Irrigation and Drainage Engineering

(Soil Water Regime Management)

(ENV-549)

4ETCS, Master option

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Platform of Hydraulic Constructions

Project: flooding in Nouakchott (Mauritania) – Groundwater modelling and numerical solutions for steady state

Water budget

- Input:
  - Precipitation
  - Groundwater recharge = definition
  - Injection
- Output:
  - Evapotranspiration
  - Pumping
  - Groundwater flow

Groundwater flow

- Groundwater = continuous variable (a value at a time depends on the value at the previous time)
- Determination of groundwater flow by measuring groundwater depth in observation wells (single point measurement)
- From groundwater depth measurements ➔ piezometric map
- Groundwater flow from the highest hydraulic heads to the lowest ➔ diffusivity equation

Diffusion equation

- Continuity equation

$$\frac{\partial \rho}{\partial t} + \rho \nabla \cdot \vec{U} = 0$$

  - 2D situation → $w = 0$
  - Steady state → $\frac{\partial}{\partial t} = 0$

$$\vec{U} = (u, v, w)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

- Darcy equation

$$\vec{U}_i = -K_i \frac{dh}{di}$$

$$\frac{\partial}{\partial x}\left(-K_x \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(-K_y \frac{\partial h}{\partial y}\right) = 0$$

Directionally homogeneous and isotropic

$$K_x \frac{\partial^2 h}{\partial x^2} + K_y \frac{\partial^2 h}{\partial y^2} = 0$$

Fully homogeneous and isotropic

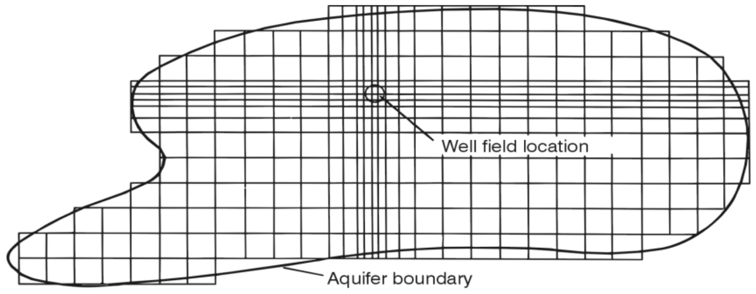
$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = \nabla^2 h = 0$$

Laplace equation

Numerical modelling

- Simplifying assumptions for the diffusion equation
- Spatial discretization (cells of a grid) and temporal discretization (time steps)

➔ Finite difference to estimate the partial derivative in each cell (one equation per cell)



Fetter, 1994

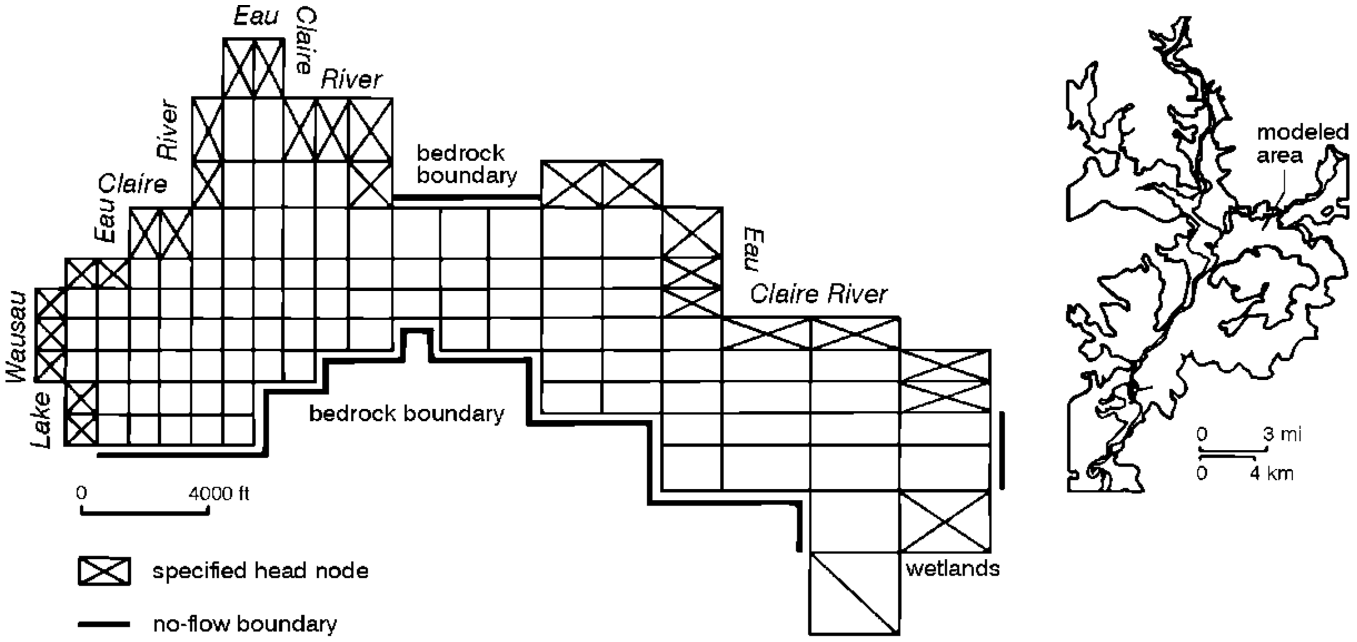
More on numerical solutions vs analytical solutions?

