i^2 & a_i\sigma_i^2 \\ a_i\sigma_i^2 & \sum_{j=1}^N a_j^2\sigma_j^2 + \sigma_Z^2 \end{pmatrix}\right)\right) \\ &= \frac{1+\exp\{w\}}{1+\exp\{\mu_W\}} \\ &\quad \times \phi\left(w, s_i, \ell \middle| \begin{pmatrix} \mu_W - \frac{\sigma_W^2}{2} \\ \mu_W - \frac{\sigma_W^2}{2} \\ \left(\sum_{j=1}^N a_j\right)\left(\mu_W - \frac{\sigma_W^2}{2}\right) \end{pmatrix}, \right. \\ &\qquad \left. \begin{pmatrix} \sigma_W^2 & \sigma_W^2 & \left(\sum_{j=1}^N a_j\right)\sigma_W^2 \\ \sigma_W^2 & \sigma_W^2 + \sigma_i^2 & \left(\sum_{j=1}^N a_j\right)\sigma_W^2 + a_i\sigma_i^2 \\ \left(\sum_{j=1}^N a_j\right)\sigma_W^2 & \left(\sum_{j=1}^N a_j\right)\sigma_W^2 + a_i\sigma_i^2 & \left(\sum_{j=1}^N a_j\right)^2\sigma_W^2 + \sum_{j=1}^N a_j^2\sigma_j^2 + \sigma_Z^2 \end{pmatrix}\right), \end{aligned}$$

where the last equality follows from well-known results on the products of normal densities.

29. Using standard transformation of random variables formulae, it is straightforward to show that this is equivalent to assuming that the probability $\widetilde{\pi}$ is distributed unconditionally according to $f_{\widetilde{\pi}}(x) = \frac{1}{1+\exp\{\mu_W\}}\frac{1}{x(1-x)^2}\phi\left(\log\left(\frac{x}{1-x}\right)|\mu_W - \frac{1}{2}\sigma_W^2, \sigma_W^2\right)\mathbb{I}\{x \in [0,1]\}$. The tilted normal prior on $\widetilde{W}$ may seem somewhat contrived at first glance, but it leads to simple, intuitive conditional distributions for $\widetilde{V}$. It is straightforward to show by integrating that $\widetilde{V}$ is unconditionally binomially distributed with success probability $\frac{\exp\{\mu_W\}}{1+\exp\{\mu_W\}}$, so $\mu_W = 0$ corresponds to an unconditional distribution that places probability $1/2$ on each state.

To determine the conditional density $f_{\widetilde{W}|\widetilde{S}_i,\widetilde{L}}$, I will use the fact that $f_{\widetilde{W}|\widetilde{S}_i,\widetilde{L}}$ must be proportional to the joint density $f_{\widetilde{W},\widetilde{S}_i,\widetilde{L}}$ derived above, where the constant of proportionality will in general depend on s_i and ℓ.

$$\begin{aligned} f_{\widetilde{W}|\widetilde{S}_i,\widetilde{L}}(w|s_i,\ell) &\propto f_{\widetilde{W},\widetilde{S}_i,\widetilde{L}}(w,s_i,\ell) \\ &\propto (1+\exp\{w\})\frac{\phi\left(w,s_i,\ell\middle|\cdot\right)}{\phi\left(s_i,\ell\middle|\cdot\right)} \end{aligned} \tag{A.37}$$

where the second line uses the expression from the joint density from the previous displayed equation and divides by a function, $\phi(s_i,\ell|\cdot)$, that depends only on s_i and ℓ, and hence does not affect the proportionality.

