

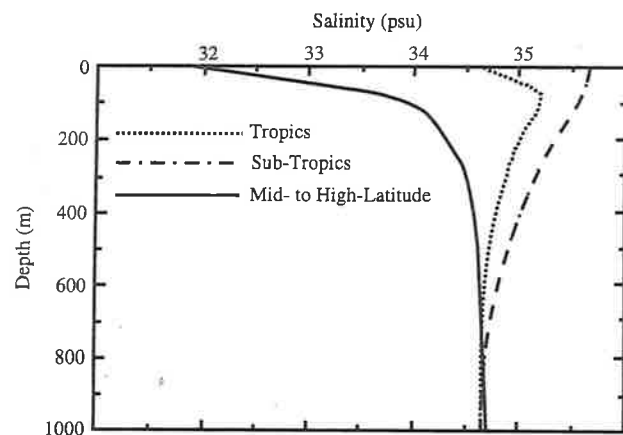


Figure 1.2 Profiles of annual mean ocean salinity in the upper ocean. High salinities near the surface are observed in subtropical latitudes because evaporation of fresh water from the ocean exceeds precipitation in these regions. In latitudes where precipitation exceeds evaporation, and hence the flux of fresh water into the ocean is positive, salinities are relatively low. In the deep ocean, salinity varies little with latitude, since evaporation and precipitation—the primary mechanisms influencing salinity—act at the surface. (Data from Levitus, 1982.)

The average salinity of the world ocean is 34.7 psu. Salinity in the open ocean ranges from about 33 to 38 psu. Higher values of salinity occur in regions of high evaporation such as the Mediterranean and Red Seas, where salinity values reach as high as 39 and 41 psu, respectively. Profiles of annual and zonal mean salinities are given in Figure 1.2. The salinity is large in the subtropical latitudes because evaporation exceeds precipitation and leaves the water enriched with salt. Salinities are low in the tropics and the mid-latitudes, where precipitation is high. The salinity of the North Atlantic Ocean averages 37.3 psu, compared with the North Pacific Ocean salinity of 35.5 psu.

In addition to dissolved salts, seawater also has dissolved gases (e.g., oxygen, carbon dioxide, and sulphur dioxide) and a variety of suspended particles (e.g., soil, atmospheric aerosol, and biogenic particulate matter).

1.3 Pressure

Pressure is defined as force per unit area, $p = \mathcal{F}/A$. The principal force contributing to pressure in the atmosphere and ocean is the gravitational force.¹ The mass per unit

¹ Bannon *et al.* (1997) estimate that the surface pressure is a factor of 0.25% less than the weight per unit area of a resting atmosphere, because of lateral pressure forces associated with a curved surface geometry. Vertical accelerations can also contribute to the pressure in a column.

area of the atmosphere is approximately 10^4 kg m^{-2} , and since the acceleration due to gravity is about 10 m s^{-2} , the surface atmospheric pressure is about $p_0 = 10^5 \text{ Pa}$. Since the mass of the world ocean is about 270 times the mass of the atmosphere, pressures in the ocean are substantially greater than those in the atmosphere. The pressure at any point in the ocean is the sum of the atmospheric pressure plus the weight of the ocean in a column above the point per unit area.

In SI units, the pascal (Pa) is the unit of pressure, where $1 \text{ Pa} = 1 \text{ N m}^{-2}$, and N is a newton. Alternative units of pressure include:

bar (bar):	1 bar = 10^5 Pa
millibar (mb):	1 mb = 10^2 Pa
torricelli (torr):	1 torr = 133.322 Pa
atmosphere (atm):	1 atm = 1.01325 bar
	= 760 torr
	= $1.01325 \times 10^5 \text{ Pa}$

While Pa is the preferred unit of pressure, torr is a unit commonly used by atmospheric chemists and chemical oceanographers. The unit mb is frequently used by meteorologists, and oceanographers commonly use decibars (db). The preferred pressure unit for meteorology is hPa (10^2 Pa), which is equivalent to mb.