➔ Bear, J., & Cheng, A. H.-D. (2010). Modeling Groundwater Flow and Contaminant Transport. Springer Netherlands. https://doi.org/10.1007/978-1-4020-6682-5

Needed information for simulation

- Hydraulic conductivity (or transmissivity) in each cell
- Pumping and groundwater recharge in each cell
- Aquifer geometry and spatial discretization
- Boundary conditions (constant head;

  specified flow, including no flow)



Anderson et Woessner, 1991

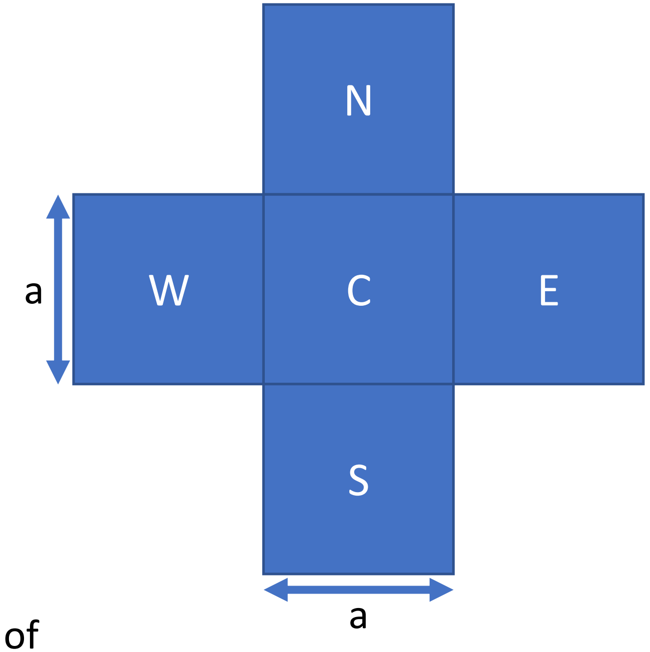
Steady state

- Finite difference solution

Conditions of application: unconfined water table, homogeneous, isotropic, steady state, horizontal two-direction flow

➔ Darcy law on a square-shaped grid

➔ Water budget for each cell of the grid



$$Q = Kab\frac{\Delta h}{L} \quad \text{so} \quad Q = Ta\frac{\Delta h}{L}$$

With b the aquifer thickness
L the distance between the center of two cells

$$\sum Q_i = 0$$

$$\Updownarrow \ L = a$$

$$\sum \left[T_i \Delta h_i \right] = 0$$

Steady state

As the cells are square shaped:

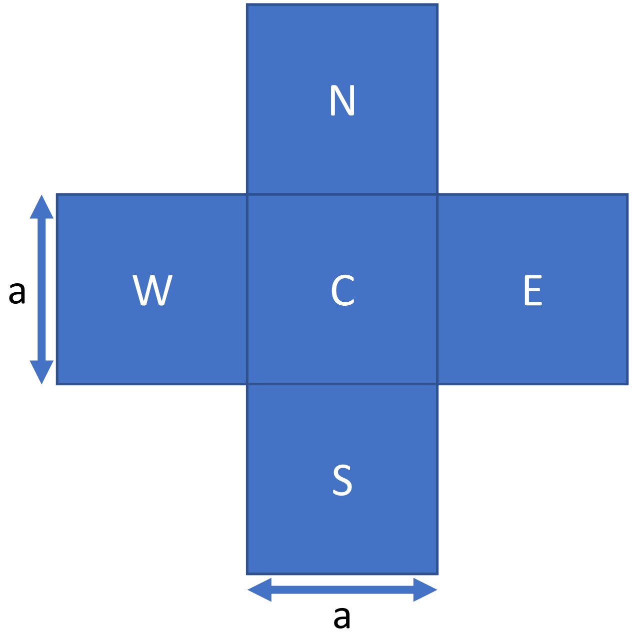
$$\sum [T_i \Delta h_i] = 0$$

$$T_{CN}(h_C - h_N) + T_{CS}(h_C - h_S) + T_{CE}(h_C - h_E) + T_{CW}(h_C - h_W) = 0$$

$$h_C = \frac{[T_{CN} h_N + T_{CS} h_S + T_{CE} h_E + T_{CW} h_W]}{[T_{CN} + T_{CS} + T_{CE} + T_{CW}]}$$

$$h_C = \frac{\sum [T_{iC} h_i]}{[\sum T_{iC}]}$$

➔As T = constant $\quad h_C = \frac{T \sum [h_i]}{4T} = \frac{\sum [h_i]}{4}$



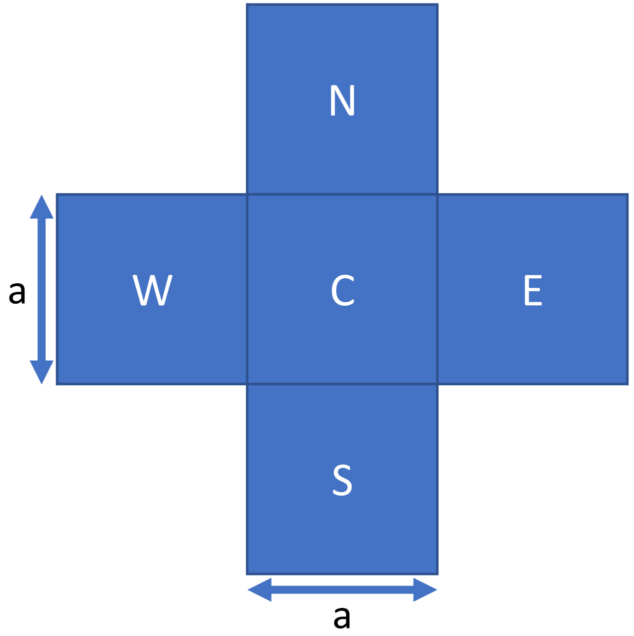
Steady state

➔ If well in cell C (pumping < 0, injection > 0)

