First order reactions

$$\frac{dC}{dt} = \pm KC \Rightarrow \frac{dC}{C} = \pm K dt$$

$$\ln C = \pm Kt + \ln C_0$$

$$C = e^{\pm Kt + \ln C_0} = C_0 e^{\pm Kt}$$

For the decay $\ominus$ we have the half-life such that

$$C(t_{1/2}) = C_0/2 = C_0 e^{-K t_{1/2}}$$

$$\Rightarrow -t_{1/2} = -\frac{\ln 2}{K} \Rightarrow t_{1/2} = \frac{\ln 2}{K}$$ Independent of C_0!

For the growth $\oplus$ we can introduce similarly a doubling period $t_{1/2}$.

1D ADR equation

$$\frac{\partial C}{\partial t} + V_{adv}\frac{\partial C}{\partial x} = D\frac{\partial^2 C}{\partial x^2} \pm KC$$

$\ominus \rightarrow$ we restrict ourselves to decay

We set $C = \varphi(x,t)\, e^{-Kt}$

We have $\frac{\partial C}{\partial t} = \frac{\partial \varphi}{\partial t} e^{-Kt} - K\varphi e^{-Kt}$

and $\frac{\partial^2 C}{\partial x^2} = e^{-Kt}\frac{\partial^2 \varphi}{\partial x^2}$

The ADR equation becomes $\frac{\partial \varphi}{\partial t} + V_{adv}\frac{\partial \varphi}{\partial x} = D\frac{\partial^2 \varphi}{\partial x^2}$ ADE

with solution $\varphi(x,t) = \frac{M}{A\sqrt{4\pi Dt}} e^{-(x-V_{adv}t)^2/4Dt}$ $\qquad \varphi(x, t\to 0) = \frac{M}{A}\delta(x)$

$$C(x,t) = \frac{M}{A\sqrt{4\pi Dt}} e^{-(x-V_{adv}t)^2/4Dt}\, e^{\pm Kt}$$