

Solution Exercise week 14 – Analysis of discharge data for hydrologic design

1 On paper

1. **Task:** Write the general equation to estimate the annual maximal discharge $Q_{est}(T)$ that corresponds to a return period T according to the frequency factor approach. Then, write the frequency factor equation for the LP3 approach.

Solution: The frequency factor general equation is:

$$x_T = \bar{x} + K_T S_x \quad (1)$$

where x_T is the estimated value corresponding to a return period T , $\bar{x}$ is the mean of the variable and S_x the standard deviation. K_T is the frequency factor, which depends on the return period and on the specifics of the particular probability distribution. The LP3 approach is based on equation 1 but with a transformed variable $y = \log_{10}(Q)$ instead of x . Thus, the discharge is estimated as:

$$Q_{est} = 10^{\bar{y} + K_T S_y} \quad (2)$$

2. **Question:** Which terms of the equation depend on the return period? Which depend(s) on the observed discharge time series?

Solution: From equation 2 we see that $\bar{y}$ and S_y only depend on the observed data. Then, the frequency factor K_T depends on both the return period and the data (in this case, through the skewness G_s) and so it can be expressed as $K(T, G_s)$.

3. **Task:** Estimate the discharge corresponding to a return period $T=50$ years, knowing that the mean and standard-deviation of the log-transformed discharge y are $\bar{y} = 0.8$ and $S_y = 0.1$ respectively, and that the skewness $G_s(y) = -0.4$. Use tables 10.4.1abcd at slide 40–42 to obtain the frequency factor $K_T(T, G_s)$.

Solution: In this case, the equation takes the form: $Q_{est}(50) = 10^{\bar{y} + K_T(50, -0.4) S_y}$. By checking the table at slide 41 we find that $K_T(50, -0.4) = 1.834$. Thus, $Q_{est}(50) = 10^{0.8 + 1.834 \cdot 0.1} = 9.63 \text{ m}^3/\text{s}$.

4. **Task:** Write a pseudo-code on paper to compute the estimated maximal discharges using the LP3 approach for an arbitrary number of return periods of interest.

Solution: A possible pseudocode is reported below:

- 1: $y \leftarrow \log_{10}(\text{discharge data})$
- 2: $\bar{y} \leftarrow \text{mean}(y)$
- 3: $S_y \leftarrow \text{standard deviation}(y)$
- 4: $G_s \leftarrow \text{skewness}(y)$
- 5: $T_{vec} \leftarrow \text{vector of return periods of interest}$
- 6: $\text{computeKTLP3}(\tau, G_s) \leftarrow \text{define a function that computes the frequency factor for the variable } y, \text{ given a return period and a skewness}$
- 7: Initialize results vector Q_{est}

```

8: for i = 1 to length of  $T_{vec}$  do
9:    $K_T \leftarrow \text{computeKTLP3}(G_s, T_{vec}(i))$ 
10:   $\log_{10} Q_{est} \leftarrow \bar{y} + K_T \times S_y$ 
11:   $Q_{est}(i) \leftarrow 10^{\log_{10} Q_{est}}$ 
12: end for

```

2 Numerical application

1. **Task:** Check how the datafile `2371_Abfluss_Tagesmittel_[...].csv` is formatted. Then, open the script `compute_yearly_maxima.m` and run it. [...]

Solution: The datafile has 8 lines of header and values are separated by a semicolon (;). There are various variables (in German): the year and the flow value (m^3/s) are at columns 7 and 9. Note that the last data points are provisional (*provisorische Daten*). The data can be imported as a Matlab table with:

```
S = readtable(<input_fname>, 'Delimiter', ';', 'NumHeaderLines', 8).
```

In the provided script, you can select a different year that will appear in the bottom plot.

2. **Task:** Now start a new script and import the yearly maxima from file `annual_maxima_LeChenit.csv`.

Solution: As usual, you can import the data into a Matlab table using the command `readtable` (for example,

```
S = readtable('annual_maxima_LeChenit.csv', 'Format', '%f%f', 'NumHeaderLines', 3).
```

3. **Task:** Implement the pseudo-code you have written on paper. It is very convenient that you create a function `computeKTLP3` that takes the variables G_s and T as input, implements the frequency factor equation (see Appendix) and returns the frequency factor K_T for the LP3 distribution.

Solution: This is an example of function that computes the frequency factor for the LP3 distribution:

```

1 function KT = computeKTLP3(T, Gs)
2
3 F = 1-1/T; % non-exceedance probability for return period T
4 z = icdf('normal', F, 0, 1); % inverse cumulative frequency for a
   standard normal distribution, evaluated in F
5 b= Gs /6; %to simplify the equation
6 KT = z + (z^2 - 1)*b + (1/3)*(z^3 - 6*z)*b^2 - (z^2 - 1)*b^3 + z*
   b^4 - (1/3)*(b)^5; % frequency factor of the LP3
7
8 end

```

4. **Task:** Test your function for a few values of T and G_s and check that they are similar to those reported in the tables at slides 40–41.