While $(\widetilde{W},\widetilde{S}_i,\widetilde{L})$ are not jointly normally distributed, the ratio of the two normal density functions in equation (A.37) coincides with the ratio that would occur when using Bayes' rule to update beliefs about the first element of a jointly normal random vector with the given means and covariance matrix. Hence, the expression in equation (A.37) can be written

$$(1+\exp\{w\})\phi\left(w\middle|\hat{\mu}_i(s_i,\ell;a)-\frac{1}{2}\hat{V}_i(a),\hat{V}_i(a)\right),$$

where

$$\begin{aligned} \hat{\mu}_i(s_i,\ell;a) &= \alpha_{i1}(a)s_i+\alpha_{i2}(a)\ell+\alpha_{i3}(a)\mu_W \\ \hat{V}_i(a) &= \frac{\sigma_W^2\sigma_i^2\left(\sum_{j=1}^N a_j^2\sigma_j^2+\sigma_Z^2-a_i^2\sigma_i^2\right)}{(\sigma_W^2+\sigma_i^2)\left[\sum_{j=1}^N a_j^2\sigma_j^2+\sigma_Z^2-a_i^2\sigma_i^2\right]+\sigma_W^2\sigma_i^2\left(\left(\sum_{j=1}^N a_j\right)-a_i\right)^2}, \end{aligned}$$

with

$$\begin{aligned} \alpha_{i1}(a) &= \frac{\sigma_W^2\left(\sum_{j=1}^N a_j^2\sigma_j^2+\sigma_Z^2-a_i\left(\sum_{j=1}^N a_j\right)\sigma_i^2\right)}{(\sigma_W^2+\sigma_i^2)\left[\sum_{j=1}^N a_j^2\sigma_j^2+\sigma_Z^2-a_i^2\sigma_i^2\right]+\sigma_W^2\sigma_i^2\left(\left(\sum_{j=1}^N a_j\right)-a_i\right)^2} \\ \alpha_{i2}(a) &= \frac{\sigma_W^2\sigma_i^2\left(\left(\sum_{j=1}^N a_j\right)-a_i\right)}{(\sigma_W^2+\sigma_i^2)\left[\sum_{j=1}^N a_j^2\sigma_j^2+\sigma_Z^2-a_i^2\sigma_i^2\right]+\sigma_W^2\sigma_i^2\left(\left(\sum_{j=1}^N a_j\right)-a_i\right)^2} \\ \alpha_{i3}(a) &= \frac{\sigma_i^2\left(\sum_{j=1}^N a_j^2\sigma_j^2+\sigma_Z^2-a_i^2\sigma_i^2\right)}{(\sigma_W^2+\sigma_i^2)\left[\sum_{j=1}^N a_j^2\sigma_j^2+\sigma_Z^2-a_i^2\sigma_i^2\right]+\sigma_W^2\sigma_i^2\left(\left(\sum_{j=1}^N a_j\right)-a_i\right)^2}. \end{aligned}$$

Since $f_{\widetilde{W}|\widetilde{S}_i,\widetilde{L}}(w|s_i,\ell)\propto(1+\exp\{w\})\phi\left(w\middle|\hat{\mu}_i(s_i,\ell)-\frac{1}{2}\hat{V}_i(a),\hat{V}_i(a)\right)$, it follows from integrating that the normalizing constant must be $1/\left[1+\exp\{\hat{\mu}_i(s_i,\ell;a)\}\right]$. The conditional density is, therefore,

$$f_{\widetilde{W}|\widetilde{S}_i,\widetilde{L}}(w|s_i,\ell)=\frac{1+\exp\{w\}}{1+\exp\{\hat{\mu}_i(s_i,\ell;a)\}}\phi\left(w\middle|\hat{\mu}_i(s_i,\ell)-\frac{1}{2}\hat{V}_i(a),\hat{V}_i(a)\right).$$

Integrating against $f_{\widetilde{V}|\widetilde{W}}$ delivers the conditional pdf of $\widetilde{V}$

$$\begin{aligned} f_{\widetilde{V}|\widetilde{S}_i,\widetilde{L}}(v|s_i,\ell) &= \int_{-\infty}^{\infty} f_{\widetilde{V}|\widetilde{W}}(v|x)f_{\widetilde{W}|\widetilde{S}_i,\widetilde{L}}(x|s_i,\ell)dx \\ &= \exp\left\{\frac{v-V_L}{V_H-V_L}\hat{\mu}_i(s_i,\ell;a)-\log\left(1+\exp\{\hat{\mu}_i(s_i,\ell;a)\}\right)\right\}\mathbb{I}\{v\in\{V_L,V_H\}\}, \end{aligned}$$

which is a binomial distribution with log-odds ratio

$$\hat{\mu}_i(s_i,\ell;a)=\alpha_{i1}(a)s_i+\alpha_{i2}(a)\ell+\alpha_{i3}(a)\mu_W$$

that is linear in the private signal, the linear statistic, and the prior mean.

