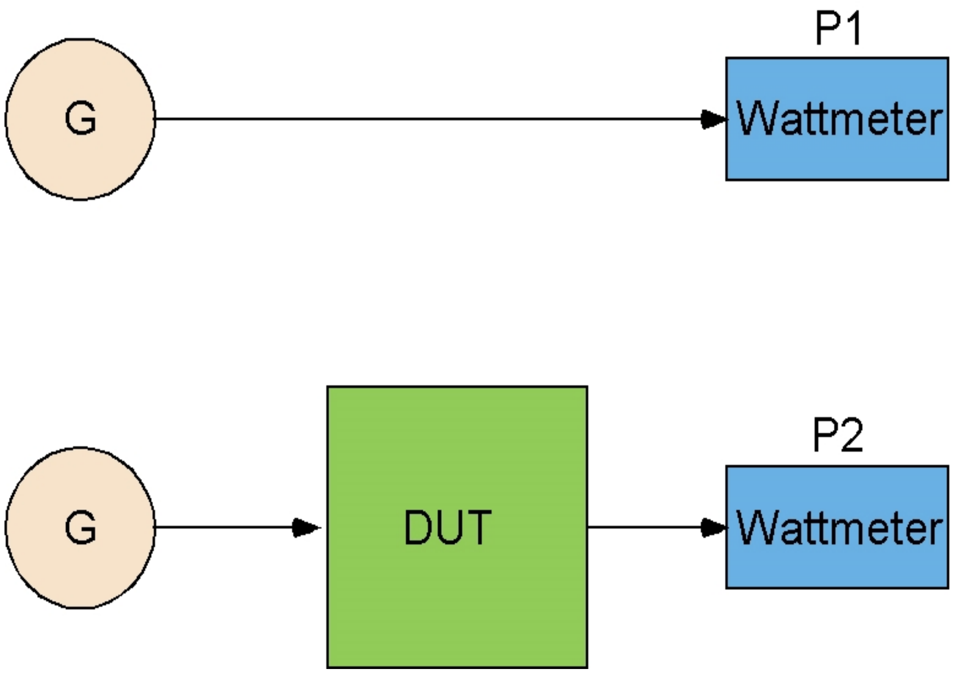
Component characterization

relative measurements

Outline

- Introduction
- Reflectometry
  - SWR measurements
  - Coupler reflectometry
- Measurement of transfer functions
- The network analyser

Introduction : an example



$P1 = 10 \pm 1.2$ mW
$P2 = 9 \pm 1.1$ mW

$P1 - P2 = 1 \pm 2.3$ mW

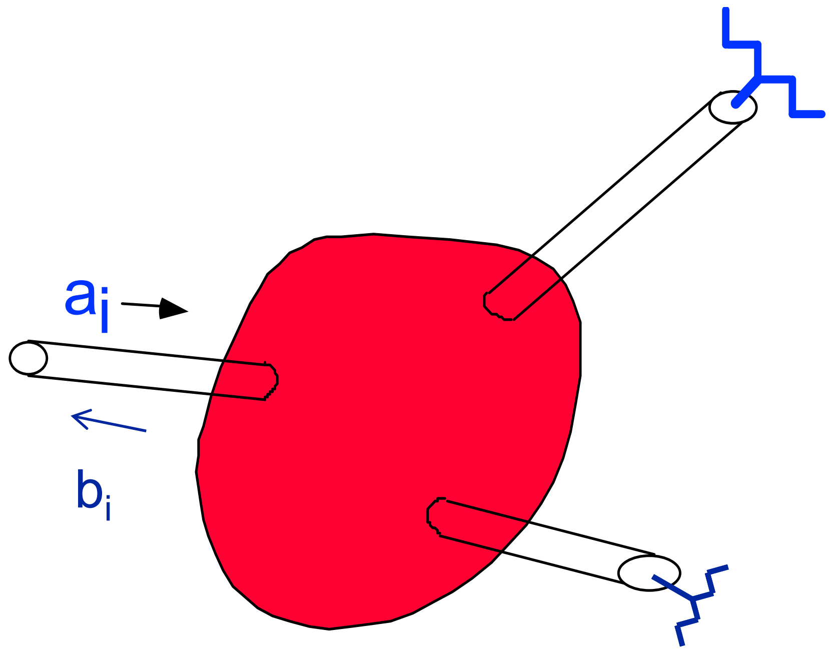
$P2/P1 = 0.9^{+0.248}_{-0.195}$ mW

Thus relative measurement are needed

Component characteristics

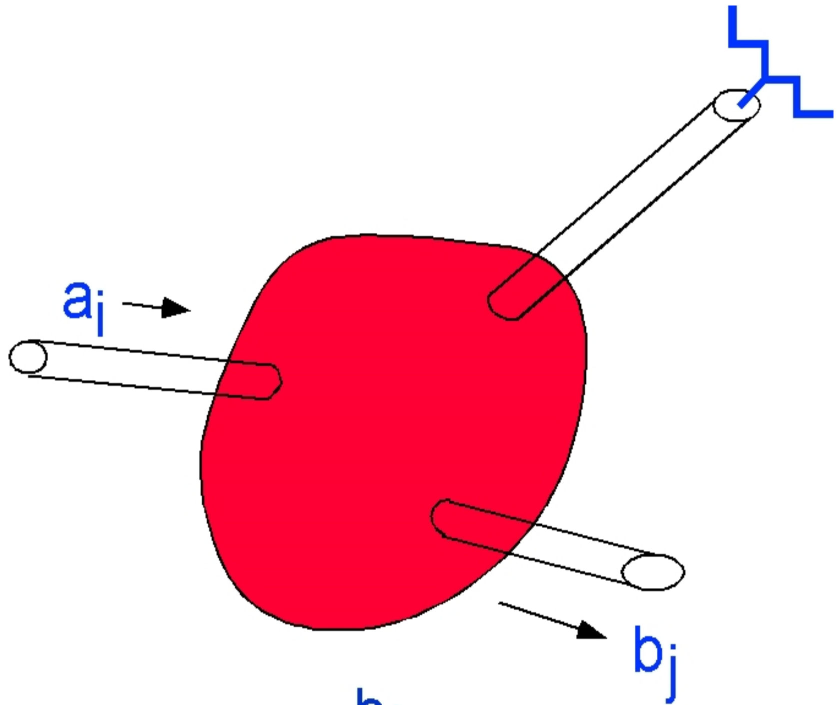
- A component is defined by its scattering matrix
- An "s" parameter is the ratio between two signals
- This ratio should be obtained by comparison, not by computations in order to minimize errors
- Two distinct cases : reflexion and transmission

