technische universität dortmund

EPFL

Optimal Control (EE-736)

Introduction

Timm Faulwasser & Yuning Jiang

ie3, TU Dortmund

timm.faulwasser@tu-dortmund.de
yuning.jiang@epfl.ch

Block course @ EPFL

Version EE736.2024.I

Block Course Optimal Control (EE-736) – Spring 2024

Teachers

- Timm Faulwasser
- Professor at TU Dortmund University
- Former postdoc at Laboratoire d'Automatique of EPFL, senior scientist at KIT
- Email: timm.faulwasser@ieee.org
- Research on optimization-based control of networked systems with applications in
  - multi-energy
  - mechatronics
  - climate economics
  - ...

Block Course Optimal Control (EE-736) – Spring 2024

Teachers

- Yuning Jiang
- Postdoc at Laboratoire d'Automatique of EPFL
- Email: yuning.jiang@epfl.ch
- Research:
  - distributed and federated optimization;
  - robust and stochastic optimization;
  - numerical optimal control and model predictive control;
  - learning theory and nonlinear system identification;
  - and their applications in autonomous systems (e.g., autonomous vehicles and smart buildings), multi-agent systems (e.g., power and energy systems, IoT and traffic networks).

Expectations

Expectations

Block Course Optimal Control (EE-736) – Spring 2024

Content

- Part I
  - Basics of nonlinear programming
  - Primer on numerical methods for NLPs

Block Course Optimal Control (EE-736) – Spring 2024

Content

- Part I
  - Basics of nonlinear programming
  - Primer on numerical methods for NLPs
- Part II.1
  - Basics of calculus of variations
  - Optimal control theory

Block Course Optimal Control (EE-736) – Spring 2024

Content

- Part I
  - Basics of nonlinear programming
  - Primer on numerical methods for NLPs
- Part II.1
  - Basics of calculus of variations
  - Optimal control theory
- Part II.2
  - Numerical optimal control
  - Tricks for problem formulation

Block Course Optimal Control (EE-736) – Spring 2024

Content

- Part I
  - Basics of nonlinear programming
  - Primer on numerical methods for NLPs
- Part II.1
  - Basics of calculus of variations
  - Optimal control theory
- Part II.2
  - Numerical optimal control
  - Tricks for problem formulation
- Part III
  - From optimal to predictive control
  - Dissipativity and turnpike properties

Prototypical problem:

Topics covered:

Topics not covered:

Part II.1 – Optimal Control Theory

Prototypical problem:

Topics covered:

Topics not covered:

Part II.2 – Numerical Optimal Control

Prototypical problem:

Topics covered:

Topics not covered:

Part III – From Optimal to Predictive Control

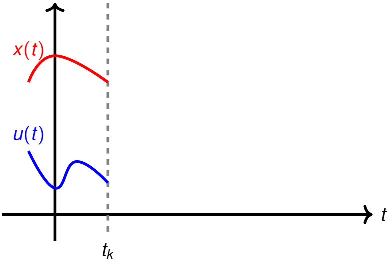
Prototypical problem:

Topics covered:

Topics not covered:

Part III – From Optimal to Predictive Control

Predictive control $\equiv$ **repeated optimal control**



1. State measurement/estimate $x(t_k)$

2. Solve $OCP(x(t_k))$:

$$\min_{u(\cdot|t_k))} \int_{t_k}^{t_k+T} \ell(x(\tau|t_k)), u(\tau|t_k))\,\mathrm{d}\tau + V_f(x(t_k+T|t_k))$$

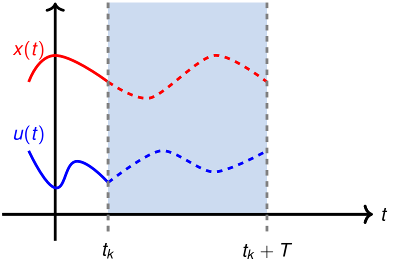
$$\begin{aligned} \forall \tau \in [t_k, t_k+T] : \frac{\mathrm{d}x(\tau|t_k)}{\mathrm{d}\tau} &= f(x(\tau|t_k), u(\tau|t_k)) \\ x(t_k|t_k) &= x(t_k) \\ x(\tau|t_k) \in \mathbb{X},\ & u(\tau|t_k) \in \mathbb{U} \\ x(t_k+T|t_k) &\in \mathbb{X}_f \end{aligned}$$