The vertical variation of pressure in the atmosphere is shown in Figure 1.3 to decrease almost exponentially with height, from a mean sea-level pressure of 1013.25 hPa. Approximately 90% of the weight of the atmosphere lies below 15 km. The

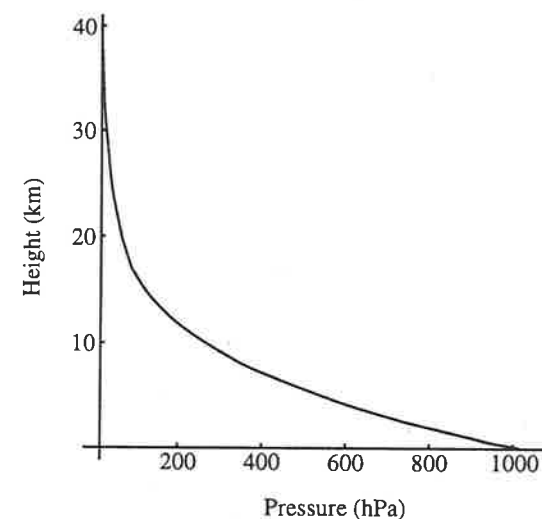


Figure 1.3 Variation of atmospheric pressure with height (U.S. Standard Atmosphere, 1976).

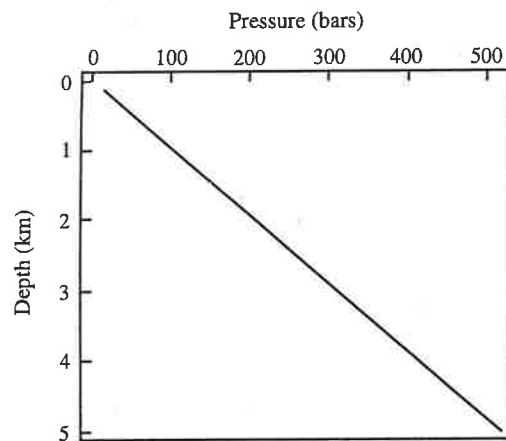


Figure 1.4 Vertical variation of pressure with depth in the ocean.

vertical variation of pressure with depth in the ocean is shown in Figure 1.4 to be approximately linear. In dealing with the pressure in the oceans, the atmospheric pressure is subtracted and the pressure at the sea surface is entered as zero. An increase of 10 m in depth in the ocean corresponds to an increase of 10^5 Pa, which is approximately 1 atm. Thus the pressure at a depth of 1 km in the ocean is equivalent to approximately 100 atm.

1.4 Density

Because of the large volumes characterizing the atmosphere and ocean, an intensive volume is desired. Such a volume, v , referred to as the *specific volume*, is given by $v = V/m$ so that the units of v are $\text{m}^3 \text{kg}^{-1}$. The specific volume is the inverse of the density, ρ , which has units of kg m^{-3} .

Figure 1.5 shows the vertical variation of density with height in the atmosphere. Density decreases with height nearly exponentially, which is related to the pressure decrease (Figure 1.3). A typical value of surface air density is 1.3 kg m^{-3} . The *mean free path* of molecules, which is determined by the frequency of intermolecular collisions, is inversely proportional to density. The mean free path increases exponentially from a value of about 10^{-7} m at the surface to the order of 1 m at 100 km.

Liquid water is almost three orders of magnitude more dense than air. Hence, the interface between the atmosphere and ocean is very stable. Since liquid water is nearly

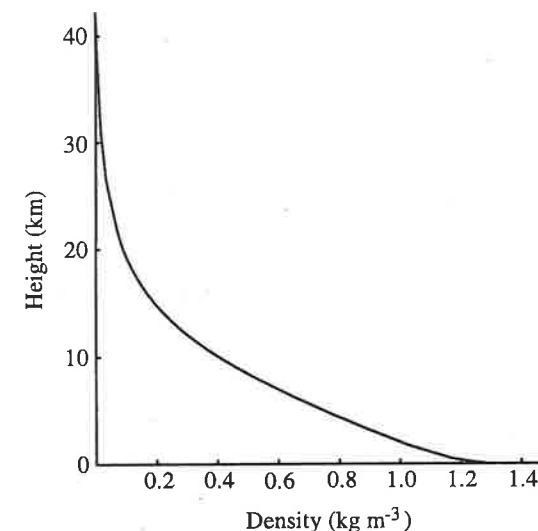


Figure 1.5 Vertical variation of density with height in the atmosphere. Density, like pressure, decreases nearly exponentially with height (U.S. Standard Atmosphere, 1976).