Reflexion measurements



$$s_{ii} = \frac{b_i}{a_i}$$

Transmission measurements

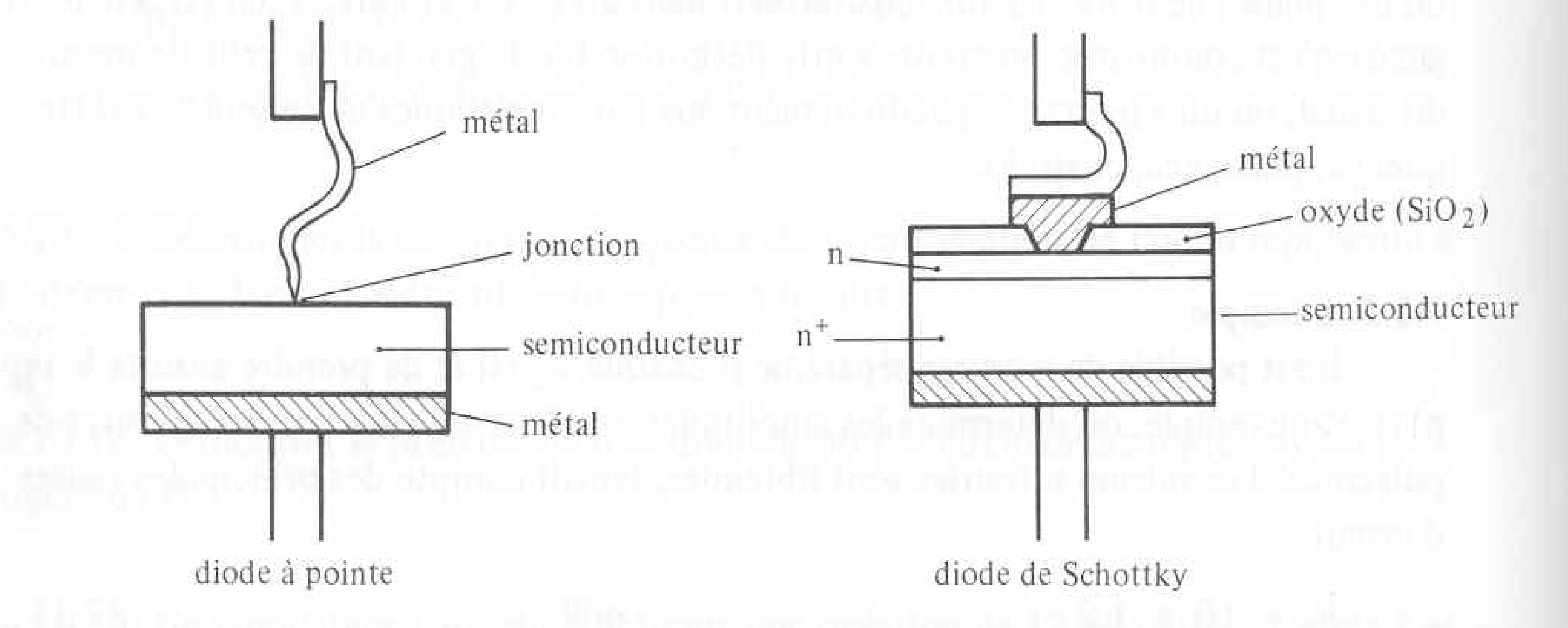


$$s_{ji} = \frac{b_j}{a_i}$$

Comparative measurements

- Search of minima and maxima
- Combining signals (sum or difference)
- Obtention of zeros and subsitution measuremnts
- Coherent frequency change and low frequency measurements

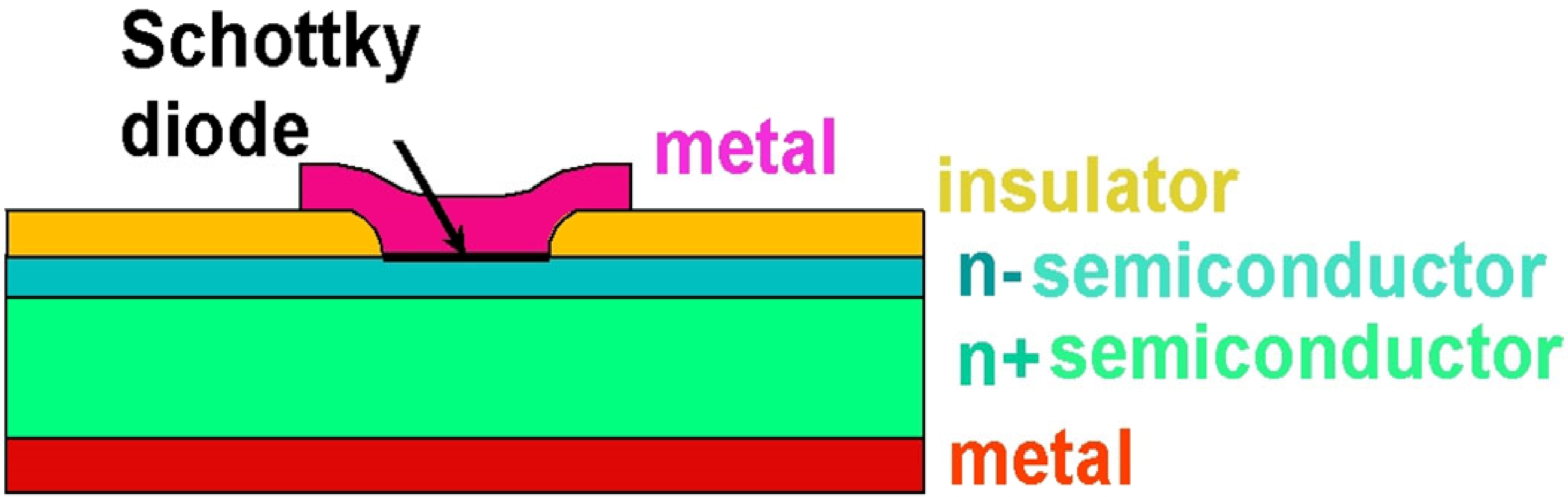
A short interlude : detectors



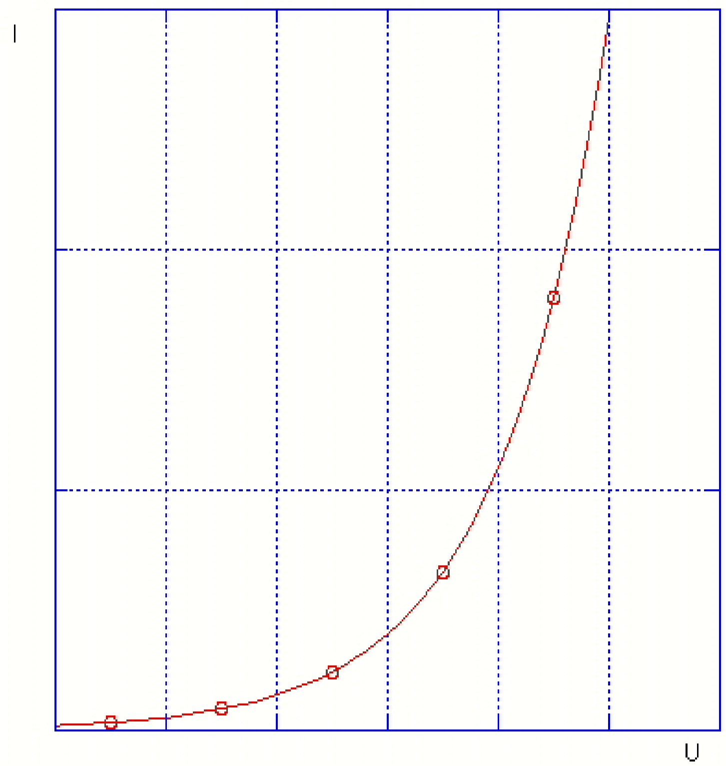
point diode

Schottky diode

Detectors


Schottky
diode
metal
insulator
n- semiconductor
n+ semiconductor
metal

Detectors



$$I = I_s\left(e^{\alpha U} - 1\right) = I_s\left[\alpha U + \frac{(\alpha U)^2}{2!} + \frac{(\alpha U)^3}{3!} + \dots\right]$$