3. Apply $u^\star(\tau|t_k)$ for $\tau \in [t_k, t_{k+1})$

- Topics covered:
- Topics not covered:

Part III – From Optimal to Predictive Control

Predictive control $\equiv$ **repeated optimal control**



1. State measurement/estimate $x(t_k)$

2. Solve $OCP(x(t_k))$:

$$\min_{u(\cdot|t_k))} \int_{t_k}^{t_k+T} \ell(x(\tau|t_k)), u(\tau|t_k))\,\mathrm{d}\tau + V_f(x(t_k + T|t_k))$$

$$\forall \tau \in [t_k, t_k + T] : \frac{\mathrm{d}x(\tau|t_k)}{\mathrm{d}\tau} = f(x(\tau|t_k), u(\tau|t_k))$$

$$x(t_k|t_k) = x(t_k)$$

$$x(\tau|t_k) \in \mathbb{X},\ \ u(\tau|t_k) \in \mathbb{U}$$

$$x(t_k + T|t_k) \in \mathbb{X}_f$$

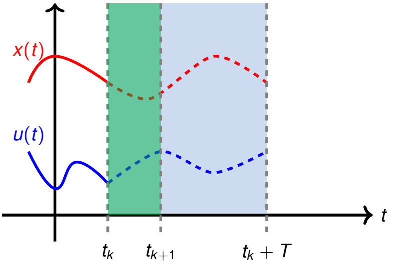
3. Apply $u^\star(\tau|t_k)$ for $\tau \in [t_k, t_{k+1})$

- Topics covered:

- Topics not covered:

Part III – From Optimal to Predictive Control

Predictive control $\equiv$ **repeated optimal control**



1. State measurement/estimate $x(t_k)$

2. Solve $OCP(x(t_k))$:

$$\min_{u(\cdot|t_k))} \int_{t_k}^{t_k+T} \ell(x(\tau|t_k)), u(\tau|t_k))\, \mathrm{d}\tau + V_f(x(t_k + T|t_k))$$

$$\forall \tau \in [t_k, t_k + T] : \frac{\mathrm{d}x(\tau|t_k)}{\mathrm{d}\tau} = f(x(\tau|t_k), u(\tau|t_k))$$
$$x(t_k|t_k) = x(t_k)$$
$$x(\tau|t_k) \in \mathbb{X},\ \ u(\tau|t_k) \in \mathbb{U}$$
$$x(t_k + T|t_k) \in \mathbb{X}_f$$

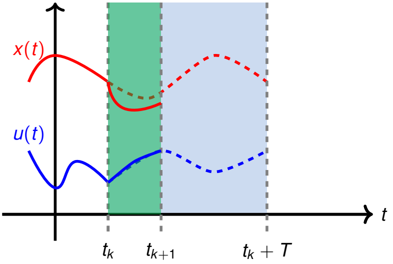
3. Apply $u^\star(\tau|t_k)$ for $\tau \in [t_k, t_{k+1})$

- Topics covered:

- Topics not covered:

Part III – From Optimal to Predictive Control

Predictive control $\equiv$ **repeated optimal control**



1. State measurement/estimate $x(t_k)$

2. Solve $OCP(x(t_k))$:

$$\min_{u(\cdot|t_k))} \int_{t_k}^{t_k+T} \ell(x(\tau|t_k)), u(\tau|t_k))\,\mathrm{d}\tau + V_f(x(t_k+T|t_k))$$

$$\forall \tau \in [t_k, t_k+T] : \frac{\mathrm{d}x(\tau|t_k)}{\mathrm{d}\tau} = f(x(\tau|t_k), u(\tau|t_k))$$

$$x(t_k|t_k) = x(t_k)$$

$$x(\tau|t_k) \in \mathbb{X}, \;\; u(\tau|t_k) \in \mathbb{U}$$

$$x(t_k+T|t_k) \in \mathbb{X}_f$$

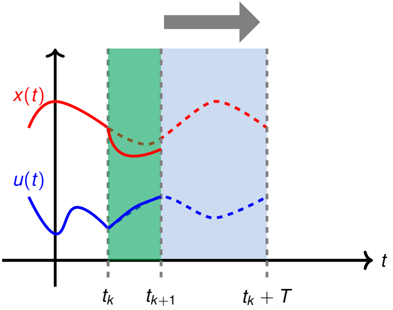
3. Apply $u^\star(\tau|t_k)$ for $\tau \in [t_k, t_{k+1})$