$$\sum_i Q = -Q_p$$

$$\sum [T_i \Delta h_i] = -Q_p$$

$$h_C = \frac{\sum[T_{iC} h_i] + Q_p}{[\sum T_{iC}]}$$



➔ If groundwater recharge in cell C

$$\sum_i Q = w$$

$$\sum [T_i \Delta h_i] = w$$

$$h_C = \frac{\sum[T_{iC} h_i] - w}{[\sum T_{iC}]}$$

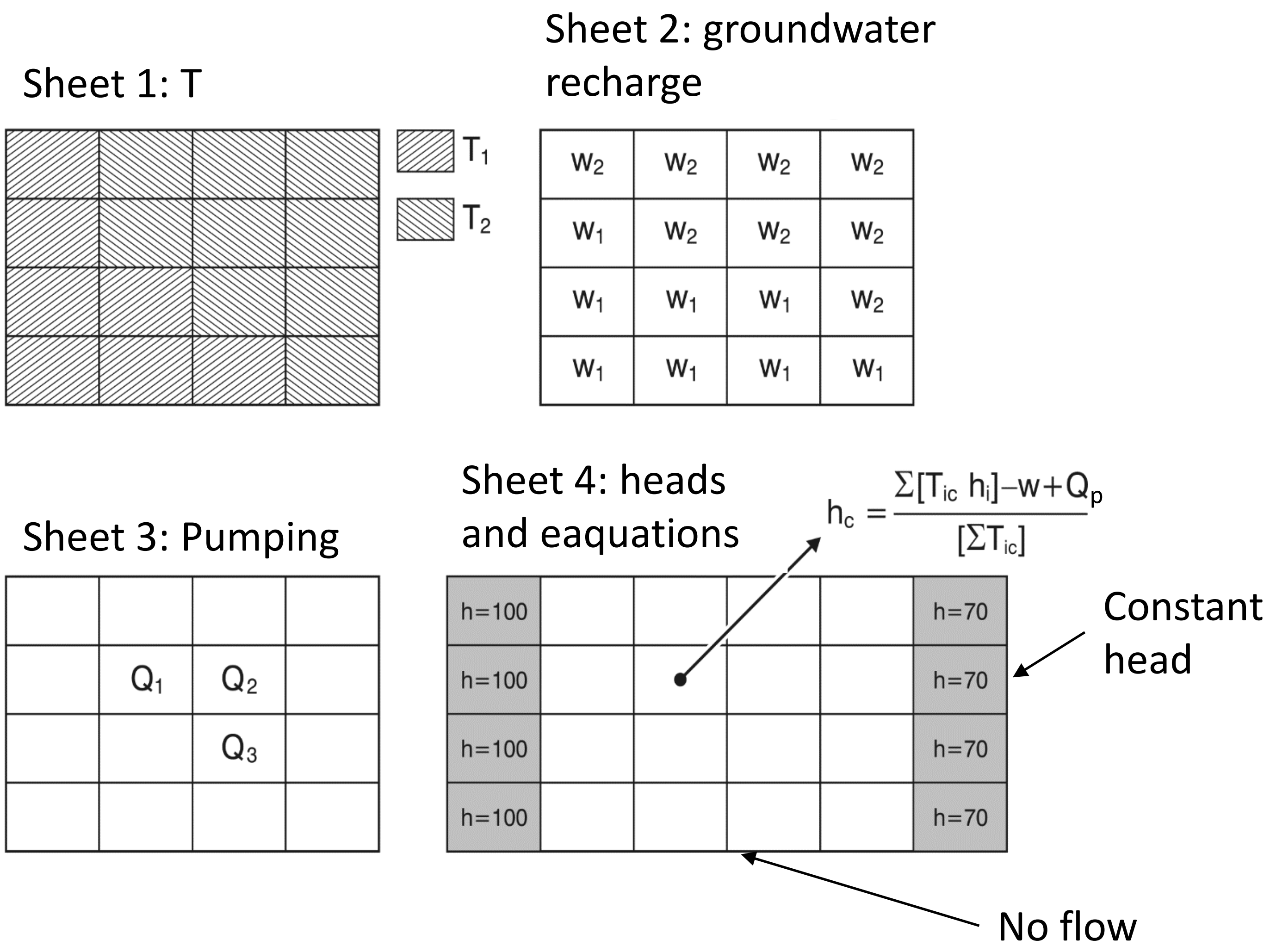
Steady state

- In Excel??

➔ One equation for each cell

➔ System with N equations

➔ Iterative computing



More on groundwater modelling?
➔ Anderson, M. P., Woessner, W. W., & Hunt, R. J. (2015). Applied Groundwater Modeling: Simulation of Flow and Advective Transport (Second Edition). Academic Press, Elsevier Inc.