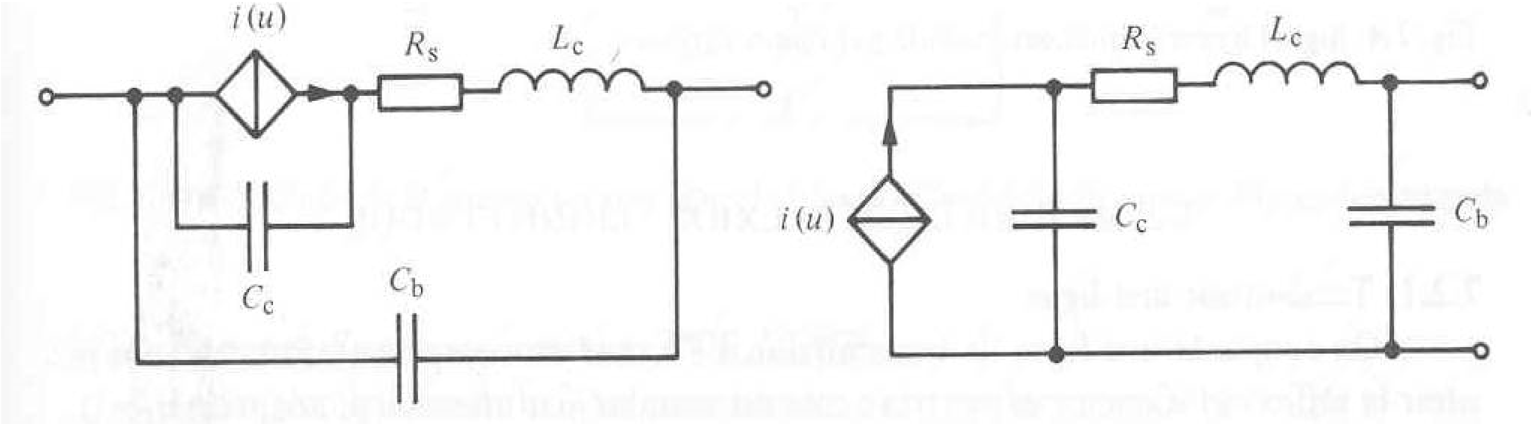
Thus, for $u = u_1 \cos \omega t$

The current will have a contiuous term given by

$$I_c = I_s\left[\frac{(\alpha u_1)^2}{4} + \frac{(\alpha u_1)^4}{64} + \frac{(\alpha u_1)^6}{2304} + \dots\right]$$

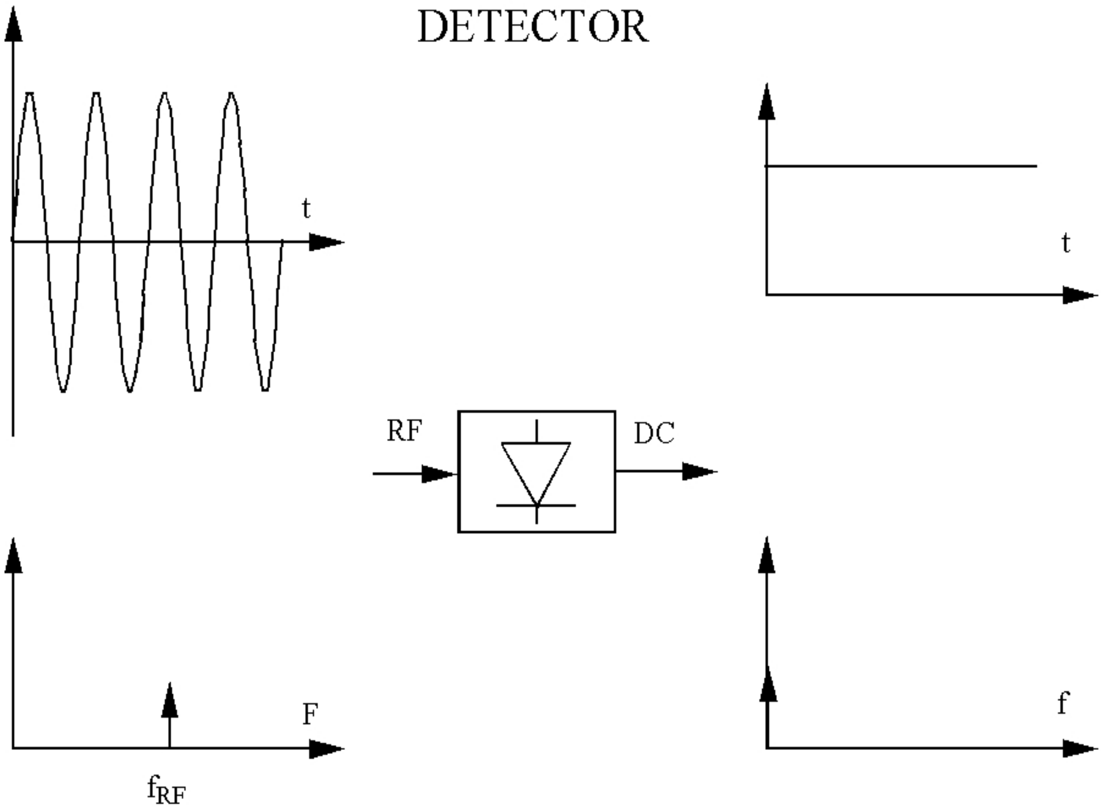
Thus Ic is proportional to P : Quadratic detector