Solution: You can check that $K_T(T = 50, G_s = -0.4) = 1.8348$, which is very similar to the value provided in the table at slide 41.

5. **Task:** Run your code over a dense range of return periods so you can obtain a smooth function $Q_{est}(T)$, and then plot it. If you want, you can also plot the measured discharge Q against the empirical return period as computed from the Weibull plotting position formula (just like you did in assignment 1 for precipitation data).

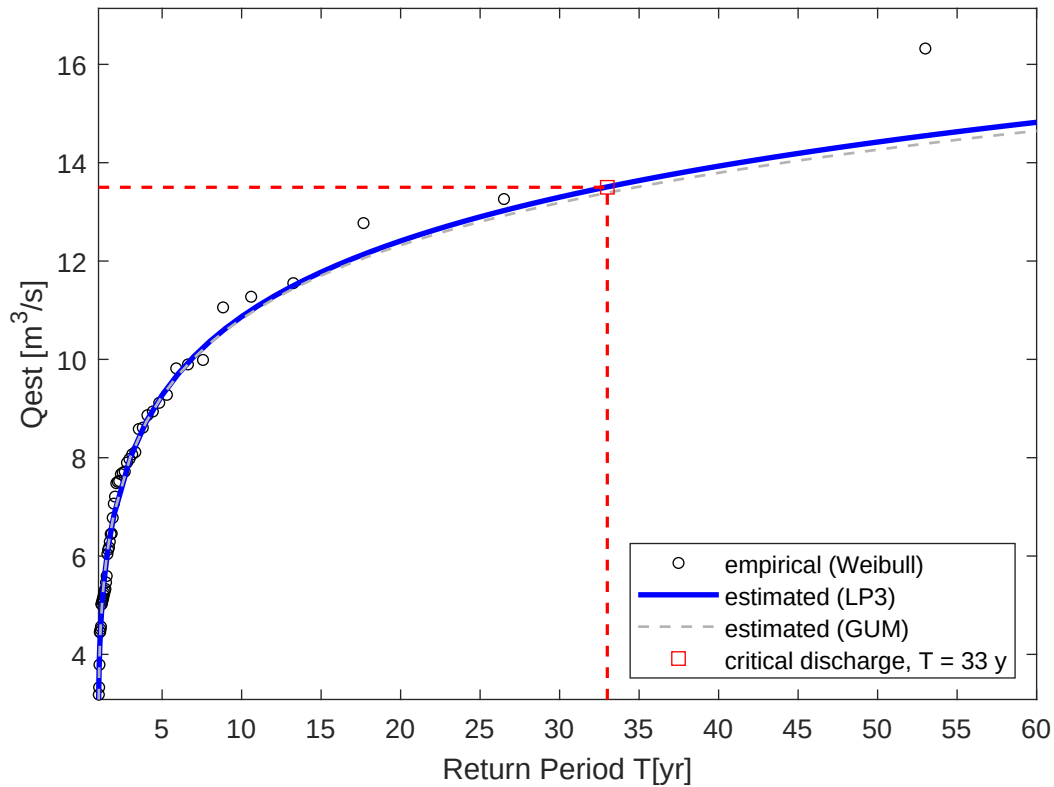


Figure 1: Estimated discharge as a function of return period

Solution: Figure 1 shows in blue the discharge Q_{est} computed for return periods in $[0.1, 60]$ (note that you cannot compute the frequency factor for $T = 1$). The plot also shows the empirical return period of the discharge data according to the Weibull plotting position approach. As you can see, there was a large event (in 2018) whose return period is estimated to be >100 years.

6. **Question:** What is the probability that the bridge will be flooded during the 5-year mandate of the mayor (*maire* in french)?

Solution: The return period corresponding to a discharge of $13.5 \text{ m}^3/\text{s}$ is approximately 33 years (see Figure 1). The probability that the bridge will be flooded during the president's mandate is therefore : $P(\text{flood in 5yr}) = 1 - P(\text{no flood in 5yr}) = 1 - (1 - 1/33)^5 = 14.3\%$.

7. **Optional Task:** Use the Gumbel distribution, instead of the LP3 distribution, for the estimation of return periods and use it to provide another estimate of the flooding probability of the bridge during 5 years.

8. **Solution:** Using the method of moments, the parameters of the Gumbel distribution are $\alpha = 0.48$ and $u = 6.13$. The distribution is plotted in Figure 1 and it is visually very close to the LP3 estimate. According to the Gumbel estimate, the critical return period for the event $Q \geq 13.5$ is 35 years and so the flooding probability over 5 years is 13.5%.