We will look into moist atmospheric processes

→ Write combined 1st + 2nd laws for a system that contains (dry + moist) air and liquid water. (saturated air).

→ We will then understand changes in Thermod. state associated with formation and dissipation of clouds.

→ Typical processes that lead to cloud formation
- Isobaric cooling (radiative fog)
- adiabatic isobaric processes
- adiabatic expansion (clouds)
- adiabatic isobaric freezing

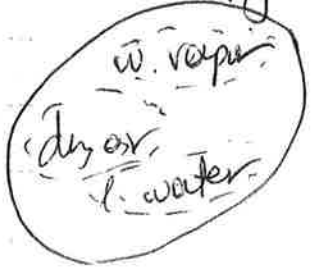
→ Most clouds are a mixture of the above, but studying each process is useful.

→ Enthalpy of a system containing moist air + liq. wat.

Q: Why enthalpy?
A: clouds are open-boundary systems. H is the appropriate representation vs. U, G, ...

panel concept.

Enthalpy: sum of all components



so: $dH = \sum dh$ (over all components) + latent heat release/loss.

Use 1 kg of air mixture: amount of water vapor is from w_v, and liquid water is w_ℓ

$$dH = (m_d c_{pd} + m_\ell c_{p\ell} + m_v c_{pv})\,dT + L_{ev}\,dm_v$$

dry air · liquid · vapor · phase change.

In the atmosphere, the mass of water vapor is much less than dry air content.

Q: How can we prove that?

A: From comparing ℓ_s vs p. Assuming ideal gas, $\frac{\ell_s}{(p-\ell_s)} = \frac{n_w}{n_d} << 1$.

Similarly $m_\ell << m_d$ so:

$$dH \simeq m_d c_{pd}\,dT + L_{ev}\,dm_v$$

condensation of liquid water

In intensive form (or $m_{total} = 1\,kg$):

$$dh \simeq c_{pd}\,dT + L_{ev}\,dw_v$$

Now, internal energy can be written as:

$$du = (c_{vd} + w_v c_{vv} + w_\ell c_{v\ell})\,dT + L_{ev}\,dw_v$$

and for a similar reason as for dh:

$$du = C_{vd}\,dT + L_{ev}\,dw_v \quad \text{(intensive)}$$

Now, the first law can be written as:

$$dq = du - v\,dp.$$
$$= C_{vd}dT + L_{ev}dw_v - v\,dp$$

In case where the system is adiabatic, $dq = 0$, so that: $-L_{ev}dw_v = C_p dT - v\,dp$

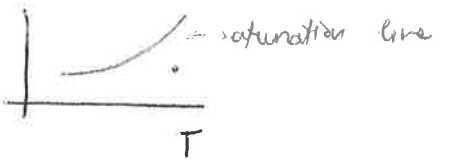
or, $$dw_v = -\frac{C_{vd}}{L_{ev}}dT + \frac{v\,dp}{L_{ev}}$$

change in water vapor content is proportional to expansion and cooling in an adiabatic air mass.

Isobaric cooling (radiative fogs, stratus clouds)

$$dq = dh = C_p dT$$

If air is RH < 100% then cooling increases RH until it is RH = 100%. At that point, the T of the parcel is called dew point



$T_D :=$ $e = e_s(T_D)$ definition

e ↑ water vapor press = saturation vapor pres @ T_D.

or $w_v = w_s(T_D)$ similarly

Same notion, define a frost point:

T_F: $e = e_{si}(T_F)$ or $w_v = w_{si}(T_F)$.

→ T_D can be determined from RH using Clausius-Clapeyron

$\frac{dp}{dT} = \frac{Lvp}{RT^2}$

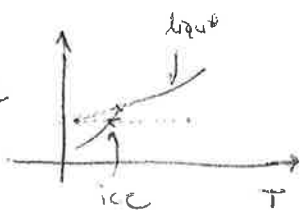
$$\frac{d(\ln e)}{dT_D} = \frac{L_v}{R_v T_D^2}$$; integration from T_D to T gives:

$$\ln\left(\frac{e_s}{e}\right) = -\ln H = \frac{L_v}{R_v}\left(\frac{1}{T_D} - \frac{1}{T}\right), \text{ or:}$$

$$\boxed{H = \exp\left\{-\frac{L_v}{R_v}\left(\frac{T - T_D}{T \cdot T_D}\right)\right\}}$$

so, if you know $T, H \Rightarrow T_D$

$T, T_D \Rightarrow RH$.



frost before liquid.