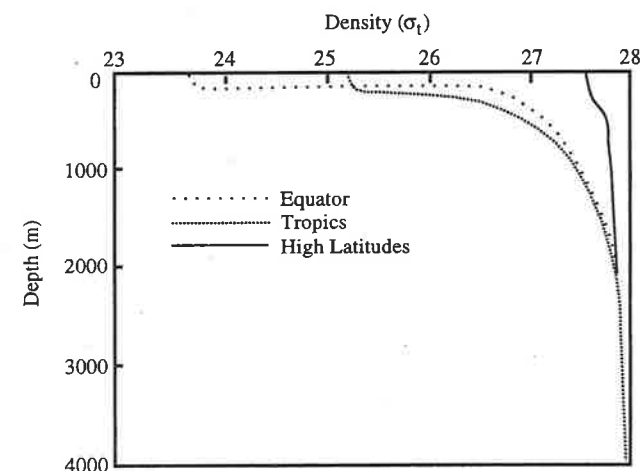


Figure 1.6 Density profiles in the ocean. Near the equator and throughout the tropics, the density increases rapidly with depth below a shallow surface layer of nearly constant density. This is due to the rapid cooling with depth in the ocean in these regions (see Figure 1.8). At very high latitudes, salinity is more important than temperature in regulating the density, and thus the σ_t profile near the surface is very different from the low-latitude profiles (see Figure 1.2). (Data from Levitus, 1982.)

1.10 Hydrostatic Equilibrium

Thus far we have examined the thermodynamic state of individual masses of air and seawater. Here we consider the state of the atmosphere and ocean in the presence of a gravitational field, particularly the height dependence of pressure, temperature, and density. The strength of the gravitational field, which depends primarily on the mass of the planet, is a central determinant of the mass of the atmosphere and ocean. Although both the atmosphere and ocean are bound to the earth by gravity, the ocean has a finite depth, while the atmosphere does not have a top and blends slowly into interplanetary space. The reason for this difference is that the atmosphere is compressible while the ocean is nearly incompressible, since the density of the atmosphere varies with pressure whereas the density of the ocean hardly varies at all.

The vertical variations of pressure in the atmosphere and ocean are observed to be much larger than either the horizontal or temporal variations. The decrease in pressure with height in the atmosphere, and the increase of pressure with depth in the ocean gives rise to a *vertical pressure gradient force*, $\mathcal{F}_p$

$$\mathcal{F}_p = -\frac{1}{\rho} \frac{\partial p}{\partial z} \quad (1.32)$$

where z is depth/height from the surface and ρ is the density.

The vertical pressure gradient force results in a vertical acceleration in the direction of decreasing pressure (upwards). The vertical pressure gradient force is generally in very close balance with the downward force due to gravitational attraction. This is called *hydrostatic balance*, and is written as

$$g = -\frac{1}{\rho} \frac{\partial p}{\partial z} \quad (1.33)$$

where g is the acceleration due to the Earth's gravity. The hydrostatic balance is applicable to most situations in the atmosphere and ocean, exceptions arising in the presence of large vertical accelerations such as are associated with thunderstorms.

Equation (1.33) can be integrated to determine a relationship between pressure and depth or height:

$$-\int dp = \int \rho g dz \quad (1.34)$$

To integrate (1.34), it is commonly assumed that g is constant; however, g varies with distance from the Earth's center and also with latitude because of the nonsphericity of the Earth.

To account for these variations in g , the *geopotential* ϕ is often introduced

$$\phi(z) = \int_0^z g dz \quad (1.35)$$

where the geopotential at sea level $\phi(0)$ is taken to be zero by convention. ϕ is the gravitational potential energy per unit mass, with units J kg^{-1} . Using the geopotential, we may write an alternative and equivalent statement of the hydrostatic balance:

$$dp = -\rho d\phi$$

The *geopotential height*, Z , can be defined for application to the atmosphere as

$$Z = \frac{\phi(z)}{g_0} = \frac{1}{g_0} \int_0^z g dz \quad (1.36a)$$

where $g_0 = 9.8 \text{ m s}^{-2}$ is the globally averaged acceleration due to gravity at the Earth's surface. The force of gravity is thus perpendicular to surfaces of constant ϕ , while not exactly perpendicular to surfaces of constant z . Geopotential height is used as a vertical coordinate in many atmospheric applications. In the lower atmosphere, Z is very nearly equal to z ; at a distance of $z = 10 \text{ km}$ above the Earth's surface at 40°N , $g = 9.771 \text{ m s}^{-2}$ and $Z = 9.986 \text{ km}$. In oceanographic applications, the *dynamic depth*, D , is used analogously to the geopotential height in the atmosphere

$$D = \frac{\phi(z)}{g_0} = \frac{1}{g_0} \int_{p_0}^p \rho dp \quad (1.36b)$$

where $D(p_0) = 0$ is assumed by convention and the *dynamic meter*, dm , is the common unit of dynamic depth. The pressure change is usually expressed in decibars (db), where $1 \text{ db} = 100 \text{ mb}$, since a pressure of 1 db is equivalent to a change of dynamic depth of about 1 dm .

