Lecture 3 – ENV 407

Review of Atmospheric Thermodynamics

Moist Processes - Hydrostatics

Applications:

- Clausius-Clapeyron Equation
- Humidity Variables
- Moist Processes in the atmosphere (lecture on whiteboard)
- Hydrostatics

Moist processes (whiteboard lecture):

- Energy content of a moist parcel
- Isobaric cooling (radiative fogs) – dew point, frost point, relative humidity in terms of the dew point temperatures
- Cooling and moistening by evaporation. Wet bulb, ice bulb temperature

Hydrostatic Balance

Although the horizontal atmosphere is in a constant state of motion, vertical velocities are fairly small (especially averaged over the large scale). Consequently, to understand the vertical structure of the atmosphere, we can approximate it to be largely steady.

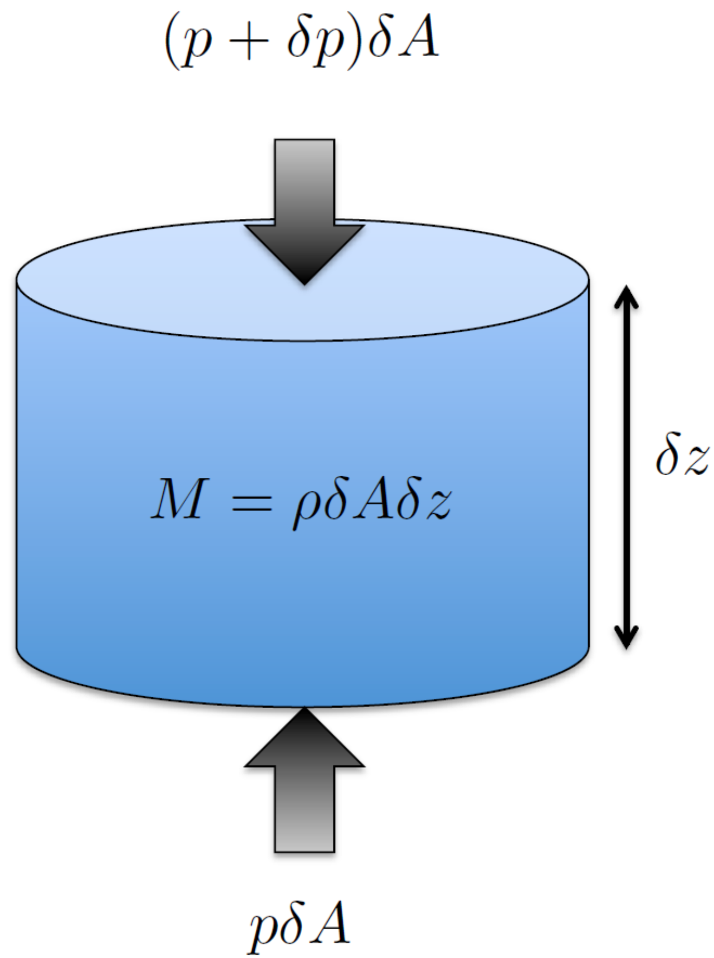


Figure: A vertical column of air of density ρ, horizontal cross-section δA, height δz and mass $M = \varrho\ \delta A\ \delta z$. The pressure at the lower surface is p, the pressure at the upper surface is $p + \delta p$.

Hydrostatic Balance

If the cylinder of air is not accelerating, it must be subject to zero net force. The vertical forces are:

Pressure acting on bottom face

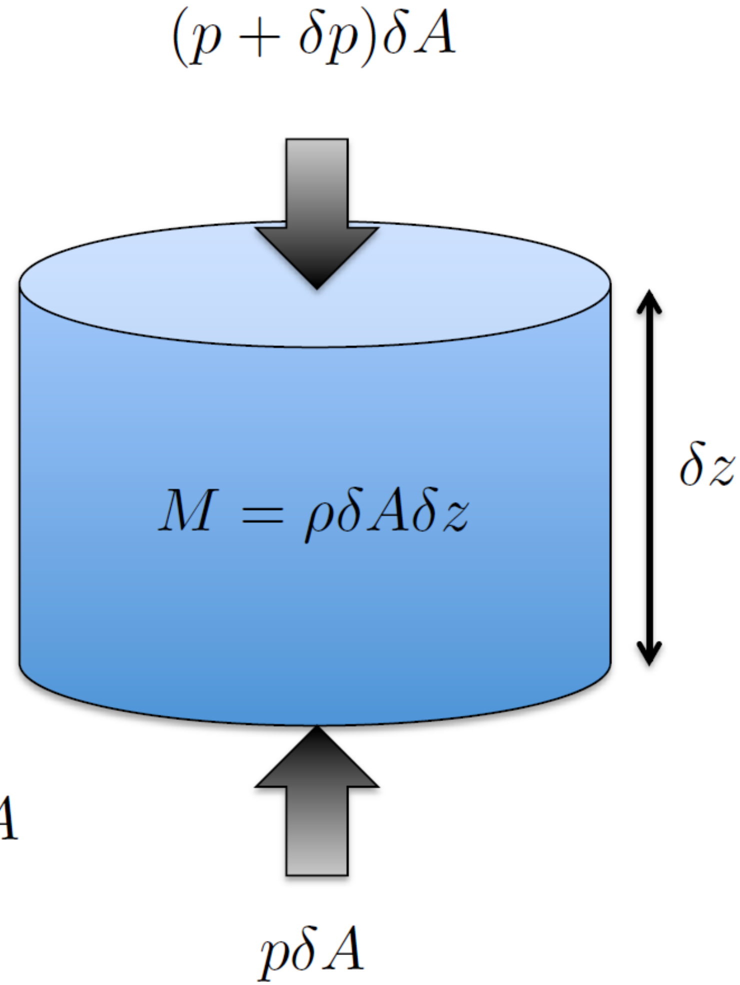
$$F_B = p\delta A$$

Gravity

$$F_g = -gM$$
$$= -g\rho\delta A\delta z$$

Pressure acting on top face

$$F_T = -(p + \delta p)\delta A$$



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Hydrostatic Balance

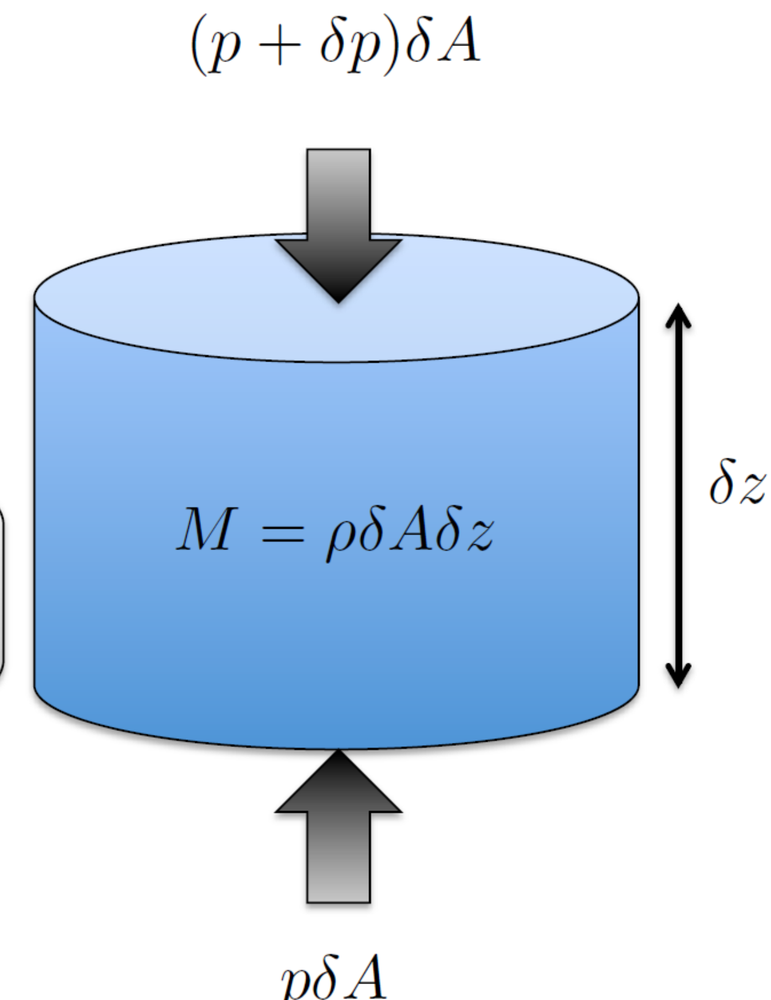
$$F_g + F_T + F_B = 0$$

$$\delta p + g\rho\delta z = 0$$

Taylor Series $\delta p \approx \frac{\partial p}{\partial z}\delta z$

Hydrostatic Balance

$$\frac{1}{\rho}\frac{\partial p}{\partial z} = -g$$



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Aside: Consider the special case of an isothermal (constant temperature) atmosphere:

$$\frac{1}{\rho}\frac{\partial p}{\partial z} = -g$$

Hydrostatic Balance

This equation does not give pressure explicitly in terms of height, since the density of air is not known.

Ideal gas law

$$\rho = \frac{p}{R_d T}$$

$$\frac{\partial p}{\partial z} + \frac{pg}{RT} = 0$$

For an isothermal atmosphere $(T = T_0)$ this equation can be exactly solved:

$$p(z) = p_s \exp\left(-\frac{zg}{R_d T}\right)$$

Exponential decay

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Aside: Consider the special case of an isothermal (constant temperature) atmosphere:

$$\Rightarrow \quad p(z) = p_s \exp\left(-\frac{gz}{R_d T_0}\right)$$

Definition: The **scale height** of an isothermal atmosphere is given by:

$$H = \frac{R_d T_0}{g}$$

$$\Rightarrow \quad p(z) = p_s \exp\left(-\frac{z}{H}\right)$$

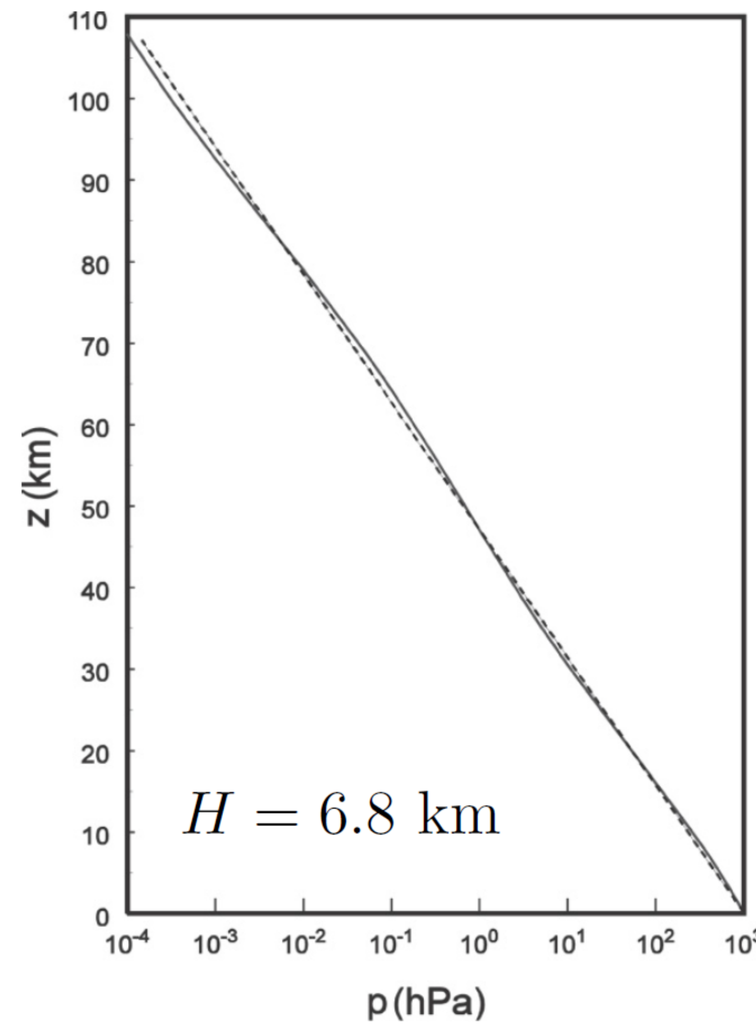
The **scale height** is an example of a quantity which imparts a notion of a "natural measuring stick" for an idealized atmosphere. This notion will generalize to more realistic atmospheric flows as well.

Aside: Consider the special case of an isothermal (constant temperature) atmosphere:

For an isothermal atmosphere $T = T_0$

$$p(z) = p_s \exp\left(-\frac{z}{H}\right)$$

Figure: Observed profile of pressure (solid) plotted against isothermal profile. Observed temperature variations only lead to small variations in the pressure from an exponential profile.



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Geopotential

$$\boxed{\nabla\Phi = g\mathbf{k}}$$

In height coordinates, geopotential is purely a function of z

$$\Rightarrow \quad \frac{d\Phi}{dz} = g$$

Integrate $\Rightarrow \quad \Phi(z) - \Phi(0) = \int_0^z g dz$

Define $\Phi(0) = 0 \quad \Rightarrow \quad \Phi(z) = \int_0^z g dz = gz$

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The use of **geopotential on constant pressure surfaces** is analogous to the use of **pressure on constant height surfaces**.

Question: How are geopotential and pressure connected?

From $\frac{d\Phi}{dz} = g$ $\Rightarrow$ $g dz = d\Phi$

From $\frac{dp}{dz} = -\rho g$ $\Rightarrow$ $g dz = -\frac{dp}{\rho}$

Hydrostatic balance

Ideal gas law $\rho = \frac{p}{R_d T}$

$$d\Phi = -\frac{R_d T dp}{p}$$

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Ideal gas law $\quad \rho = \dfrac{p}{R_d T}$

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$$d\Phi = -\frac{R_d T dp}{p}$$

Recall: From elementary calculus, $\frac{dp}{p} = d\ln p$

$$d\Phi = -R_d T d\ln p$$

Integrate over a layer

$$\Phi(z_2) - \Phi(z_1) = -R_d \int_{p_1}^{p_2} T d\ln p$$

Use geopotential height

$$Z_2 - Z_1 = -\frac{R_d}{g} \int_{p_1}^{p_2} T d\ln p$$

$Z_2 - Z_1$ is the thickness of the layer bounded above by p_2 and below by p_1
This thickness is proportional to the temperature of the layer.

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Hypsometric Equation

From hydrostatic balance and the ideal gas law we have

$$Z_2 - Z_1 = -\frac{R_d}{g}\int_{p_1}^{p_2} T\,d\ln p$$

If the temperature in a layer is constant then

$$\boxed{h = Z_2 - Z_1 = \frac{R_d T}{g}\ln\left(\frac{p_1}{p_2}\right)}$$

Hypsometric Equation

This is the relationship between layer thickness and temperature.