→ Once cooling goes beyond T_D, then you have a liquid phase appearing.

→ Q: Does T decrease when you cool beyond T_D?
A: Yes! only that $\mathcal{H} = 1$ always.

→ Q: How does T vary then?
A: Use Thermodynamics!

$$dq = dh = c_p\, dT + L_v\, dw_v \quad (4)$$

(cooling rate)

@ $\mathcal{H} = 1$, $w_v = w_s$ always. However, we also have appearance of a liquid water phase we can define a liquid water mass ratio similar to w_v:

$$w_l = \frac{m_l}{m_{dry}}, \text{ so that the total}$$

amount of water, $w_t = w_v + w_l = \text{const.}$ for a closed system. Because of that,

$$dw_t = 0 = dw_v + dw_l \rightarrow dw_l = -dw_v \quad (1)$$

This simply says the liquid water that appears is from condensation from gas phase.

(from previous chat, $\varepsilon = \frac{M_v}{M_d} \approx 0.62$ (2))

Since $dw_s = +\varepsilon \frac{de_s}{p} = + \frac{\varepsilon L_{ev} e_s}{p R_v T^2} dT$ (2)

(Clausius Clapeyron)

Then from (1) + (2):

$$dw_\ell = - dw_s = - \frac{\varepsilon L_{ev} e_s}{p R_v T^2} dT \quad (3)$$

Combining (3), (1) and (4):

$$dq = c_p dT + L_w \frac{\varepsilon L_w e_s}{p R_v T^2} dT \Rightarrow$$

$$\Rightarrow \boxed{dq = \left\{ c_p + \frac{L_{ev}^2 e_s \varepsilon}{p R_v T^2} \right\} dT} \quad (5)$$

Temperature drop as a function of dq

at $H < 1$, $dq = c_p dT$

$H \geq 1$, $dq =$ from (5).

$\longrightarrow dT(H<1) > dT(H \geq 1)$ for equal dq

Let Tom calculate for $dq = 1$. dT more

$\Longrightarrow$ Slower for a cooling with cloud. That is why the T drop when rays are covered is

much slower than when you are in (clear sky (at night).

Recap: We are studying thermodynamics of moist air

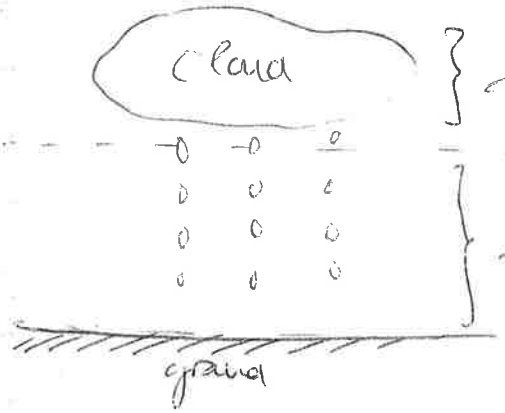
→ Isobaric cooling. We saw that the temperature drop related to a cooling rate, is given by (for moist air):

$$dq = \left\{ c_p + \frac{L_w^2 e_s}{P R_d T^2} \right\} dT$$, and that the amount of liq. water released is:

$$dw_l = -\left\{ \frac{L_w e_s}{c_p P R_d T^2 + L_w e_s} \right\} dq \quad ; \quad e_s(T)$$

→ The exact equations apply to isobaric heating (e.g. Thermal dissipation of fogs by sun). This mechanism happens near coastal and foggy areas.