Detectors

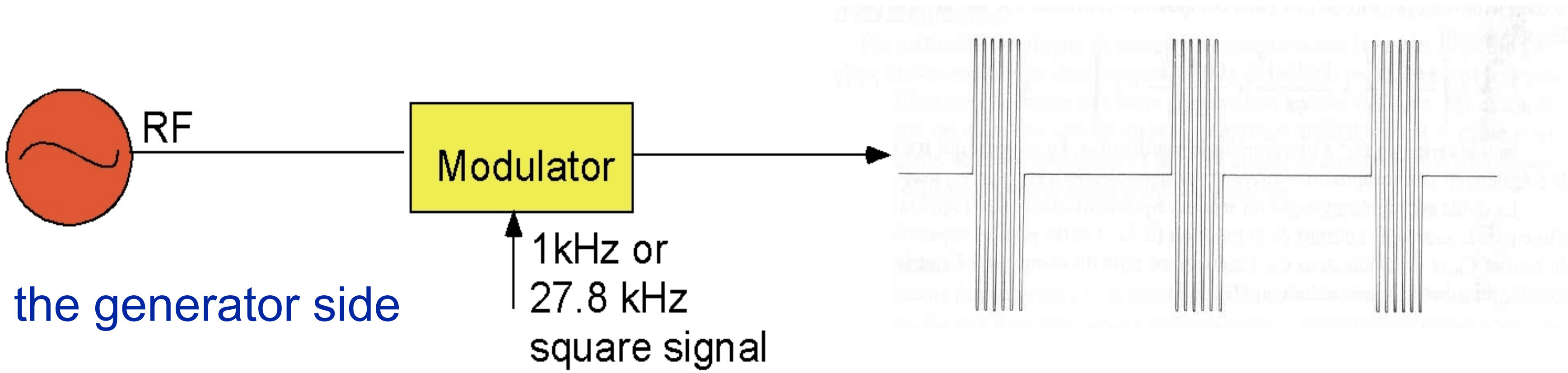


The detector itself has a low pass equivalent circuit

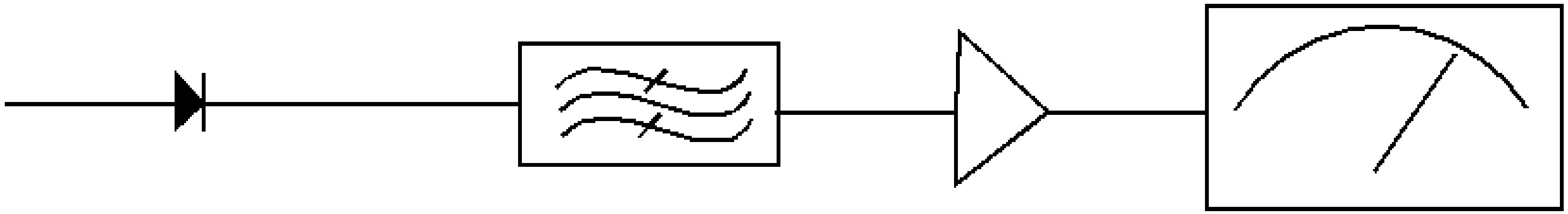
Detectors



Detectors : modulation

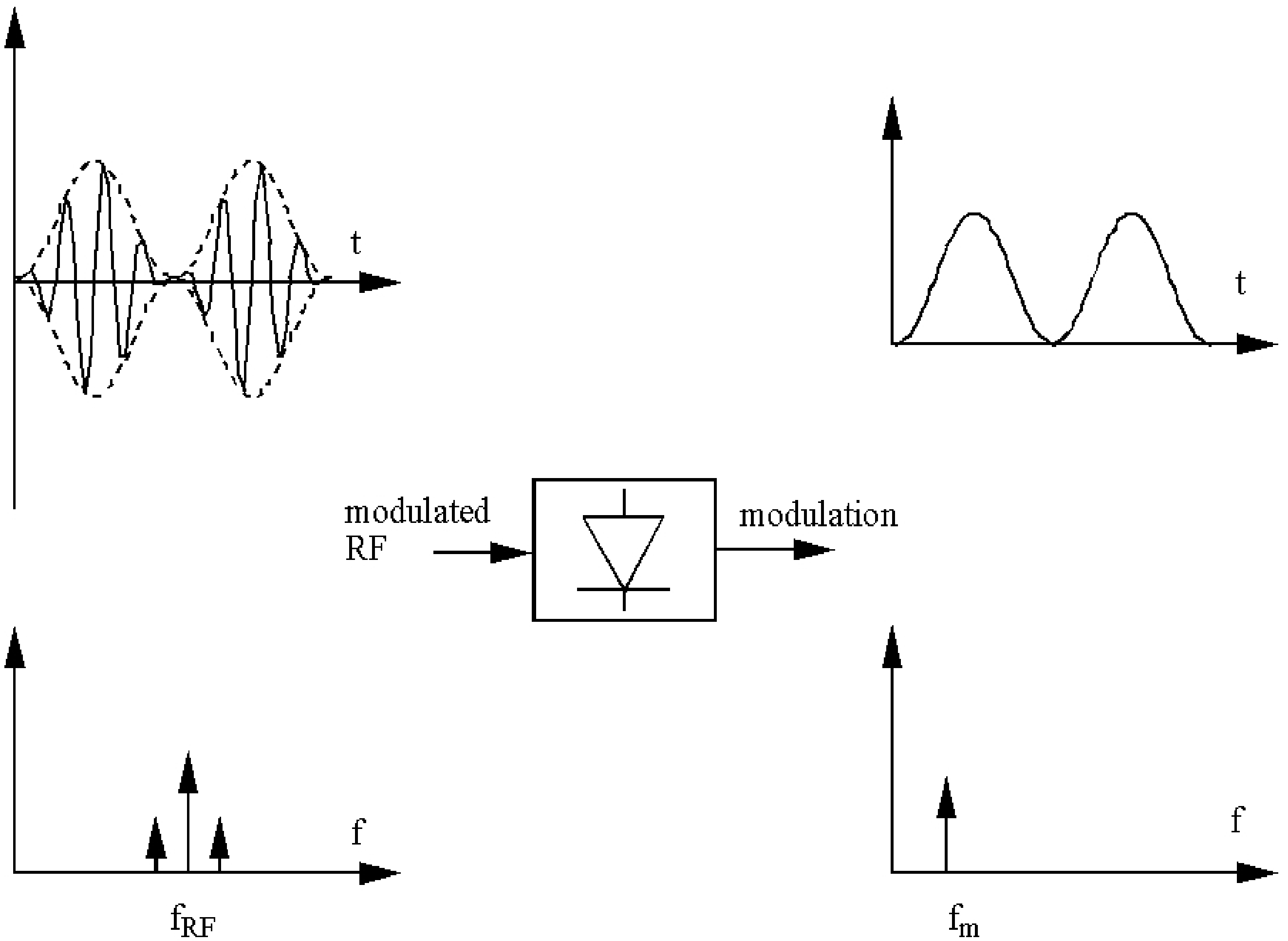


on the generator side



on the detector side

Detectors : modulation



Modulated detection

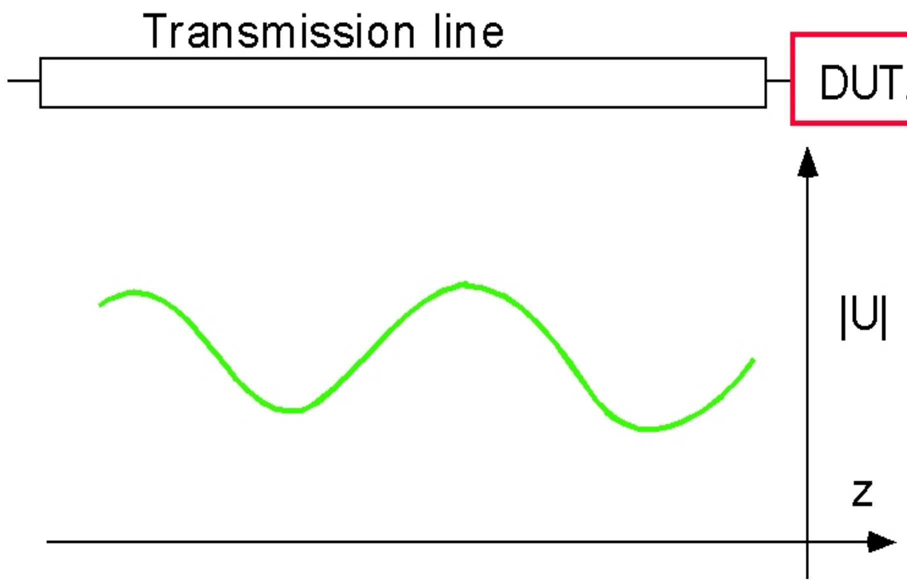
Detectors : modulation

- Advantages :
  - the band pass filter is tunable, and can be very selective
  - The amplifiers are coupled in AC and not in DC, which takes care of the drift problems

- Do not forget to adjust the frequency of the filter or of the modulating signal !