To place the pdf in the form from Assumption 10, define $k_{i1}(a)=\frac{1}{V_H-V_L}\alpha_{i1}(a)$, $k_{i2}(a)=\frac{1}{V_H-V_L}\alpha_{i2}(a)$, and $k_{i3}(a)=\frac{1}{V_H-V_L}\alpha_{i3}(a)$, $b_i(\ell;a)=\ell$ and write

$$\begin{aligned} f_{\widetilde{V}|\widetilde{S}_i,\widetilde{L}}(v|s_i,\ell) &= \exp\Big\{(v-V_L)(k_{i1}(a)s_i+k_{i2}(a)\ell+k_{i3}(a)\mu_W) \\ &\quad -\log(1+\exp\{(V_H-V_L)(k_{i1}(a)s_i+k_{i2}(a)\ell+k_{i3}(a)\mu_W)\})\Big\}\mathbb{I}\{v\in\{V_L,V_H\}\} \\ &= \exp\Big\{\hat{L}_i(s_i,\ell)v-V_L\left(\hat{L}_i(s_i,\ell)+k_{i3}(a)\mu_W\right) \\ &\quad -\log\left(1+\exp\left\{(V_H-V_L)\left(\hat{L}_i(s_i,\ell)+k_{i3}(a)\mu_W\right)\right\}\right)+vk_{i3}(a)\mu_W\Big\}\mathbb{I}\{v\in\{V_L,V_H\}\} \end{aligned} \tag{A.38}$$

where the first line uses the expression for $\hat{\mu}_i(\cdot)$ and cancels the resulting $V_H - V_L$ terms from the α_{ij}'s, and the second line writes $\hat{L}_i(s_i,\ell) = k_{i1}(a)s_i + k_{i2}(a)\ell$ and rearranges.

This is in the desired form with $k_{i1}(a), k_{i2}(a)$, and $b_i(\ell,a)$ as given above, and

$$g_i(\hat{L};a) = V_L\left(\hat{L}_i(s_i,\ell)+k_{i3}(a)\mu_W\right)+\log\left(1+\exp\left\{(V_H-V_L)\left(\hat{L}_i(s_i,\ell)+k_{i3}(a)\mu_W\right)\right\}\right)$$

$$dH_i(v;a) = \begin{cases}\exp\{vk_{i3}(a)\mu_W\} & v\in\{V_L,V_H\}\\ 0 & v\notin\{V_L,V_H\}\end{cases} \qquad \text{or, equivalently,}$$

$$H_i(v;a) = \begin{cases}0 & v<V_L\\ \exp\{V_Lk_{i3}(a)\mu_W\} & V_L\le v<V_H\\ \exp\{V_Lk_{i3}(a)\mu_W\}+\exp\{V_Hk_{i3}(a)\mu_W\} & V_H\ge v.\end{cases}$$

With the conditional distribution in the desired form, it remains to demonstrate existence of a solution to the price information equations and then characterize the price function. Given the expression for $k_{i1}(a)$, the price information equations are

$$a_i = \frac{\tau_i}{V_H-V_L}\frac{\sigma_W^2}{\sigma_W^2+\sigma_i^2}\frac{\sum_{j=1}^N a_j^2\sigma_j^2+\sigma_Z^2-a_i\left(\sum_{j=1}^N a_j\right)\sigma_i^2}{\left[\sum_{j=1}^N a_j^2\sigma_j^2+\sigma_Z^2-a_i^2\sigma_i^2\right]+\frac{\sigma_W^2\sigma_i^2}{\sigma_W^2+\sigma_i^2}\left(\left(\sum_{j=1}^N a_j\right)-a_i\right)^2}, \quad i\in\{1,\ldots,N\}. \tag{A.39}$$

The following Lemma demonstrates the existence of a solution to this system of equations.