→ Cooling and moistening by evaporation

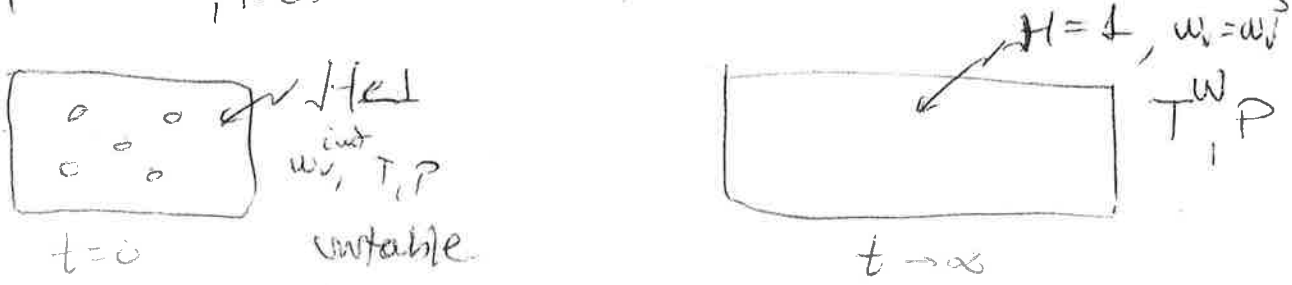


What if the cloud starts raining? water will fall into unsaturated air. That means

→ You are putting liquid water into a system that wants to be in a vapor phase state (i.e gas, water vapor). As a result, you will have evaporation.

→ Because of evaporation, T will drop (of the air) and H will increase. The maximum T-drop that could happen would correspond to the maximum liquid water that will evaporate, i.e., until the air becomes saturated ($H=1$).

→ We can calculate the change. Assume we have w_v, $H<1$ and also that we have $P=\text{const}$, $T=T_{initial}$ and an amount of liquid water, just enough to completely evaporate (even if we had more, it wouldn't matter as it would not evaporate more). Assume $P=\text{const}$, i.e.:



Assume no external heat sources $dq=0$, so that 1st law ⇒ for 1kg of that air:

$$dq = dh - v\,dp \;\Rightarrow\; dh = 0 \quad \text{(isenthalpic process)}$$

so $dh = c_p dT - L_{ev} dw_p = c_p dT + L_{ev} dw_s$

by integrating from w_v to $w_v^{final} = w_v + w_l$

(w_v: initial amount; w_v^{final}: final amount of liquid water)

we get:

$$c_p \int_T^{T_w} dT = -L_{ev} \int_{w_v}^{w_v^{final} = w_v + w_l} dw_s \quad \Rightarrow$$

$$\Rightarrow c_p (T_w - T) = -L_{ev} \left[w_v^{final}(T_w) - w_v \right]$$

Now, since w_v^{final} = saturation w_v @ T_w, we can write:

$$\boxed{T_w = T - \frac{L_{ev}}{c_p} \left\{ w_s(T_w) - w_v \right\} \quad (1)}$$

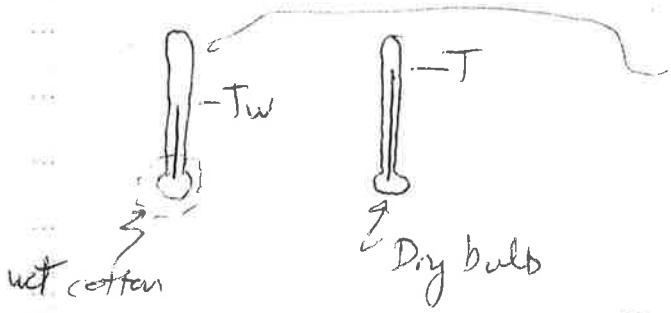
This is called "wet-bulb" temperature

Above is a non-linear eqn w.r.t T_w, as w_s depends on T_w.

→ Now, if we know T_w, T, we can determine w_v from (1).

↳ and $H = \frac{w_v}{w_s(T)}$

T_w can be easily determined by a "wet-bulb" Thermometer, which is just a Thermometer who's bulb is kept continuously wet



The wet bulb evaporates water continuously, until the air around it becomes saturated. Since heat is taken away from it's surroundings until $H=1$ in the vicinity of the Thermometer, that means when the wet Thermometer stabilizes, it will give an indication that corresponds to T_w

So, to measure H, all you need is 2 Thermometer and some patience, w_v is calculated from (1).