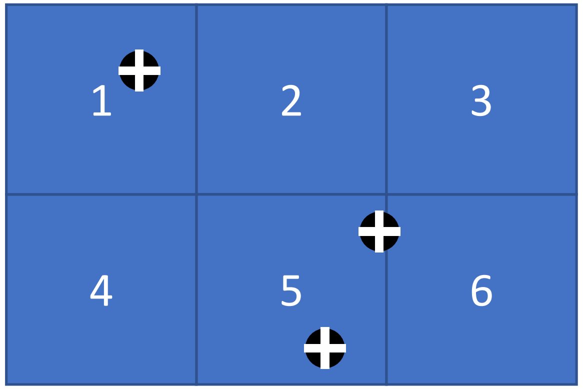
Model precision

What do we use to compute the precision of a simulation?

➔ The comparison of the simulated and observed variable(s) of interest

Which data for the Nouakchott project?

➔ Simulated hydraulic head (groundwater level) and measured hydraulic heads
  - ➔ Simulated hydraulic head = 1 value for each cell of the simulated area (hydraulic head constant within each cell)
  - ➔ Measured hydraulic head = 3 observation wells + 1



Objective functions

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Objective functions ➔ indication on the performance of the model

« how well did the model perform? »

➔ Comparison between simulated and observed variables

$$ME = \frac{1}{n}\sum_{t=1}^{n}(\mathrm{sim}_t - \mathrm{obs}_t)$$

$$AME = \frac{1}{n}\sum_{t=1}^{n}|\mathrm{sim}_t - \mathrm{obs}_t|$$

With
n the number of measurements
Sim the simulated value
Obs the observed value

An objetive function is optimized ⇔ maximization or minimization depending on the formula

Name	Description	Formula*
DRMS	Daily Root Mean Squared Error	$\sqrt{\frac{1}{n}\sum_{t=1}^{n}(d_t - o_t(\theta))^2}$
TMVOL	Total Mean Monthly Volume Squared Error	$\sum_{i=1}^{n_{\mathrm{month}}}\left(\frac{1}{n_{\mathrm{day}}(i)}\sum_{t=1}^{n_{\mathrm{day}(i)}}(d_t - o_t(\theta))\right)^2$
ABSERR	Mean Absolute Error	$\frac{1}{n}\sum_{t=1}^{n}\lvert d_t - o_t(\theta)\rvert$
ABSMAX	Maximum Absolute Error	$\max_{1\le t\le n}\lvert d_t - o_t(\theta)\rvert$
NS	Nash-Sutcliffe Measure	$1 - \frac{\frac{1}{n}\sum_{t=1}^{n}[d_t - o_t(\theta)]^2}{\frac{1}{n}\sum_{t=1}^{n}(d_t - \bar{d})^2}$
BIAS	Bias (mean daily error)	$\frac{1}{n}\sum_{t=1}^{n}(d_t - o_t(\theta))$
PDIFF	Peak Difference	$\max_{1\le t\le n}\{d_t\} - \max_{1\le t\le n}\{o_t(\theta)\}$
RCOEF	First Lag Autocorrelation	$\frac{\frac{1}{n}\sum_{t=1}^{n}(d_t - o_t(\theta))(d_{t+1} - o_{t+1}(\theta))}{\sigma_d\sigma_{o(\theta)}}$
NSC	Number of Sign Changes	(Count the number of times the sequence of residuals changes sign)

*Minimize with respect to θ.

Vijai Gupta et al. (1998)

Darcy law

➔ Calculate the groundwater flux between two points

In the Nouakchott model, as the cells are square shaped:

$$Q = Kab\frac{\Delta h}{L}$$

$$Q = Ta\frac{\Delta h}{L}$$

With K the hydraulic conductivity ($m.s^{-1}$)
a the side of the cell (m)
b the thickness of the aquifer (m)
Δh the difference in hydraulic head between the center of 2 cells
L the distance between the center of 2 cells (m)
T the transmissivity ($m^2.s^{-1}$) (T = K*b)