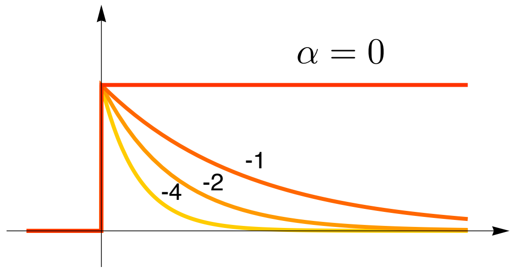
Construction of exponential B-spline

- Differential operator $\mathrm{L} = \mathrm{D} - \alpha \mathrm{Id} = \mathrm{P}_\alpha$

  Green's function: $\rho_\alpha(r) = \mathbb{1}_+(r) e^{\alpha r}$



- Discrete version of operator (weighted differences)

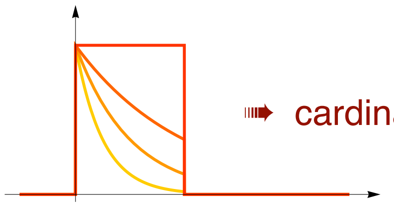
$$\mathrm{L_d} s(r) = \Delta_\alpha s(r) = s(r) - \mathrm{e}^\alpha s(r-1)$$

Fourier multiplier: $1 - \mathrm{e}^\alpha \mathrm{e}^{-\mathrm{j}\omega}$

- First-order exponential B-spline

$$\beta_\alpha(r) = \Delta_\alpha \rho_\alpha(r) = \mathbb{1}_{[0,1)}(r) \mathrm{e}^{\alpha r} \quad \xleftrightarrow{\mathcal{F}} \quad \frac{1 - \mathrm{e}^\alpha \mathrm{e}^{-\mathrm{j}\omega}}{\mathrm{j}\omega - \boxed{\alpha}} = \frac{\widehat{L}_\mathrm{d}(\omega)}{\widehat{L}(\omega)}$$

pole



➠ cardinal (exponential) splines of minimal support

- Space of cardinal P_α-splines: $\quad V_0 = \mathrm{span}\{\beta_\alpha(\cdot - k)\}_{k \in \mathbb{Z}}$

  β_α generates an *orthogonal* basis for the first-order cardinal exponential splines