- Topics covered:

- Topics not covered:

Part III – From Optimal to Predictive Control

Predictive control $\equiv$ **repeated optimal control**



1. State measurement/estimate $x(t_k)$

2. Solve $OCP(x(t_k))$:

$$\min_{u(\cdot|t_k))} \int_{t_k}^{t_k+T} \ell(x(\tau|t_k)), u(\tau|t_k))\, \mathrm{d}\tau + V_f(x(t_k+T|t_k))$$

$$\forall \tau \in [t_k, t_k+T] : \frac{\mathrm{d}x(\tau|t_k)}{\mathrm{d}\tau} = f(x(\tau|t_k), u(\tau|t_k))$$

$$x(t_k|t_k) = x(t_k)$$

$$x(\tau|t_k) \in \mathbb{X}, \;\; u(\tau|t_k) \in \mathbb{U}$$

$$x(t_k+T|t_k) \in \mathbb{X}_f$$

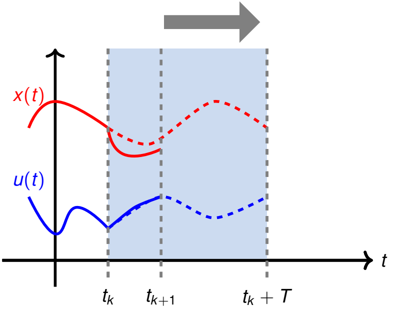
3. Apply $u^{\star}(\tau|t_k)$ for $\tau \in [t_k, t_{k+1})$

- Topics covered:

- Topics not covered:

Part III – From Optimal to Predictive Control

Predictive control $\equiv$ **repeated optimal control**



1. State measurement/estimate $x(t_k)$

2. Solve $OCP(x(t_k))$:

$$\min_{u(\cdot|t_k))} \int_{t_k}^{t_k+T} \ell(x(\tau|t_k)), u(\tau|t_k))\,\mathrm{d}\tau + V_f(x(t_k+T|t_k))$$

$$\forall \tau \in [t_k, t_k+T] : \frac{\mathrm{d}x(\tau|t_k)}{\mathrm{d}\tau} = f(x(\tau|t_k), u(\tau|t_k))$$

$$x(t_k|t_k) = x(t_k)$$

$$x(\tau|t_k) \in \mathbb{X},\ \ u(\tau|t_k) \in \mathbb{U}$$

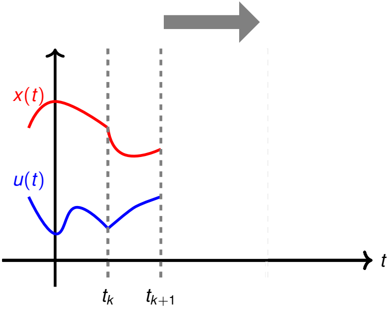
$$x(t_k+T|t_k) \in \mathbb{X}_f$$

3. Apply $u^\star(\tau|t_k)$ for $\tau \in [t_k, t_{k+1})$

- Topics covered:

- Topics not covered:

Predictive control $\equiv$ **repeated optimal control**



1. State measurement/estimate $x(t_k)$

2. Solve $OCP(x(t_k))$:

$$\min_{u(\cdot|t_k))} \int_{t_k}^{t_k+T} \ell(x(\tau|t_k)), u(\tau|t_k)) \,\mathrm{d}\tau + V_f(x(t_k + T|t_k))$$

$$\forall \tau \in [t_k, t_k + T] : \frac{\mathrm{d}x(\tau|t_k)}{\mathrm{d}\tau} = f(x(\tau|t_k), u(\tau|t_k))$$

$$x(t_k|t_k) = x(t_k)$$

$$x(\tau|t_k) \in \mathbb{X}, \;\; u(\tau|t_k) \in \mathbb{U}$$

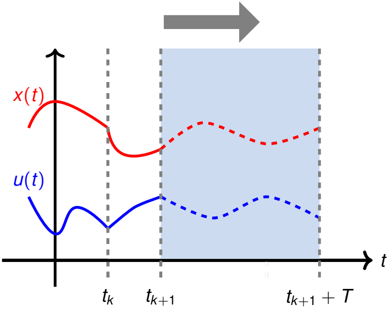
$$x(t_k + T|t_k) \in \mathbb{X}_f$$

3. Apply $u^\star(\tau|t_k)$ for $\tau \in [t_k, t_{k+1})$

- Topics covered:
- Topics not covered:

Part III – From Optimal to Predictive Control

Predictive control $\equiv$ **repeated optimal control**



1. State measurement/estimate $x(t_k)$

2. Solve $OCP(x(t_k))$:

$$\min_{u(\cdot|t_k))} \int_{t_k}^{t_k+T} \ell(x(\tau|t_k)), u(\tau|t_k))\, \mathrm{d}\tau + V_f(x(t_k+T|t_k))$$

$$\forall \tau \in [t_k, t_k+T] : \frac{\mathrm{d}x(\tau|t_k)}{\mathrm{d}\tau} = f(x(\tau|t_k), u(\tau|t_k))$$

$$x(t_k|t_k) = x(t_k)$$

$$x(\tau|t_k) \in \mathbb{X},\ \ u(\tau|t_k) \in \mathbb{U}$$

$$x(t_k+T|t_k) \in \mathbb{X}_f$$

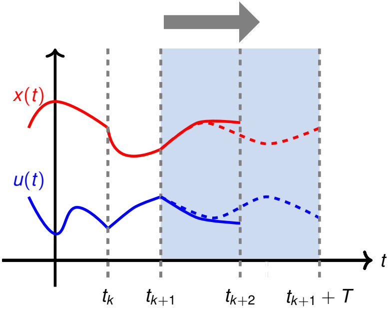
3. Apply $u^\star(\tau|t_k)$ for $\tau \in [t_k, t_{k+1})$

- Topics covered:

- Topics not covered:

Part III – From Optimal to Predictive Control

Predictive control $\equiv$ **repeated optimal control**



1. State measurement/estimate $x(t_k)$

2. Solve $OCP(x(t_k))$:

$$\min_{u(\cdot|t_k))} \int_{t_k}^{t_k+T} \ell(x(\tau|t_k)), u(\tau|t_k))\, \mathrm{d}\tau + V_f(x(t_k+T|t_k))$$

$$\forall \tau \in [t_k, t_k+T] : \frac{\mathrm{d}x(\tau|t_k)}{\mathrm{d}\tau} = f(x(\tau|t_k), u(\tau|t_k))$$

$$x(t_k|t_k) = x(t_k)$$

$$x(\tau|t_k) \in \mathbb{X},\ \ u(\tau|t_k) \in \mathbb{U}$$

$$x(t_k+T|t_k) \in \mathbb{X}_f$$

3. Apply $u^\star(\tau|t_k)$ for $\tau \in [t_k, t_{k+1})$

- Topics covered:

- Topics not covered:

Goal – Master Optimization-Based Control

4. Real-time NMPC Design

- Stability? State estimation? Robustness? ...

Goal – Master Optimization-Based Control

4. Real-time NMPC Design

- Stability? State estimation? Robustness? ...

3. Solve OCP

- Choose a numerical strategy to obtain an (approximated) solution.
- Implement numerical solution. Use available toolboxes.
- Construct feasible initial guess $\rightarrow$ Solve OCP

Goal – Master Optimization-Based Control

4. Real-time NMPC Design

- Stability? State estimation? Robustness? ...

3. Solve OCP

- Choose a numerical strategy to obtain an (approximated) solution.
- Implement numerical solution. Use available toolboxes.
- Construct feasible initial guess $\rightarrow$ Solve OCP

2. Problem Analysis

- Does the model have a solution for any admissible control?
- Can the constraints be met? $\rightarrow$ feasibility
- Can the system outputs be observed/measured?
- Existence of optimal solutions?

Goal – Master Optimization-Based Control

4. Real-time NMPC Design

- Stability? State estimation? Robustness? ...

3. Solve OCP

- Choose a numerical strategy to obtain an (approximated) solution.
- Implement numerical solution. Use available toolboxes.
- Construct feasible initial guess → Solve OCP

2. Problem Analysis

- Does the model have a solution for any admissible control?
- Can the constraints be met? → feasibility
- Can the system outputs be observed/measured?
- Existence of optimal solutions?

1. Problem Formulation

- Model of system dynamics and constraints (physical restrictions)?
- Choose a performance criterion (objective functional).
- Choose the decision variables.