Lemma A15. *There exists a solution $a^*=(a_1^*,\ldots,a_N^*)$ to the price-information equations in equation* (A.39). *The solution satisfies*

$$0<a_i^*<\frac{\tau_i}{V_H-V_L}\frac{\sigma_W^2}{\sigma_W^2+\sigma_i^2}, \quad i\in\{1,\ldots,N\}.$$

Proof The proof closely follows that of Lemma 3.1 in Hellwig (1980). I begin by showing that any solution must satisfy the given inequality. Let a be any solution and let $I_0=\{i:a_i\le 0\}$ be the set of investors for which the lower inequality is violated. Suppose that this set is non-empty. Then since I_0 is finite, there exists some $i_0\in I_0$ such that for all $k\in I_0$, one has $a_{i_0}\sigma_{i_0}^2\ge a_k\sigma_k^2$. Therefore, the numerator of the fraction involving the a's in the price-information equation corresponding to i_0 satisfies

$$\begin{aligned}\sum_{j=1}^N a_j^2\sigma_j^2+\sigma_Z^2-a_i\left(\sum_{j=1}^N a_j\right)\sigma_i^2 &> \sum_{j=1}^N a_j^2\sigma_j^2-a_i\left(\sum_{j=1}^N a_j\right)\sigma_i^2\\ &\ge \sum_{k\in I}a_k^2\sigma_k^2-a_i\left(\sum_{k\in I}a_k\right)\sigma_i^2\\ &= \sum_{k\in I}a_k(a_k\sigma_k^2-a_i\sigma_i^2)\\ &\ge 0.\end{aligned}$$

Therefore, the numerator and the denominator of the price-information equation for i_0 are strictly positive, which implies $a_{i_0}>0$. This contradicts $i_0\in I_0$. It follows that $a_i>0$ for all $i\in\{1,\ldots,N\}$. Therefore, returning to equation (A.39), it is immediate that $a_i<\frac{\tau_i}{V_H-V_L}\frac{\sigma_W^2}{\sigma_W^2+\sigma_i^2}$ since the previous result implies $a_i\left(\sum_{j=1}^N a_j\right)>a_i^2$, which means that the numerator in the fraction in equation (A.39) is strictly less than the denominator.

To complete the proof, I will use Brouwer's fixed point theorem to demonstrate the existence of a solution to equation (A.39). Let the set $Y=\prod_{i=1}^N\left[0,\frac{\tau_i}{V_H-V_L}\frac{\sigma_W^2}{\sigma_W^2+\sigma_i^2}\right]$ be the product of the intervals within which any solution must lie. Define a function $T_0:Y\to\mathbb{R}^N$ by

$$(T_0[a])_i=\frac{\tau_i}{V_H-V_L}\frac{\sigma_W^2}{\sigma_W^2+\sigma_i^2}\frac{\sum_{j=1}^N a_j^2\sigma_j^2+\sigma_Z^2-a_i\left(\sum_{j=1}^N a_j\right)\sigma_i^2}{\left[\sum_{j=1}^N a_j^2\sigma_j^2+\sigma_Z^2-a_i^2\sigma_i^2\right]+\frac{\sigma_W^2\sigma_i^2}{\sigma_W^2+\sigma_i^2}\left(\left(\sum_{j=1}^N a_j\right)-a_i\right)^2} \quad i\in\{1,\ldots,N\},$$

and define a function $T_1: Y \to Y$, to which I will apply Brouwer's theorem, by

$$(T_1[a])_i = \begin{cases} 0 & \text{if } (T_0[a])_i < 0 \\ (T_0[a])_i & \text{if } 0 \leq (T_0[a])_i \leq \tau_i \frac{\sigma_W^2}{\sigma_W^2+\sigma_i^2} \\ \frac{\tau_i}{V_H-V_L} \frac{\sigma_W^2}{\sigma_W^2+\sigma_i^2} & \text{if } (T_0[a])_i > \tau_i \frac{\sigma_W^2}{\sigma_W^2+\sigma_i^2}. \end{cases}$$