Measurement of the reflexion coefficient

The good old method : the VSWR meter



The voltage along a tranmission line can be expressed as the sum of an incident and a reflected wave

$$U_i = \sqrt{Z_{ci}}\left(a_i + b_i\right) = \sqrt{Z_{ci}}\left(a_i + s_{ii}a_i\right)$$

$$U_i\left(z\right) = \sqrt{Z_{ci}}\,a_i\left(z\right)\left[1 + s_{ii}e^{2j\beta z}\right]$$

$$\left|U_i\left(z\right)\right| = \sqrt{Z_{ci}}\left|a_i\left(z\right)\right|\sqrt{\left[1+\left|s_{ii}\right|\cos\left(\varphi+2\beta z\right)\right]^2 + \left|s_{ii}\right|^2\sin^2\left(\varphi+2\beta z\right)}$$

$$\left|U_i\left(z\right)\right| = \sqrt{Z_{ci}}\left|a_i\left(z\right)\right|\sqrt{1+\left|s_{ii}\right|^2 + 2\left|s_{11}\right|\cos\left(\varphi+2\beta z\right)}$$

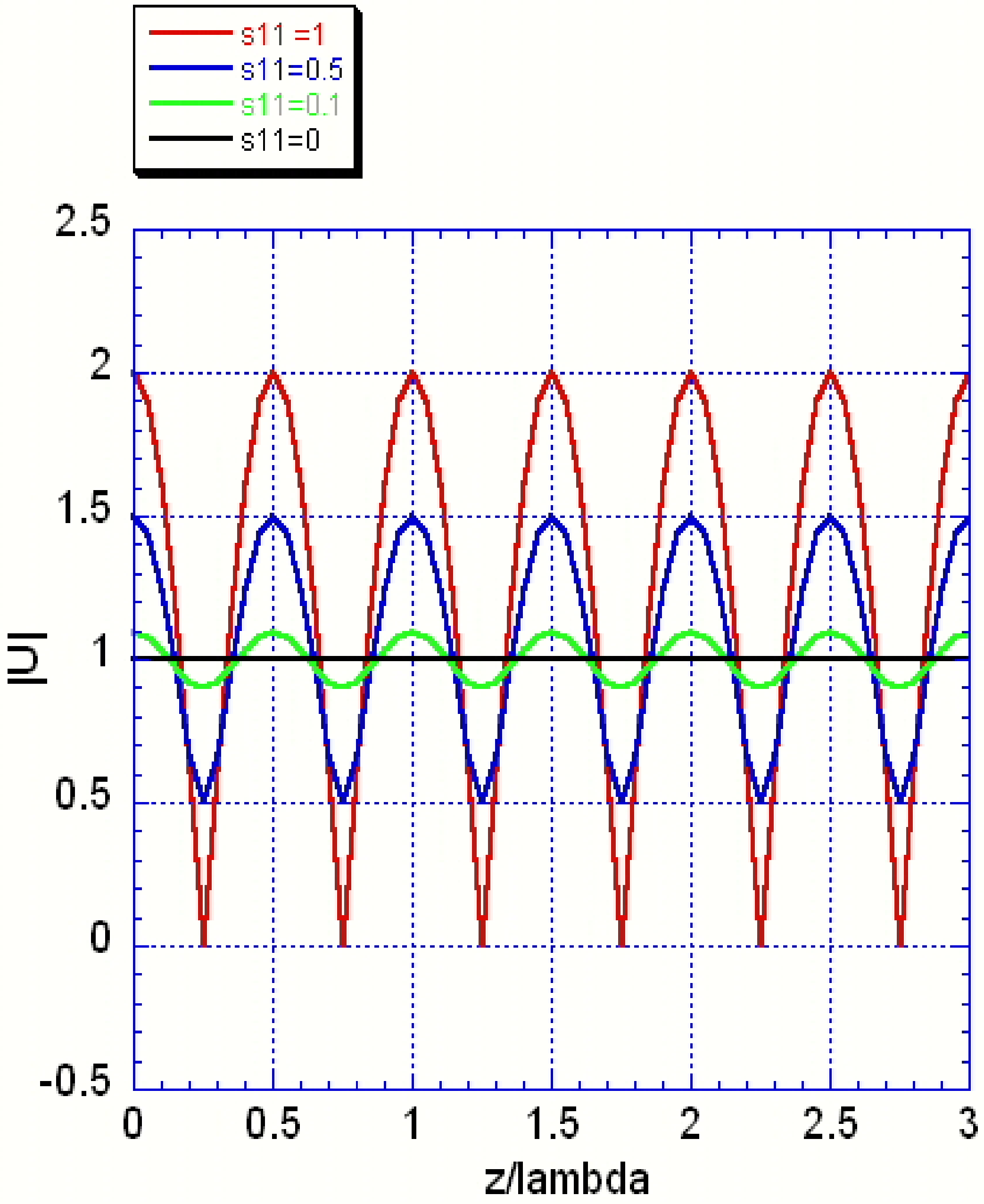
VSWR : definition

Extrema of $|U_i|$:

$$U_{\max} = \sqrt{Z_{ci}}\,|a_i|\left(1+|s_{ii}|\right) \quad at \;\; \varphi + 2\beta z_i = 2n\pi$$

$$U_{\min} = \sqrt{Z_{ci}}\,|a_i|\left(1-|s_{ii}|\right) \quad at \;\; \varphi + 2\beta z_i = (2n+1)\pi$$

