ENV 407 – Atmospheric Processes: From Clouds to Global Scales
Hydrostatic Equilibrium – Hypsometric Equation

1. Using the equation we derived for barometric pressure as a function of altitude calculate H for a dry atmosphere with an effective temperature of 273°K.

assume isothermal, dry atmosphere

$$\frac{dp}{dz} = -\frac{\rho g}{R_d T} \Rightarrow \ln\frac{P}{P_0} = \frac{-gz}{R_d T} \Rightarrow H = \frac{RT}{g} = \frac{\frac{8.314}{29\times10^{-3}}\,273}{9.81} \Rightarrow$$

$$\Rightarrow \boxed{H = 7.98\text{ km}}$$

2. For the value of H calculated above determine the altitude where P = 0.5 atm.

$$\ln\frac{P}{P_0} = -\frac{z}{H} \Rightarrow z = H\ln\frac{P_0}{P} \Rightarrow z = 7.98\ln 2 \Rightarrow$$

assume $P_0 = 1$ atm

$$\Rightarrow \boxed{z = 5.53\text{ km}}$$

3. For the value of H calculated above what is the air pressure in atmospheres at the top of Mt Everest (8.85km)?

$$\ln\frac{P}{P_0} = -\frac{z}{H} = -\frac{8.85}{7.98} \Rightarrow P = P_0\exp(-1.1) = P_0\,0.33 \Rightarrow$$

$$\Rightarrow \boxed{P = 0.33\text{ atm}}$$

4. On Mars the atmosphere is mainly CO2, the temperature is 220°K and the acceleration of gravity is 3.7 m/s2. What is the scale height of the Martian atmosphere? Compare the scale height to the Earth's atmosphere and explain why they scale heights are different.

Mw of CO_2

$$H = \frac{R_v T}{g} = \frac{\frac{8.314}{44\times10^{-3}}\,220}{3.7} = 11.24\text{ km}$$

So P drops less with altitude for Mars ($H = 11.24$ km) compared to Earth (7.98 km). That is most likely because of the reduced gravity of Mars, because its atmosphere has "heavier" molecules (CO_2 vs N_2/O_2).

This is a very famous experiment by Toricelli 1643. Atmospheric pressure measurement for the first time!

5. A barometer with a cross sectional area of 1.00 cm2 at sea level measures a pressure of 76.0 cm of mercury. The pressure exerted by this column of mercury is equal to the pressure exerted all the air on 1 cm2 of earth's surface. Given the density of mercury of 13.6 g/cm3 and the average radius of the earth of 6371 km, calculate the total mass of the Earth's atmosphere in kilograms.

$P = \frac{\text{weight}}{\text{Area}} = \frac{mg}{\text{Area}} \rightarrow$ for air that means m of air over 1 column of atmosphere

is equal to $m = \frac{P \cdot \text{Area}}{g}$ ① atmospheric pressure.

density, mercury height

Atmospheric pressure $P = \frac{\text{weight of mercury}}{\text{Surface area}} = \frac{\rho \times 1cm^2 \times h \times g}{1cm^2}$

$= 13.6 \times 10^3 \, kg\,m^{-3} \times 0.76m \times 9.81 m\,s^{-2}$

$= 1.014 \times 10^5 \, Pa$

So $m_{atmosphere} = \frac{1.014 \times 10^5 \times [4\pi (6371 \times 10^3)^2]}{9.81} = 5.27 \times 10^{18} \, kg$

6. An air bubble with a radius of 1.5 cm at the bottom of a lake where the temperature is 8.4°C and the pressure is 2.8 atm rises to the surface, where the temperature is 25°C and the pressure is 1.0 atm. Calculate the radius of the bubble when it reaches the surface. Assume ideal gas behavior. The volume of a sphere is (4/3)πr3 where r is the radius.

dont have to do this.