The set Y is the product of compact sets and is, therefore, itself compact, and by construction the function T_1 is continuous and maps Y into itself. By Brouwer's theorem, T_1 has a fixed point in Y. It remains to verify that the fixed point lies in the interior of Y. Suppose that $a_i^* = 0$ for some i. Then, $(T_0[a^*])_i = \frac{\tau_i}{V_H-V_L} \frac{\sigma_W^2}{\sigma_W^2+\sigma_i^2}$, which implies $a_i^* = (T_1[a^*])_i = \frac{\tau_i}{V_H-V_L} \frac{\sigma_W^2}{\sigma_W^2+\sigma_i^2}$, which contradicts $a_i^* = 0$. It follows that $a_i^* > 0$ for all i. This in turn implies that $(T_0[a^*])_i < \frac{\tau_i}{V_H-V_L} \frac{\sigma_W^2}{\sigma_W^2+\sigma_i^2}$, which means $(T_1[a^*])_i < \frac{\tau_i}{V_H-V_L} \frac{\sigma_W^2}{\sigma_W^2+\sigma_i^2}$. Therefore, a_i^* satisfies $0 < a_i^* = (T_1[a^*])_i < \frac{\tau_i}{V_H-V_L} \frac{\sigma_W^2}{\sigma_W^2+\sigma_i^2}$. ‖

To complete the derivation of the equilibrium price function, one requires the investors' demand functions. The price reaction function G_i is the inverse of

$$g_i'(\hat{L}_i; a) = V_L + (V_H - V_L) \frac{\exp\left\{(V_H - V_L)\left(\hat{L}_i(s_i, \ell) + k_{i3}(a)\mu_W\right)\right\}}{1 + \exp\left\{(V_H - V_L)\left(\hat{L}_i(s_i, \ell) + k_{i3}(a)\mu_W\right)\right\}},$$

Hence,

$$G_i(p; a) = \frac{1}{V_H - V_L} \log\left(\frac{p - V_L}{V_H - p}\right) - k_{i3}(a)\mu_W.$$

Therefore, the demand function of investor i is

$$X_i(s_i, \ell, p) = \tau_i \left(k_{i1}(a)s_i + k_{i2}(a)\ell + k_{i3}(a)\mu_W - \frac{1}{V_H - V_L} \log\left(\frac{p - V_L}{V_H - p}\right)\right).$$

Let $\tau = \sum_{i=1}^N \tau_i$ be the aggregate risk tolerance. The market-clearing condition requires that when the demand functions are evaluated at an equilibrium price $P(\cdot)$,

$$\sum_{i=1}^N \tau_i \left(k_{i1}(a)s_i + k_{i2}(a)\ell + k_{i3}(a)\mu_W - \frac{1}{V_H - V_L} \log\left(\frac{P(\cdot) - V_L}{V_H - P(\cdot)}\right)\right) = z + \bar{z}$$

$$\Rightarrow \sum_{i=1}^N \tau_i k_{i1}(a)s_i - z + \ell \sum_{i=1}^N \tau_i k_{i2}(a) + \mu_W \sum_{i=1}^N \tau_i k_{i3}(a) - \bar{z} = \frac{\tau}{V_H - V_L} \log\left(\frac{P(\cdot) - V_L}{V_H - P(\cdot)}\right)$$

$$\Rightarrow P(\cdot) = V_L + (V_H - V_L) \frac{\exp\left\{\frac{V_H - V_L}{\tau}\left[\sum_{i=1}^N \tau_i k_{i1}(a)s_i - z + \ell \sum_{i=1}^N \tau_i k_{i2}(a) + \mu_W \sum_{i=1}^N \tau_i k_{i3}(a) - \bar{z}\right]\right\}}{1 + \exp\left\{\frac{V_H - V_L}{\tau}\left[\sum_{i=1}^N \tau_i k_{i1}(a)s_i - z + \ell \sum_{i=1}^N \tau_i k_{i2}(a) + \mu_W \sum_{i=1}^N \tau_i k_{i3}(a) - \bar{z}\right]\right\}},$$

where the second line pulls the summation inside the parentheses, and the final line rearranges to solve for $P(\cdot)$.