Since the focus of this text is on the ocean and lower atmosphere, we will assume that $g = g_0$ is a constant, which simplifies the integration and evaluation of (1.34). However, integration of (1.34) also requires some assumption about the vertical variation of the density, ρ .

Because seawater is nearly incompressible, density is nearly constant within the ocean, and thus there is a nearly linear relationship of pressure with depth (Figure 1.4). In the ocean, the following integrated form of the hydrostatic equation is used:

$$p(-z) = p_0 + \rho g z \quad (1.37a)$$

where p_0 is the atmospheric pressure. If we assume that $\rho = 1036 \text{ kg m}^{-3}$ and $g = 9.8 \text{ m s}^{-2}$, we may then write

$$p(-z) = p_a + 10153 z \quad (1.37b)$$

where z is in meters and p is in Pa. Because $10^5 \text{ Pa} = 1 \text{ bar}$, it is easily seen that the oceanic pressure increases at approximately 1 db per meter of depth. Ocean pressures given in db are numerically equivalent to the depth in meters to within 1–2%. However, if seawater were actually incompressible, the sea level would rise by more than 30 m, because the hydrostatic pressure in the deep ocean is so great.

Because air is compressible and density decreases with height in the atmosphere (Figure 1.5), integration of (1.34) for the atmosphere is more complicated than for the ocean. However, useful insights can be derived from examining an idealized *homogeneous atmosphere*, where density is assumed constant. Consideration of a homogeneous atmosphere with finite surface pressure implies a finite total height for the atmosphere, which is called the *scale height* H . Assuming that density is constant, we can integrate (1.34) from sea level, where the pressure is p_0 , to a height H , where the pressure is zero, to obtain

$$p_0 = \rho g H \quad (1.38)$$

The height of the homogeneous atmosphere (often referred to as the *scale height*) is therefore

$$H = \frac{p_0}{\rho g} = \frac{R_d T_0}{g} \quad (1.39)$$

where T_0 is the surface temperature and H can be evaluated from the surface temperature and known constants to be approximately 8 km. From the ideal gas law, it is easily inferred that temperature must decrease with height in the homogeneous atmosphere. The lapse rate of the homogeneous atmosphere is obtained by differentiating the ideal gas law with respect to z , holding density constant

$$\frac{\partial p}{\partial z} = \rho R_d \frac{\partial T}{\partial z} \quad (1.40)$$

Combining (1.40) with the hydrostatic equation (1.33) leads to the result

$$\Gamma = -\frac{\partial T}{\partial z} = \frac{g}{R_d} = 34.1^\circ\text{C km}^{-1} \quad (1.41)$$

Thus the lapse rate of a homogeneous atmosphere is constant and about six times larger than the lapse rate normally observed in the atmosphere (which is $\Gamma \approx 6.5^\circ\text{C km}^{-1}$). The lapse rate for the homogeneous atmosphere is referred to as the *autoconvective lapse rate* for the following reason: if the lapse rate exceeds the autoconvective value, it is implied that the lower air is less dense than the air above, causing the atmosphere to overturn and the spontaneous initiation of convection. Values of the atmospheric lapse rate as large as the autoconvective value are observed over desert surfaces in summer when the solar heating is high; however, lapse rates in the atmosphere typically do not exceed $\Gamma \approx 10^\circ\text{C km}^{-1}$.

Further insight is gained by examining the characteristics of yet another idealized atmosphere, called the *isothermal atmosphere*. After substitution of the ideal gas law for density, we can write the hydrostatic equation in the following form:

$$\partial p = -\frac{p g}{R_d T} \partial z \quad (1.42)$$

This equation is easily integrated for a constant temperature from sea level ($z = 0$, $p = p_0$) to some arbitrary height z

$$\int_{p_0}^p \frac{dp}{p} = -\frac{g}{R_d T} \int_0^z dz \quad (1.43)$$

or

$$\ln \frac{p}{p_0} = -\frac{g z}{R_d T} \quad (1.44a)$$

Taking antilogs and using $H = RT/g$, we have

$$p = p_0 \exp(-z/H) \quad (1.44b)$$

Thus pressure decreases exponentially with height in an isothermal atmosphere, and there is no definite upper boundary to this atmosphere. Note that when $z = H$, the pressure is $1/e$ of its surface value. The isothermal atmosphere resembles the real atmosphere more closely than does the homogeneous atmosphere; however, (1.44b) is not applicable to the real atmosphere except when applied over a shallow layer above the ground.

