6

The Diagnosis of Mid-Latitude Synoptic-Scale Vertical Motions

Objectives

Regions of upward vertical motion are often associated with clouds and precipitation since rising air cools by expansion. This cooling increases the relative humidity of the air which can eventually lead to condensation and cloud formation. Regions of rising air are also often associated with mass divergence in an atmospheric column and, consequently, surface pressure falls and cyclogenesis. Regions of downward vertical motion are often cloud free as air dries and warms upon being compressed as it sinks to higher pressure. Mass convergence into an atmospheric column, characteristic of regions of downward vertical motion, results in surface pressure rises and surface anti-cyclogenesis. As a result of the fundamental nature of these relationships, it is not an exaggeration to say that determination of where, when, and to what degree the air is rising or sinking is of fundamental importance for accurately diagnosing the current weather or predicting its future state. In this chapter we will investigate a number of different methods for diagnosing synoptic-scale vertical motions in typical mid-latitude weather systems.

Some of these diagnostic methods will derive from careful consideration of the ageostrophic wind vector itself. Several others (the Sutcliffe development theorem as well as the traditional and $\vec{Q}$-vector forms of the quasi-geostrophic omega equation) will arise from simultaneously solving the quasi-geostrophic vorticity and thermodynamic energy equations for the vertical motion, ω, and will make reference only to the instantaneous mass distribution. Taken together, the collection of diagnostics to be developed in this chapter will provide us with a formidable set of tools for understanding the synoptic-scale behavior of mid-latitude weather systems. We begin our investigation by considering the ageostrophic wind.

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6.1 The Nature of the Ageostrophic Wind: Isolating the Acceleration Vector

Recall that the geostrophic wind is non-divergent on an f plane. In fact, under such conditions only departures from geostrophy contribute to horizontal divergence and, through the continuity of mass, to vertical motions as shown in (4.9). For this reason it is extremely important to examine means by which the ageostrophic motions in the mid-latitude atmosphere might be diagnosed. We begin with the frictionless equation of motion

$$\frac{d\vec{V}}{dt} = -f\hat{k} \times \vec{V} - \nabla\phi, \tag{6.1}$$

and take the vertical cross-product of this expression to obtain

$$\frac{\hat{k}}{f} \times \frac{d\vec{V}}{dt} = \frac{\hat{k}}{f} \times (-f\hat{k} \times \vec{V}) - \frac{\hat{k}}{f} \times \nabla\phi. \tag{6.2a}$$

The right hand rule dictates that $\hat{k} \times \hat{k} \times \vec{A} = -\vec{A}$, and $\vec{V}_g = (\hat{k}/f) \times \nabla\phi$, so

$$\frac{\hat{k}}{f} \times \frac{d\vec{V}}{dt} = \vec{V} - \vec{V}_g = \vec{V}_{ag}. \tag{6.2b}$$

The famous British meteorologist R. C. Sutcliffe[1] reasoned that surface pressure falls resulted from vertical differences in mass divergence in a column. Larger mass divergence aloft than at the surface resulted in surface pressure falls and vice versa for surface pressure rises. Such differences in divergence could be related to differential accelerations at the surface and aloft through application of (6.2b). Thus, Sutcliffe argued that isolation of the acceleration vector could give insights into the sense of the vertical motion in an atmospheric column. Before presenting the elegant theory of Sutcliffe (1939), let us endeavor to isolate the acceleration vector, and its ageostrophic consequences, in two rather simple cases. These cases correspond to the two broad classes of circumstances in which geostrophic balance is violated: the presence of along-flow speed change and curvature in the flow.

The canonical synoptic example of along-flow speed change is the isolated jet streak. Shown in Figure 6.1 is the isotach distribution associated with an isolated wind speed maximum at 300 hPa in the northern hemisphere. The dashed line drawn perpendicular to the jet axis divides the jet into the so-called entrance region to its left and the exit region to its right. A parcel of air located on the western edge of the entrance region (indicated by the solid circle in Figure 6.1) would quite obviously experience an acceleration in the direction of the flow at that location. Hence, the

[1] R. C. Sutcliffe (1904–1991) received his Ph.D. in statistics but found employment with the British Meteorological Office in 1927. He worked with the famous Tor Bergeron while in Malta where he opined that weather forecasting was scarcely worthy of description as a scientific activity. He was among the first, and greatest, atmospheric scientists to insist that weather forecasting and diagnosis should proceed from the equations of motion. This insistence led to what might be considered the first major breakthrough in modern synoptic–dynamic meteorology and his most famous contribution, 'A contribution to the problem of development' (Sutcliffe 1947).