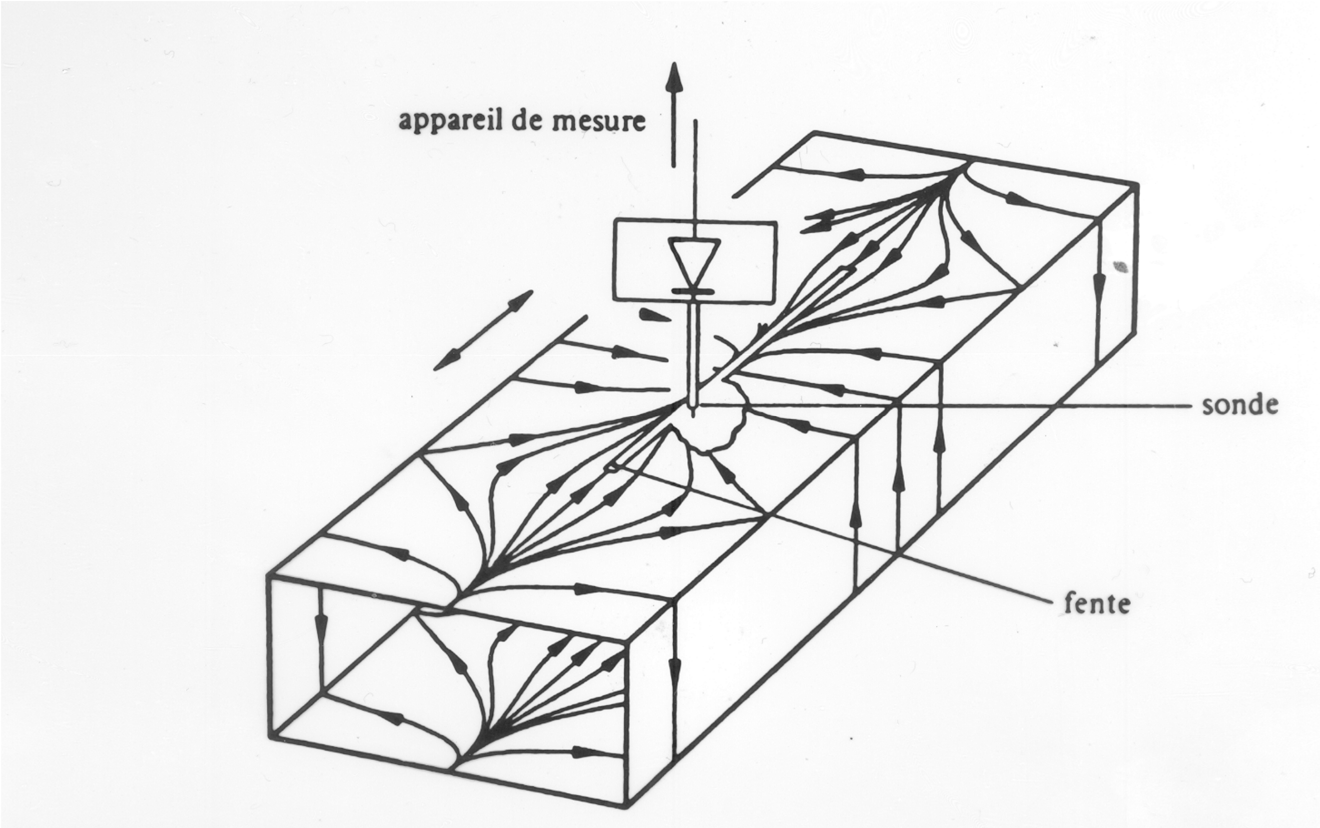
$$ROS = VSWR = \frac{|U_{\max}|}{|U_{\min}|} = \frac{1+|s_{ii}|}{1-|s_{ii}|}$$


s11 =1
s11=0.5
s11=0.1
s11=0
2.5
2
1.5
1
0.5
0
-0.5
|U|
0
0.5
1
1.5
2
2.5
3
z/lambda