Reading & Exercises

Parts I-II follow the lecture notes:
B. Chachuat. *Nonlinear and Dynamic Optimization - From Theory to Practice*. EPFL, 2009. URL: https://infoscience.epfl.ch/record/111939/files/Chachuat_07(IC32).pdf

Further references are provided on the slides.

Reading & Exercises

Parts I-II follow the lecture notes:
B. Chachuat. *Nonlinear and Dynamic Optimization - From Theory to Practice*. EPFL, 2009. URL: https://infoscience.epfl.ch/record/111939/files/Chachuat_07(IC32).pdf

Further references are provided on the slides.

Part III (incomplete lecture notes) T. Faulwasser. "Nonlinear Model Predictive Control – Theory and Applications". Lecture notes, ie3, TU Dortmund University. Incomplete and preliminary version can be downloaded from the moodle page of the course. 2021

Exercises

- Directly integrated into course
- Pen and paper as well as coding
- Bring your own laptop with MATLAB
- We will use CasADi (open source Matlab and Python toolbox)
- Coding Exercises can be also done in Python

Block Course Optimal Control (EE-736) – Spring 2024

Dates for course

- Block I
  - Mar 06 2024, 09:15 – 12:00 Intro & NLPs
  - Mar 07 2024, 09:15 – 12:00 & 14:15 – 16:00 NLPs & Casadi Intro

Block Course Optimal Control (EE-736) – Spring 2024

Dates for course

- Block I
  - Mar 06 2024, 09:15 – 12:00 Intro & NLPs
  - Mar 07 2024, 09:15 – 12:00 & 14:15 – 16:00 NLPs & Casadi Intro
- Block II
  - Mar 11 2024, 09:15 – 12:00 & 14:15 – 16:00 Optimal Control
  - Mar 12 2024, 09:15 – 12:00 & 14:15 – 18:00 Optimal Control & Consultation on projects
  - Mar 13 2024, 09:15 – 12:00 & 14:15 – 16:00 Numerical Optimal Control
  - Mar 14 2024, 09:15 – 12:00 & 14:15 – 16:00 Dissipativity & NMPC
  - Mar 15 2024, 09:15 – 12:00 & 14:15 – 16:00 NMPC & Wrap-Up

Block Course Optimal Control (EE-736) – Spring 2024

Dates for course

- Block I
  - Mar 06 2024, 09:15 – 12:00 Intro & NLPs
  - Mar 07 2024, 09:15 – 12:00 & 14:15 – 16:00 NLPs & Casadi Intro
- Block II
  - Mar 11 2024, 09:15 – 12:00 & 14:15 – 16:00 Optimal Control
  - Mar 12 2024, 09:15 – 12:00 & 14:15 – 18:00 Optimal Control & Consultation on projects
  - Mar 13 2024, 09:15 – 12:00 & 14:15 – 16:00 Numerical Optimal Control
  - Mar 14 2024, 09:15 – 12:00 & 14:15 – 16:00 Dissipativity & NMPC
  - Mar 15 2024, 09:15 – 12:00 & 14:15 – 16:00 NMPC & Wrap-Up
- Project Presentations: t.b.d.

Additional exercises on demand (webinar sessions): to be discussed

Have Laptop with recent Matlab version at hand

Block Course Optimal Control (EE-736) – Spring 2024

Moodle page

- Lecture slides
- Additional material
- Exercises (tasks and solutions)
- `https://moodle.epfl.ch/course/view.php?id=14143`
- PDFs of slides will be provided on moodle page the evening before the course is given.
  - *.pdf PDF handout for note taking on digital devices
  - PrintableHandout_*.pdf PDF handout for printing (4 slides per page)

Block Course Optimal Control (EE-736) – Spring 2024

Moodle page

- Lecture slides
- Additional material
- Exercises (tasks and solutions)
- `https://moodle.epfl.ch/course/view.php?id=14143`
- PDFs of slides will be provided on moodle page the evening before the course is given.
  - *.pdf PDF handout for note taking on digital devices
  - PrintableHandout_*.pdf PDF handout for printing (4 slides per page)

Credits and Examination

- 3 ECTS
- Project-based examination
- Pick project related to your research
- Write report (max 20 pages)
- Present project to the class (15-20 minutes)

Questions?