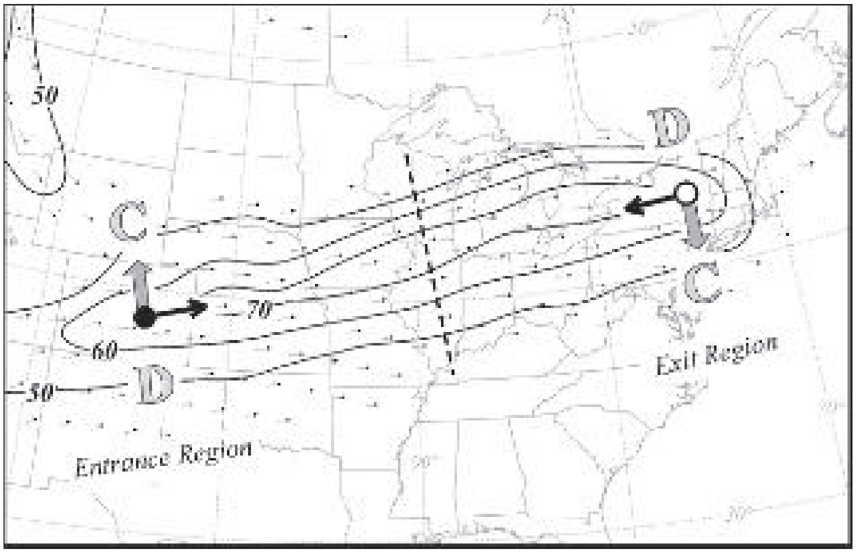

Figure 6.1 The 300 hPa isotachs (solid lines) and wind vectors associated with a straight jet at 0000 UTC 12 November 2003 from NCEP's Eta model analysis. Isotachs are labeled in m s^{-1} and contoured every 10 m s^{-1} starting at 50 m s^{-1}. Only wind vectors greater than 40 m s^{-1} are shown. Thick black arrows indicate the direction of the acceleration vector $d\vec{V}/dt$ at the entrance region (solid black circle) and exit regions (open circle) of the jet. The gray shaded arrow is the resultant ageostrophic wind vector, $\vec{V}_{ag}$, at both locations. C and D represent the locations of 300 hPa ageostrophic convergence and divergence, respectively

vector $d\vec{V}/dt$ points eastward toward the center of the jet streak. Consequently, the ageostrophic wind vector, $\vec{V}_{ag}$, points northward at the indicated point. The result of this distribution of ageostrophic winds in the entrance region of the jet is that there is convergence of air at 300 hPa to the north of the indicated position and divergence of air at 300 hPa to the south of the indicated position. Given that 300 hPa is nearly at the top of the troposphere, upper-level divergence (convergence) is associated with upward (downward) vertical motion in the intervening column and so a thermally direct vertical circulation generally exists in the entrance region of a straight jet streak.

A parcel of air located on the eastern edge of the exit region (indicated by the open circle in Figure 6.1) would quite obviously experience a deceleration in the direction opposite the flow at that location. Hence, the vector $d\vec{V}/dt$ points westward toward the center of the jet streak. Consequently, the ageostrophic wind vector, $\vec{V}_{ag}$, points southward at the indicated point. The result of this distribution of ageostrophic winds in the exit region of the jet is that there is convergence of air at 300 hPa to the south of the indicated position and divergence of air at 300 hPa to the north of the indicated position. Upward vertical motion occurs in the column beneath the upper divergence maxima and, thus, a thermally indirect vertical circulation generally exists in the exit region of a straight jet streak.

Curvature in the flow is also a circumstance that violates the geostrophic assumption. Consider flow through an upper tropospheric trough–ridge couplet where the

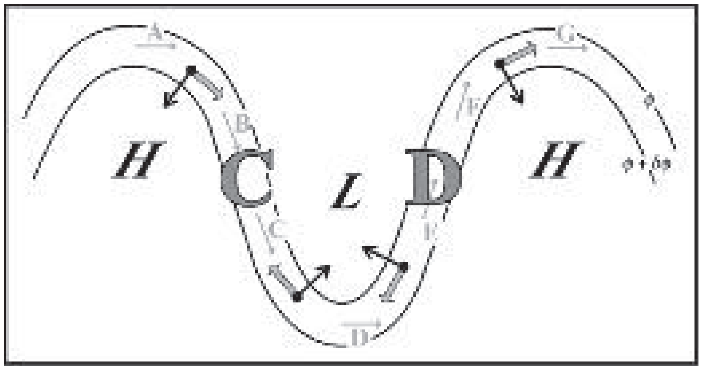

Figure 6.2 Schematic upper tropospheric trough–ridge wave train in which the speed of the flow is the same everywhere. Thick black arrows represent the acceleration vectors, $d\vec{V}/dt$, at the indicated points determined graphically by finite differencing between adjacent wind arrows (in gray and labeled as described in the text). Gray shaded arrows represent resultant ageostrophic winds, $\vec{V}_{ag}$, at the indicated points. Convergence and divergence are indicated by C and D, respectively

wind speed is constant and everywhere parallel to the geopotential height lines as shown in Figure 6.2. Under such circumstances, the acceleration of the wind will be entirely a consequence of directional changes. Thus, between points A and B in Figure 6.2, a southwestward-directed acceleration is required to turn the wind from westerly at point A to northwesterly at point B. There is no direction change between points B and C and, thus, no acceleration vector. A northeastward-directed acceleration is required to turn the northwesterly wind at point C to a westerly direction at point D. In order to turn the westerly at point D to a southwesterly at point E, a northwestward-directed acceleration is required. No change in direction exists between points E and F but a change from southwesterly at F to westerly at point G requires a southeastward-directed acceleration as shown. Given the four acceleration vectors drawn in Figure 6.2, it is simple to draw the ageostrophic winds in this trough–ridge couplet. The ageostrophic winds clearly converge on the western side of the upper trough (on its upstream side) leading to downward vertical motion in the column in that location. Meanwhile, the divergence of the ageostrophic winds on the downstream side of the upper trough is associated with upward vertical motions in the column in that location. This result provides a first insight into the physical reason why inclement weather is often found downstream of upper-level trough axes while clear skies are often found downstream of upper-level ridge axes. This basic relationship lies at the heart of understanding the distribution of sensible weather in the middle latitudes.

6.1.1 Sutcliffe's expression for net ageostrophic divergence in a column

Having examined the distribution of the ageostrophic winds in these canonical synoptic examples, let us now turn our attention to the insightful work of Sutcliffe