PROBLEM SET

Problem 1: Logarithmic velocity profile (Part I.)

In exercise set 7 you were provided with atmospheric turbulence data measured with a sonic anemometer and a hygrometer during a field campaign at the Seedorf Lake (FR) to investigate turbulent fluxes over water bodies. All measurements were taken at a height of $\mathbf{2m}$

Previously you were asked to computed some statistical properties of the flow, such as the standard deviations of the various time-series, as well as the turbulent fluxes of momentum, $(\overline{u'w'})$, and water vapor, $(\overline{u'q'})$.

Now we want to examine the data set to learn more about the boundary layer.

i. Using the results of the statistical analysis performed in the previous exercise set, find the value of $\boldsymbol{u_*}$. Assume that the measurements are taken in the surface layer.
ii. Based on the value of u_* determine the aerodynamic roughness length z_0. What can we tell about the surrounding environment? Is it more likely to be a coastal area? Farmland? A forest?
iii. Predict the velocity at a height of 10m. Can predict the velocity of the wind for any height? Why or why not?
iv. Plot the velocity as a function of height up to 100m

Note: It is interesting to observe that a lot of information can be determined regarding the boundary layer using only a time-series of data recorded at a single point in space.

SOLUTION:

i. We take advantage of the relationship between the vertical flux of momentum and friction velocity.

$$\overline{u'w'} = -u_*^2$$

Using the value for $\overline{u'w'}$ calculated in exercise set 7 ($\overline{u'w'} = -0.05\ \mathrm{m^2/s^2}$) we find $\boldsymbol{u_* = 0.23\ \mathrm{m/s}}$**.**

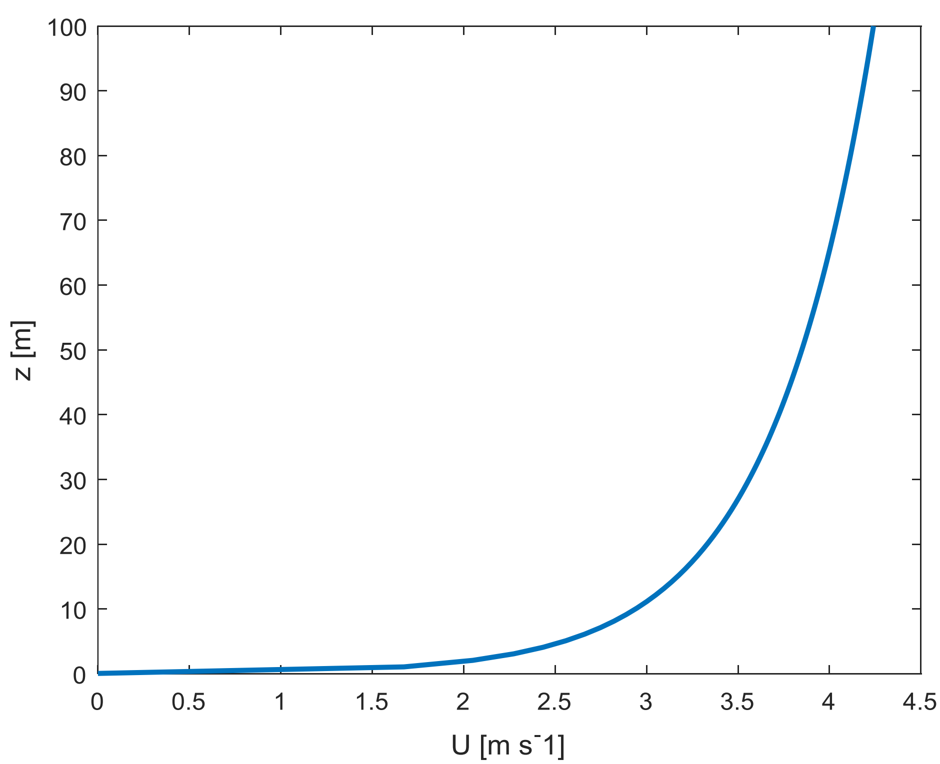
ii. We approximate the boundary layer as a logarithmic velocity distribution of the form

$$\frac{U}{u_*} = \frac{1}{k} \ln \frac{z}{z_0}$$

Using the value of for U calculated in exercise set 7 ($U = 2.03\ \mathrm{m/s}$) and the height of the sonic anemometer ($z = 2\mathrm{m}$), we calculate $\boldsymbol{z_0 = 5.52 \times 10^{-2}\mathrm{m}}$. This corresponds roughly to the aerodynamic roughness length of farmland. We can assume it is likely that there are some scattered trees and hedges around, or perhaps crops.

iii. Using again the logarithmic representation of the boundary layer we calculate the value of U for $z = 10\mathrm{m}$ and find $\boldsymbol{U(10\mathrm{m}) = 2.94\mathrm{m/s}}$. The logarithmic boundary layer approximation is only applicable to the lowest $10 - 20\%$ of the boundary layer depth, and only above the viscous sublayer. Therefore, we can only make predictions about the velocity for heights within this range.

iv.



Problem 2: Logarithmic velocity profile (Part II.)

Given the following wind speeds measured at various heights in a neutral boundary layer, find the

i. Boundary layer depth (H)
ii. Aerodynamic roughness (z_0)
iii. Friction velocity (u_*)
iv. Shear stress at the ground (τ) (in units of N/m^2).