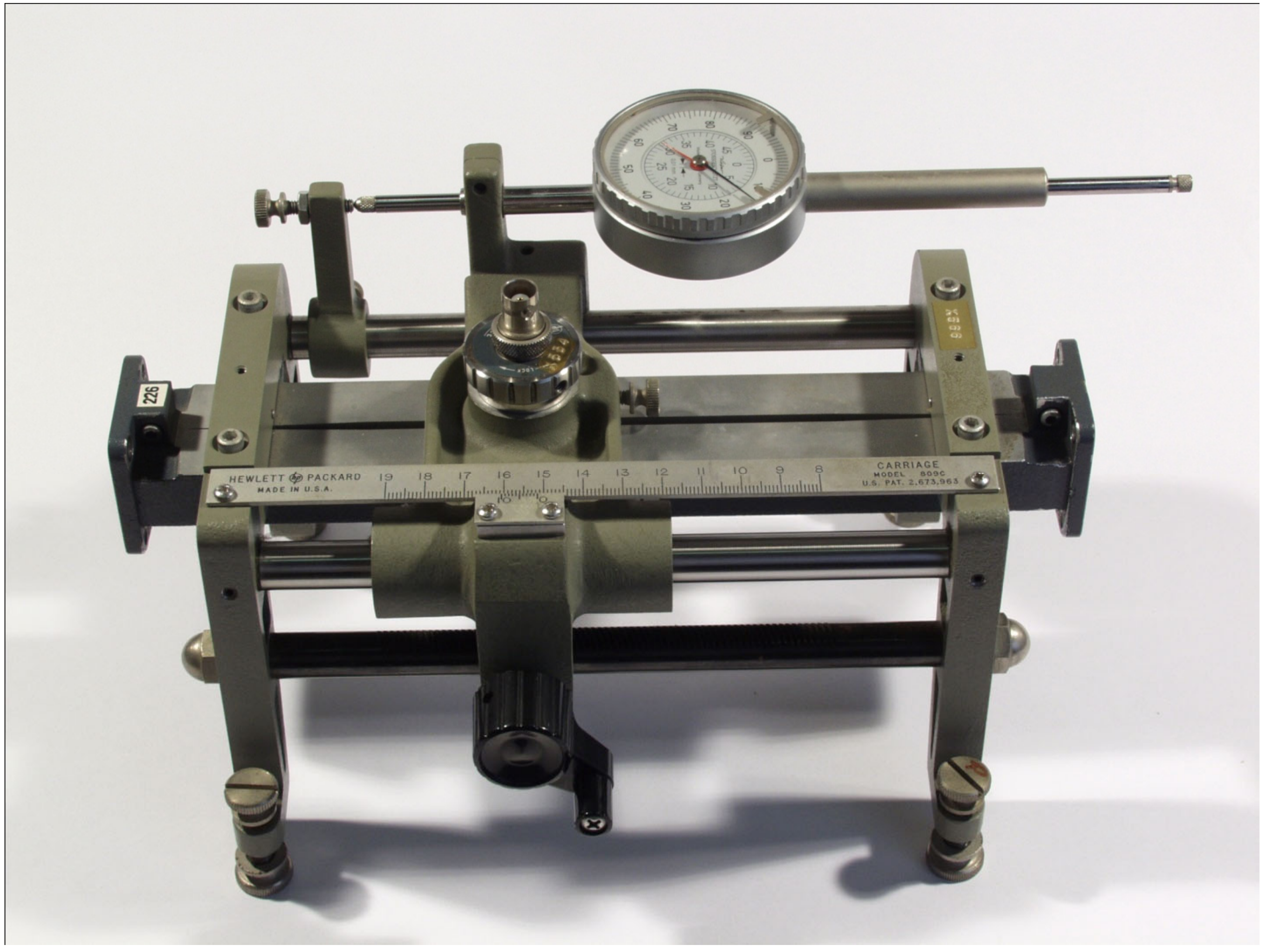
Measurement of the VSWR

- vary z
- vary β (thus the frequency)
- vary both

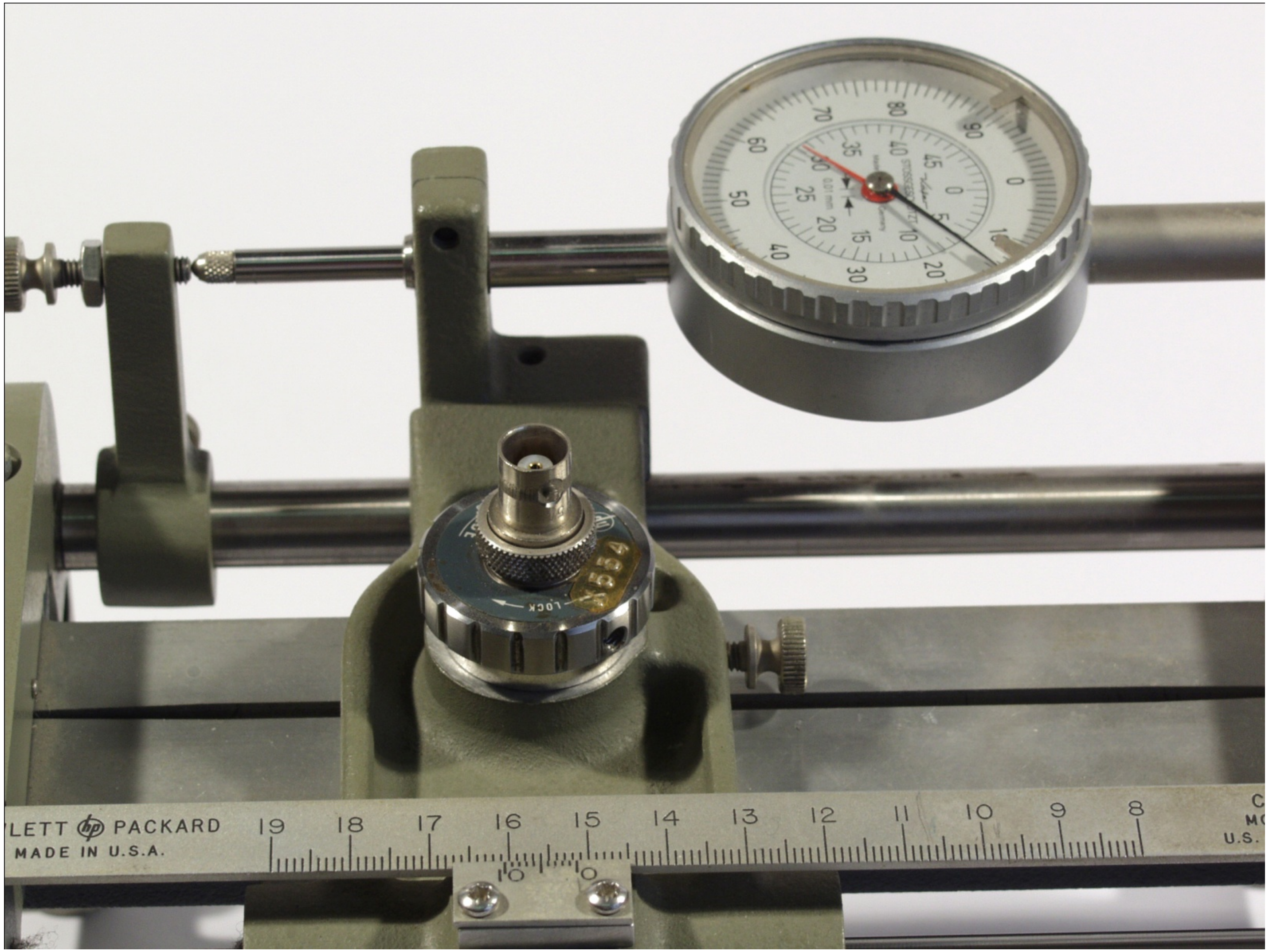
Slotted line



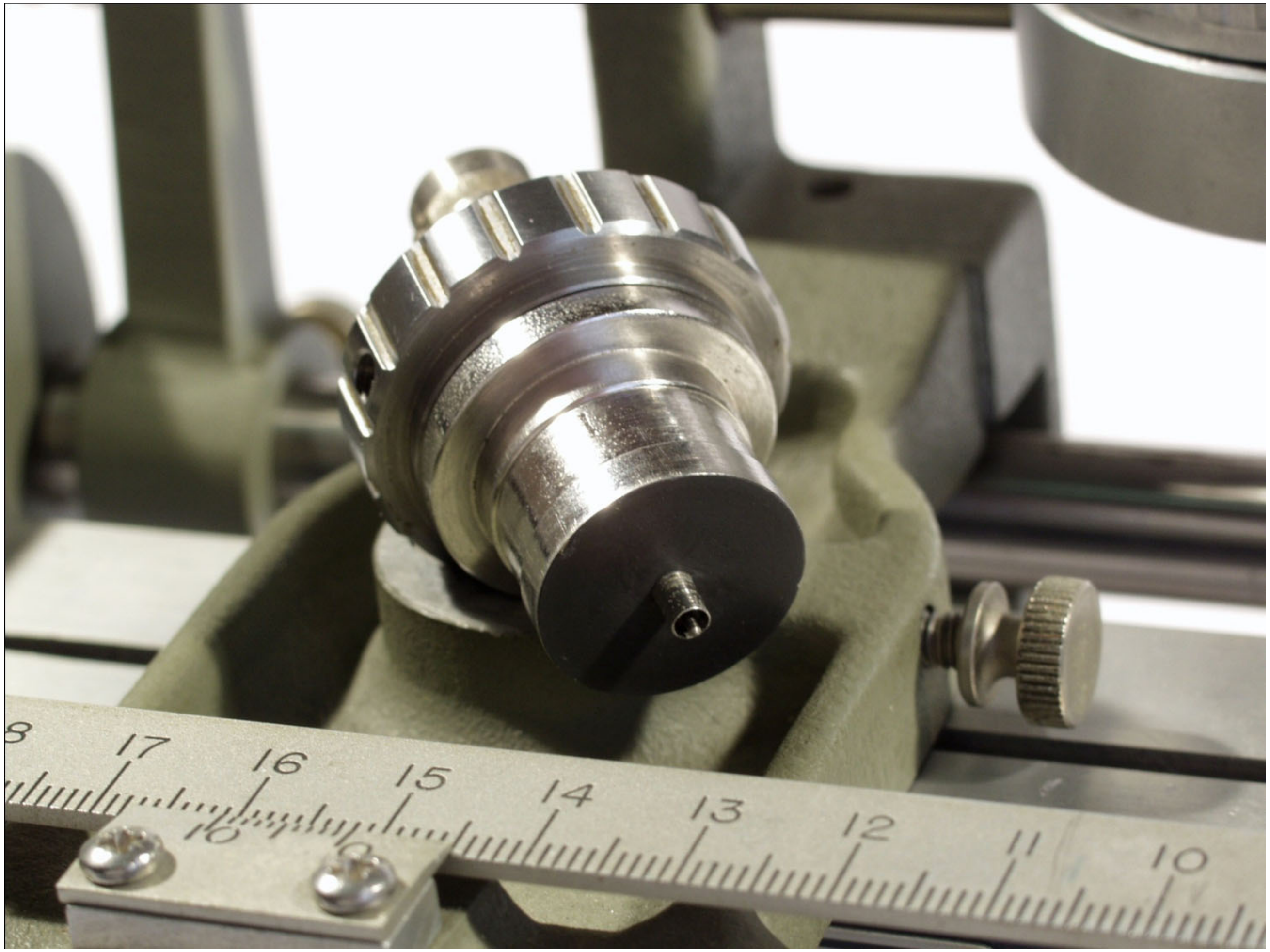
Slotted line