Since the solution a^* to the price-information equations satisfies $\tau_i k_{i1}(a^*) = a_i^*$, one can substitute to write the price function explicitly in terms of the equilibrium statistic L^*

$$P(\cdot) = V_L + (V_H - V_L) \frac{\exp\left\{\frac{V_H - V_L}{\tau}\left[L^*(s,z)\left(1 + \sum_{i=1}^N \tau_i k_{i2}(a^*)\right) + \mu_W \sum_{i=1}^N \tau_i k_{i3}(a^*) - \bar{z}\right]\right\}}{1 + \exp\left\{\frac{V_H - V_L}{\tau}\left[L^*(s,z)\left(1 + \sum_{i=1}^N \tau_i k_{i2}(a^*)\right) + \mu_W \sum_{i=1}^N \tau_i k_{i3}(a^*) - \bar{z}\right]\right\}}.$$

Defining k_2 as the aggregate (risk-tolerance-weighted) responsiveness to public information

$$k_2(a^*) \equiv \sum_{i=1}^N \tau_i k_{i2}(a^*),$$

and k_3 as the aggregate responsiveness to prior information

$$k_3(a^*) \equiv \sum_{i=1}^N \tau_i k_{i3}(a^*),$$

the price function is in the form of the general expression in Proposition 7

$$P(L^*(s,z)) = V_L + (V_H - V_L) \frac{\exp\left\{\frac{V_H - V_L}{\tau}\left[L^*(s,z)(1 + k_2(a^*)) + \mu_W k_3(a^*) - \bar{z}\right]\right\}}{1 + \exp\left\{\frac{V_H - V_L}{\tau}\left[L^*(s,z)(1 + k_2(a^*)) + \mu_W k_3(a^*) - \bar{z}\right]\right\}}.$$

Clearly this function is well defined and increasing for any $L^* \in \mathbb{R}$, as required. ‖

Proof (Example 2). Each informed investor receives an additive signal $\widetilde{S}_i = \widetilde{V} + \widetilde{\varepsilon}_i$, where $\widetilde{\varepsilon}_i \sim N(0, \sigma_i^2)$ are independently distributed. The supply shock is distributed $\widetilde{Z} \sim N(0, \sigma_Z^2)$. For simplicity, there are no uninformed investors, though they can be easily accommodated by allowing some of the σ_i^2 to tend to ∞.

To begin the derivation of equilibrium, I place the investors' beliefs in the exponential family form from Assumption 10. Let $L(s,z) \equiv \sum_{j=1}^N a_j s_j - z$. The joint distribution of $(\widetilde{V}, \widetilde{S}_i, L(\widetilde{S}, \widetilde{Z}))$ is

$$\begin{aligned}
& dF_{\widetilde{V},\widetilde{S}_i,\widetilde{L}}(v, s_i, \ell) \\
&= dF_{\widetilde{V}}(v) f_{\widetilde{S}_i,\widetilde{L}|\widetilde{V}}(s_i, \ell | v) ds_i d_\ell \\
&= dF_{\widetilde{V}}(v) \phi\left(s_i, \ell \middle| \left(\begin{pmatrix} v \\ \left(\sum_{j=1}^N a_j\right) v \end{pmatrix}\right), C_i(a)\right) ds_i d\ell \\
&= dF_{\widetilde{V}}(v) \frac{1}{2\pi\sqrt{|C_i|}} \\
&\quad \times \exp\left\{-\frac{1}{2}\begin{pmatrix} s_i \\ \ell \end{pmatrix}' C_i^{-1} \begin{pmatrix} s_i \\ \ell \end{pmatrix} + \begin{pmatrix} 1 \\ \sum_{j=1}^N a_j \end{pmatrix}' C_i^{-1} \begin{pmatrix} s_i \\ \ell \end{pmatrix} v - \frac{1}{2} \begin{pmatrix} 1 \\ \sum_{j=1}^N a_j \end{pmatrix}' C_i^{-1} \begin{pmatrix} 1 \\ \sum_{j=1}^N a_j \end{pmatrix} v^2 \right\} ds_i d\ell,
\end{aligned}$$

where $C_i(a) = \begin{pmatrix} \sigma_i^2 & a_i \sigma_i^2 \\ a_i \sigma_i^2 & \sum_{j=1}^n a_j^2 \sigma_j^2 + \sigma_Z^2 \end{pmatrix}$.

