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Office fédéral de météorologie et de climatologie MétéoSuisse
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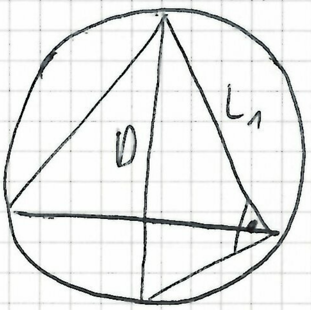
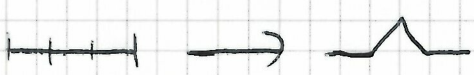
Incomplete self-similarity (Koch snowflake)





Dimensional analysis is trivial: $\frac{P_n}{D} = f\left(\frac{L_n}{D}\right)$

Let us compute f analytically

We have:
$$L_1 = \cos(30^\circ)\, D = \frac{\sqrt{3}}{2}\, D$$
$$L_2 = L_1/3$$
$$L_{n+1} = L_n/3$$
$$P_{n+1} = \frac{4}{3}\, P_n$$

$$\Rightarrow P_n = 3 \left(\frac{4}{3}\right)^{n-1} L_1$$

$$L_n = \left(\frac{1}{3}\right)^{n-1} L_1 \Rightarrow (n-1) = \frac{\log(L_1/L_n)}{\log 3}$$

Thus
$$\frac{P_n}{D} = 3 \left(\frac{4}{3}\right)^{n-1} \frac{L_1}{D}$$
$$= 3\; 10^{(n-1)(\log 4 - \log 3)}\; \frac{L_1}{D}$$
$$= 3\; 10^{\log(L_1/L_n)\, \underbrace{\left(\frac{\log 4 - \log 3}{\log 3}\right)}_{=:\,\alpha}}\; \frac{L_1}{D}$$
$$= 3 \left(\frac{L_1}{L_n}\right)^{\alpha} \frac{\sqrt{3}}{2} = 3 \left(\frac{\sqrt{3}}{2}\right)^{\alpha+1} \underbrace{\left(\frac{L_n}{D}\right)^{-\alpha}}_{\to\, \infty}$$

$\hookrightarrow \frac{L_n}{D} \to 0$