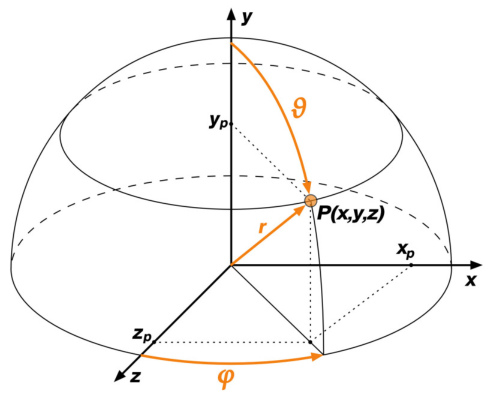
Exercise: Optimal Control of a Robot Arm

This exercise aims at steering the robot arm to perform time-optimal point-to-point motions.



We model the robot arm in 3D as follows

$$
\begin{aligned}
\ddot{\varphi}(t) &= \frac{u_\varphi(t)}{r \cdot m} - \underbrace{2\cot(\vartheta(t))\dot{\varphi}(t)\dot{\vartheta}(t)}_{\text{due to coriolis forces}} \\
\ddot{\vartheta}(t) &= \underbrace{\sin(\vartheta(t))g}_{\text{due to gravitation}} + \frac{u_\vartheta(t)}{r \cdot m} + \underbrace{\sin(\vartheta(t))\cos(\vartheta(t))\dot{\varphi}(t)^2}_{\text{due to centrifugal forces}}
\end{aligned}
$$

with gravitational constant $g = 9.81\,[\mathrm{m/s^2}]$ and mass $m = 1\,[\mathrm{kg}]$ and arm length $r = 1\,[\mathrm{m}]$. Then, we can summarize our dynamic system as

$$
\dot{x}(t) = f(x(t), u(t)) \quad \text{with} \begin{cases} x(t) = (\varphi(t), \dot{\varphi}(t), \vartheta(t), \vartheta(t)) \in \mathbb{R}^4 \\ u(t) = (u_\varphi(t), u_\vartheta(t)) \in \mathbb{R}^2 \end{cases}
$$

where the control inputs are defined by the motor torques $u_\varphi(t)$ and $u_\vartheta(t)$ in $[\mathrm{N \cdot m}]$. The optimal control problem is formulated in continuous time below

$$
\begin{array}{rll}
\text{minimize} & T & \textsc{Objective} \\
\text{subject to} & \forall t \in [0, T], \quad \dot{x}(t) = f(x(t), u(t)) & \textsc{Nonlinear dynamics} \\
& x(0) = \begin{pmatrix} 0 \\ 0 \\ \frac{\pi}{2} \\ 0 \end{pmatrix}, \; x(T) = \begin{pmatrix} \frac{\pi}{2} \\ 0 \\ \frac{\pi}{2} \\ 0 \end{pmatrix} & \textsc{Boundary constraints} \\
& -10\,[\mathrm{N \cdot m}] \le u_\varphi(t) \le 10\,[\mathrm{N \cdot m}] & \textsc{Input} \\
& -20\,[\mathrm{N \cdot m}] \le u_\vartheta(t) \le 20\,[\mathrm{N \cdot m}] & \textsc{constraints}
\end{array}
$$

Implementation tasks:

1. consider piece-wise constant control inputs and then, discretize the continuous-time dynamic by implementing a RK4 integrator;

2. consider the input constraints at each sampling point and then, formulate the OCP as an NLP by using `Casadi`.

3. Solve the resulting NLP by using `IPOPT` interfaced in `Casadi`, and visualize the results.