To determine the conditional density of $\widetilde{V}$ given $\widetilde{S}_i = s_i$ and $L(\widetilde{S}, \widetilde{Z}) = \ell$, I will use the fact that $dF_{\widetilde{V}|\widetilde{S}_i,\widetilde{L}}$ must be proportional to the joint distribution $dF_{\widetilde{V},\widetilde{S}_i,\widetilde{L}}$ derived above, where the constant of proportionality will depend on s_i and ℓ.

$$\begin{aligned}
dF_{\widetilde{V}|\widetilde{S}_i,\widetilde{L}}(v | s_i, \ell) &\propto dF_{\widetilde{V},\widetilde{S}_i,\widetilde{L}}(v, s_i, \ell) \\
&\propto dF_{\widetilde{V}}(v) \exp\left\{ \begin{pmatrix} 1 \\ \sum_{j=1}^N a_j \end{pmatrix}' C_i^{-1} \begin{pmatrix} s_i \\ \ell \end{pmatrix} v - \frac{1}{2} \begin{pmatrix} 1 \\ \sum_{j=1}^N a_j \end{pmatrix}' C_i^{-1} \begin{pmatrix} 1 \\ \sum_{j=1}^N a_j \end{pmatrix} v^2 \right\}. \qquad \text{(A.40)}
\end{aligned}$$

Integrating this expression with respect to v gives the constant of proportionality as

$$\left[\int_{\mathcal{V}} \exp\left\{ \begin{pmatrix} 1 \\ \sum_{j=1}^N a_j \end{pmatrix}' C_i^{-1} \begin{pmatrix} s_i \\ \ell \end{pmatrix} x - \frac{1}{2} \begin{pmatrix} 1 \\ \sum_{j=1}^N a_j \end{pmatrix}' C_i^{-1} \begin{pmatrix} 1 \\ \sum_{j=1}^N a_j \end{pmatrix} x^2 \right\} dF_{\widetilde{V}}(x) \right]^{-1}.$$

After computing the inverse of C_i:

$$C_i^{-1}(a) = \begin{pmatrix} \frac{\sum_{j=1}^N a_j^2 \sigma_j^2 + \sigma_Z^2}{\sigma_i^2\left[\left(\sum_{j=1}^N a_j^2 \sigma_j^2\right) + \sigma_Z^2 - a_i^2 \sigma_i^2\right]} & \frac{-a_i}{\left(\sum_{j=1}^N a_j^2 \sigma_j^2\right) + \sigma_Z^2 - a_i^2 \sigma_i^2} \\ \frac{-a_i}{\left(\sum_{j=1}^N a_j^2 \sigma_j^2\right) + \sigma_Z^2 - a_i^2 \sigma_i^2} & \frac{1}{\left(\sum_{j=1}^N a_j^2 \sigma_j^2\right) + \sigma_Z^2 - a_i^2 \sigma_i^2} \end{pmatrix},$$

and substituting, it follows after some tedious algebra that the density is in the desired form, with

$$k_{i1}(a) = \frac{1}{\sigma_i^2} \frac{\sum_{j=1}^N a_j^2 \sigma_j^2 + \sigma_Z^2 - a_i \sigma_i^2 \sum_{j=1}^N a_j}{\left(\sum_{j=1}^N a_j^2 \sigma_j^2\right) + \sigma_Z^2 - a_i^2 \sigma_i^2}$$

$$k_{i2}(a) = \frac{\sum_{j=1}^N a_j - a_i}{\left(\sum_{j=1}^N a_j^2 \sigma_j^2\right) + \sigma_Z^2 - a_i^2 \sigma_i^2}$$

$$b_i(\ell, a) = \ell$$

and

$$g_i(\hat{L}_i; a) = \log\left(\int_{\mathcal{V}} \exp\left\{ \hat{L}_i x - \frac{1}{2} \begin{pmatrix} 1 \\ \sum_{j=1}^N a_j \end{pmatrix}' C_i^{-1} \begin{pmatrix} 1 \\ \sum_{j=1}^N a_j \end{pmatrix} x^2 \right\} dF_{\widetilde{V}}(x)\right)$$