Many meteorological applications require an accurate relationship between atmospheric pressure and height, which necessitates considering the variation of temperature with height. These applications include: determination of the elevation at which

the observations of pressure, temperature, and humidity are obtained from balloons carrying radiosondes; conversion between pressure and height as a vertical coordinate in numerical models of the atmosphere; reduction of surface pressure to sea-level pressure over land; and determination of the thickness between pressure levels. The vertical variations of the temperature profile can be accounted for by integrating (1.42) in a piecewise manner, between height levels that are close enough so that a mean atmospheric temperature in the layer can be defined. Thus we have

$$\int_{z_1}^{z_2} g \, dz = - \int_{p_1}^{p_2} \frac{R_d \bar{T}_v}{p} dp$$

Assuming that $\bar{T}_v$ is constant within the layer, we can integrate to obtain

$$z_2 - z_1 = - \frac{R_d \bar{T}_v}{g} \ln \frac{p_2}{p_1} \quad (1.45)$$

or

$$p_2 = p_1 \exp \left[\frac{g}{R_d \bar{T}_v} (z_1 - z_2) \right] \quad (1.46)$$

Equation (1.45) is referred to as the *hypsothetic equation*. From (1.45), it is seen that the *thickness* $\Delta z = z_2 - z_1$ of a layer bounded by two isobaric surfaces is proportional to the average virtual temperature of the layer ($\bar{T}_v$). Figure 1.12 shows the variation with latitude of the relative thickness of isothermal atmospheric layers. Since temperature decreases with latitude away from the equator, the distance between two isobaric surfaces decreases from equator to pole.

An additional application of the hydrostatic equation to the atmosphere is integration under the assumption of a constant lapse rate. Assuming that temperature varies linearly with height with a lapse rate Γ , we have

$$T = T_0 - \Gamma z \quad (1.47)$$

Substituting (1.47) into (1.42) yields

$$\frac{dp}{p} = - \frac{g}{R_d} \frac{dz}{T_0 - \Gamma z}$$

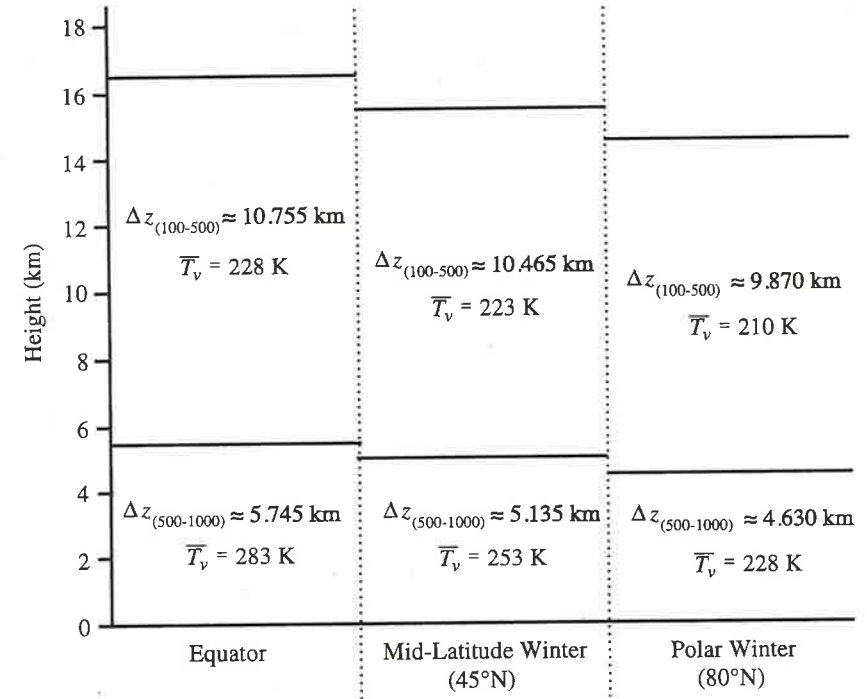


Figure 1.12 Representation of the thicknesses of the 1000–500 hPa and 500–100 hPa layers and their variation with latitude. The thickness of the layer between two isobaric surfaces is determined by the mean virtual temperature in the layer, according to (1.45), resulting in layers of decreasing thickness from equator to pole.

This equation is easily integrated between the limits ($z = 0, p = p_0$) and (z, p) to obtain

$$\ln \frac{p}{p_0} = \frac{g}{R_d \Gamma} \ln \left(\frac{T_0 - \Gamma z}{T_0} \right)$$

or

$$p = p_0 \left(\frac{T}{T_0} \right)^{g/R_d \Gamma} \quad (1.48)$$

Note that the exponent in (1.48) is equal to the ratio of the autoconvective lapse rate (1.41) to the actual lapse rate.