Exericises: NLP algorithm

1. *Newton-type methods for a boundary-value problem* Regarding the scalar dynamic

$$x_{k+1} = \frac{1}{10}(11x_k + x_k^2 + u), k = 0, ..., N-1$$

with conditions

$$x_0 = 0.1, \;\; x_N = 0.$$

We fix the horizon length $N = 30$.

   a) Formulate and solve the problem with Newton's method (exact Jacobian) initialized at $u = 0$. Plot the state trajectories in each iteration. Also plot the values x_N and the variable u as a function of the Newton iteration index.

2. *SQP for equality constrained NLP* Consider NLP

$$\min_{x \in \mathbb{R}^2} \;\; \exp(-x_1^2 - x_2^2) \quad \text{subject to} \;\; x_2^3 + x_1^2 - 1 = 0$$

   a) Implement SQP method using exact Hessian matrix with initial guess $[-1; -1]^\top$ and $[1; 1]^\top$. Plot $\|x^k - x^*\|$ with respect to the iteration index k.

   b) Implement the globalization strategy with L-1 penalty function for the SQP implementation in a). Then, test the code by using initial point $[100, 100]$.

3. *Interior Point Method for Optimal Control* Based on the dynamic in Question 2, we formulate the optimal control problem

$$\begin{aligned} \min_{\{x_k\},\{u_k\}} \quad & \sum_{k=0}^{N-1} x_k^2 + u_k^2 \\ \text{subject to} \quad & x_{k+1} = \frac{1}{10}(11x_k + x_k^2 + u_k), k = 0, ..., N-1 \\ & x_0 = 0.5 \\ & x_k \in [-1, 1], \; u_k \in [-1, 1] \end{aligned}$$

with $N = 100$. Implement the barrier method to solve the OCP and plot the trajectory. (*Hint:* solve the equality-constrained subNLPs via `fmincoin` in MATLAB.)