$$dH_i(v; a) = \exp\left\{-\frac{1}{2} \begin{pmatrix} 1 \\ \sum_{j=1}^N a_j \end{pmatrix}' C_i^{-1} \begin{pmatrix} 1 \\ \sum_{j=1}^N a_j \end{pmatrix} t^2 \right\} dF_{\widetilde{V}}(v),$$

or, equivalently,

$$H_i(v; a) = \int_0^v \exp\left\{-\frac{1}{2} \begin{pmatrix} 1 \\ \sum_{j=1}^N a_j \end{pmatrix}' C_i^{-1} \begin{pmatrix} 1 \\ \sum_{j=1}^N a_j \end{pmatrix} x^2 \right\} dF_{\widetilde{V}}(x).$$

With the beliefs in hand, the following Lemma establishes existence of a solution to the price-information equations. The proof is almost identical to that of Lemma A15 above, so I omit it.

Lemma A16. *There exists a solution a^* to the price information equations*

$$a_i^* = \frac{\tau_i}{\sigma_i^2} \frac{\sum_{j=1}^N (a_j^*)^2 \sigma_j^2 + \sigma_Z^2 - a_i^* \sigma_i^2 \sum_{j=1}^N a_j^*}{\sum_{j=1}^N (a_j^*)^2 \sigma_j^2 + \sigma_Z^2 - (a_i^*)^2 \sigma_i^2}, \quad i \in \{1, \ldots, N\}.$$

The solution satisfies

$$0 < a_i^* < \frac{\tau_i}{\sigma_i^2}, \quad i \in \{1, \ldots, N\}.$$

To complete the derivation of equilibrium, one requires the price reaction functions. They are not available in closed form without additional assumptions on $F_{\widetilde{V}}$ but are characterized as the inverses of g_i'

$$g_i'(\hat{L}; a) = \frac{\int_{\mathcal{V}} x \exp\left\{\hat{L}_i x - \frac{1}{2}\left(\frac{1}{\sum_{j=1}^N a_j}\right)' C_i^{-1} \left(\frac{1}{\sum_{j=1}^N a_j}\right) x^2\right\} dF_{\widetilde{V}}(x)}{\int_{\mathcal{V}} \exp\left\{\hat{L}_i x - \frac{1}{2}\left(\frac{1}{\sum_{j=1}^N a_j}\right)' C_i^{-1} \left(\frac{1}{\sum_{j=1}^N a_j}\right) x^2\right\} dF_{\widetilde{V}}(x)},$$

and the aggregate price reaction function $G(p; a) = \sum_{i=1}^N \tau_i G_i(p; a)$ is the risk-tolerance-weighted average of the individual functions.

Corollary 2 implies that with a^* defined by the price information equations, the price function, if it exists, must be given by

$$P(\cdot) = G^{-1}\left(\left[1 + \sum_{i=1}^N \frac{\tau_i \left(\sum_{j=1}^N a_j^* - a_i^*\right)}{\sum_{j=1}^N (a_j^*)^2 \sigma_j^2 + \sigma_Z^2 - (a_i^*)^2 \sigma_i^2}\right] L^*(s, z) - \bar{z}; a^*\right).$$

To guarantee that such a price function in fact exists, it suffices to show that $G^{-1}(\cdot; a^*)$ is monotone and maps the entire real line onto $(\underline{V}, \overline{V})$. This guarantees that an equilibrium price will be well defined for any realization of L^*. Because investor beliefs are of the exponential family form, an argument identical to that used in Lemma A10 implies that each g_i' (which is monotone) maps the entire real line onto the set $(\underline{V}, \overline{V})$. Hence, the inverses $G_i = (g_i')^{-1}$ are monotone functions that map $(\underline{V}, \overline{V})$ onto $\mathbb{R}$ so that $G = \sum_{i=1}^N \tau_i (g_i')^{-1}$ must also also be monotone and map $(\underline{V}, \overline{V})$ onto $\mathbb{R}$. Inverting once again, G^{-1} is monotone and maps $\mathbb{R}$ onto $(\underline{V}, \overline{V})$ as required. ‖

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Supplementary Data

Supplementary data are available at *Review of Economic Studies* online.

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