What would you estimate the wind speeds to be at 2m and at 10m above the ground? Assume that the von Karman constant is 0.4 and density $\rho = 1.25$ $[\text{kg/m}^3]$.

z	(m)	1	4	10	20	50	100	200	300	500	800	1000	2000
U	(m/s)	3.7	5	5.8	6.5	7.4	8	8.9	9.5	10.5	10.8	11	11

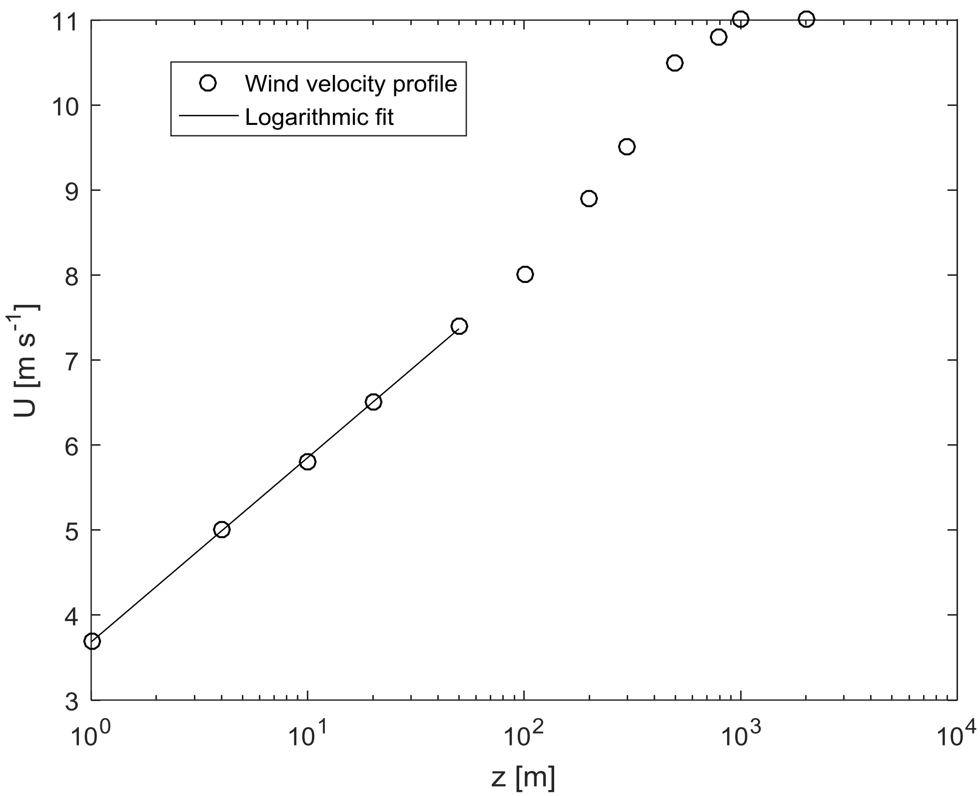
Assuming that the friction velocity measured at the same location on the next day is 0.6 $[\text{m/s}]$, what would you estimate the wind speeds to be at 2m and at 10m above the ground?

On which day (first day or second day) is the wind velocity at 2 m above the ground larger? Comment on the relation between the friction velocity and the wind speed.

SOLUTION:

From the above wind profile, the boundary layer depth (H) is approximately 1000 (m).

In general, the log-law layer depth is around 10~20 (%) of the boundary-layer depth. Here, we consider the log-law layer depth is about 100 (m) above the ground and select the points between 1 (m) to 100 (m) to do the regression of the log wind profile.



$$U = \frac{u_*}{\kappa}\ln\left(\frac{z}{z_o}\right) \Rightarrow U = \frac{u_*}{\kappa}\ln(z) - \frac{u_*}{\kappa}\ln(z_o)$$

$$\frac{u_*}{\kappa} = 0.9388 \Rightarrow \frac{u_*}{0.4} = 0.9388 \Rightarrow u_* = 0.376 \quad [m/s]$$

$$-\frac{u_*}{\kappa}\ln(z_o) = 3.6882 \quad \Rightarrow \quad -0.9388\ln(z_o) = 3.6882 \quad \Rightarrow \quad z_o = 0.0197 \quad [m]$$

$$\tau = -\rho u_*^2 = -\left(1.25 \times 0.376^2\right) \approx -0.177 \quad \left[N/m^2\right]$$

The first day

$$U_{z=2\ [m]} = \frac{u_*}{\kappa}\ln\left(\frac{z}{z_o}\right) = \frac{0.376}{0.4}\ln\left(\frac{2}{0.0197}\right) \approx 4.34\ \ [m/s]$$

$$U_{z=10\ [m]} = \frac{u_*}{\kappa}\ln\left(\frac{z}{z_o}\right) = \frac{0.376}{0.4}\ln\left(\frac{10}{0.0197}\right) \approx 5.86\ \ [m/s]$$

The second day

Because the two measurements were taken at the same location, the aerodynamic roughness calculated based on the first-day measurement can be used to estimate the wind speed at the second day.

$$U_{z=2\ [m]} = \frac{u_*}{\kappa}\ln\left(\frac{z}{z_o}\right) = \frac{0.6}{0.4}\ln\left(\frac{2}{0.0197}\right) \approx 6.93\ \ [m/s]$$

$$U_{z=10\ [m]} = \frac{u_*}{\kappa}\ln\left(\frac{z}{z_o}\right) = \frac{0.6}{0.4}\ln\left(\frac{10}{0.0197}\right) \approx 9.34\ \ [m/s]$$

The wind velocity at 2 m above the ground is larger at the second day.

Based on the log law, the friction velocity is proportional to the wind speed. Therefore, at the second day, when the wind speed increases, the friction velocity also increases.