→ If ice is evaporating, you can also have an ice bulb T, T_I defined similarly as:

$$T_I = T - \frac{L_{iv}}{c_p}\left\{w_{si}(T_I) - w_v\right\}$$

Q: is $T_I >$ or $<$ than T_w?

$$T_I = T + \frac{L_{iv}}{c_p} w_v - \frac{L_{iv}}{c_p} w_{si}(T_I)$$

$$T_W - T + \frac{L_W}{c_p} w_v - \frac{L_W}{c_p} w_s(T_W)$$

So:

$$T_I - T_W = \frac{1}{c_p}\left\{(L_{iv} - L_W) w_v - \left(L_{iv} w_s(T_W) - L_W w_s(T_W)\right)\right\}$$

→ Saturation by Adiabatic, Isobaric Mixing

Mix 2 airmasses, so that total water is conserved; assuming adiabatic ($dq = 0$) and ($dp = 0$) isobaric (eg. at the tailpipe of a car exhaust on a cold morning) you can have formation of a cloud.

→ Saturation by Adiabatic Cooling from expansion

By far the most important condensation mechanism in the atmosphere.

Recall first law for an adiabatic process:

$d\phi = c_p dT - v dp$, from which we derived a potential temperature θ if there was no condensation and an adiabatic lapse rate $\Gamma_{dry} = \frac{g}{c_p} \simeq 10$ K

As air rises and expands, $T \downarrow$ and $H \uparrow$. When $H = 1$ (saturation level), clouds form. We can determine $?T$ where that happens as:

$H = \frac{e}{e_s} \Rightarrow d(\ln H) = d(\ln e) - d(\ln e_s)$ (2)

but

$$dP = de$$

(because as the gas expands, all components "decompress" equally, $\frac{e_1}{P_1} = \frac{e_2}{P_2}$)

So substitution into the 1st law gives

$$d(\ln e) = \frac{c_p}{R_d} d(\ln T) \quad (3)$$

from C-C diagram: $d(\ln e_s) = \frac{L_{ev}}{R_v T} d(\ln T)$ (4)

So, substituting (3), (4) into (2):

$$d(\ln H) = \left(\frac{c_p}{R_d} - \frac{\varepsilon L_{ev}}{R_d T}\right) d(\ln T)$$

So, integrating from $H = H^{init}$ to $H = 1$:

$$\boxed{H = \exp\left\{-\left(\frac{c_p}{R_d} \ln \frac{T_s}{T}\right) - \frac{\varepsilon L_{ev}}{R_d}\left(\frac{1}{T_s} - \frac{1}{T}\right)\right\}} \quad (4)$$

→ So from ④, if you know H, T you can determine $\underline{\underline{T_s}}$ (T at saturation).

→ The saturation pressure can be determined from ideal gas law:

$$\ln\frac{P_s}{P} = \frac{C_p}{R_d}\ln\frac{T_s}{T} \rightarrow \boxed{P_s = P\left(\frac{T_s}{T}\right)^{C_p/R_d}}$$

from hypsometric equation we can write:

$$\left.\begin{aligned} d(\ln e) &= -\frac{g}{R_d T}\,dz \\ \text{and}\quad d(\ln e) &= \frac{L_{ev}}{R_v T_D^2}\,dT_D \end{aligned}\right\} \rightarrow$$

$$\Rightarrow \boxed{\frac{dT_D}{dz} = -\frac{T_D^2\, g}{\varepsilon L_{ev} T} = \frac{T_D^2 C_p}{\varepsilon L_{ev} T}\,\Gamma_d}$$

dew point depression.

→ The height for which condensation occurs is the Rising condensation level.

determine from $\Gamma_d = \frac{T - T_s}{z_s - z}$, $\Rightarrow \boxed{z_s = \frac{T - T_s}{\Gamma_d}}$