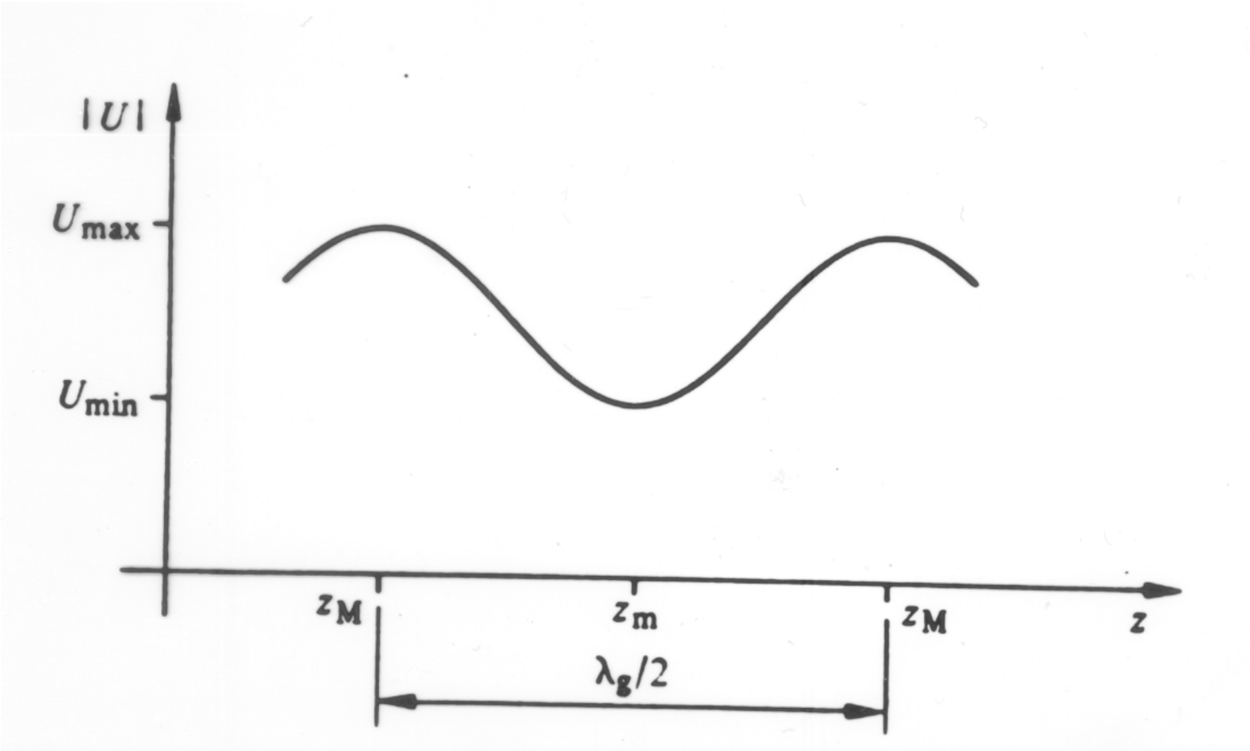
Slotted line



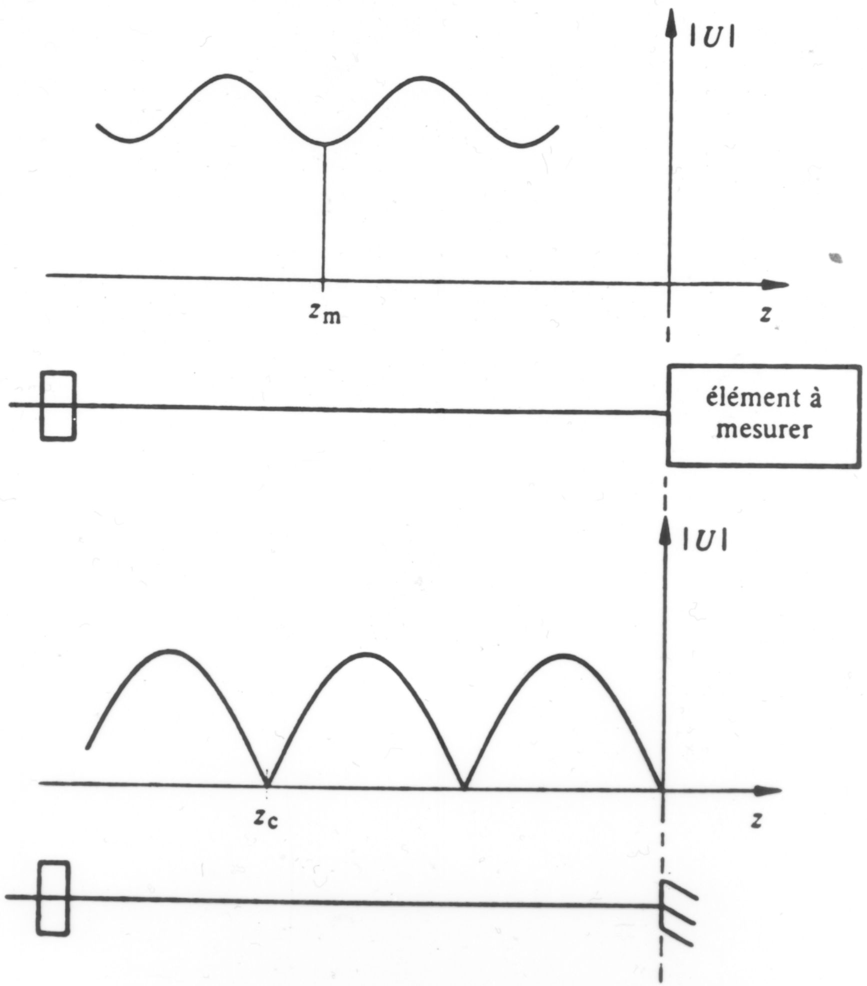
Slotted line



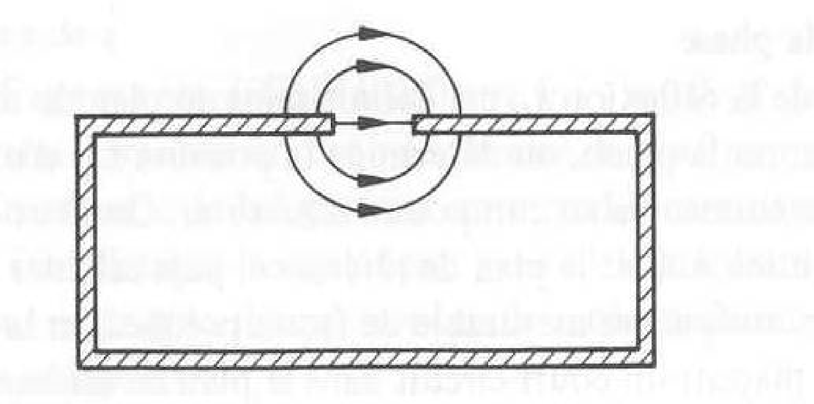
Slotted line



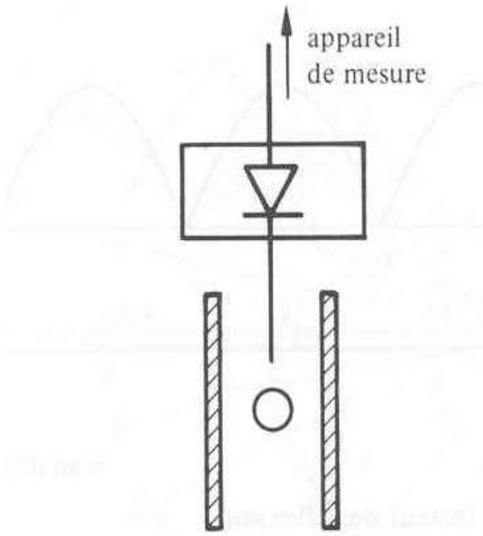
slotted line



slotted line



The slot mode



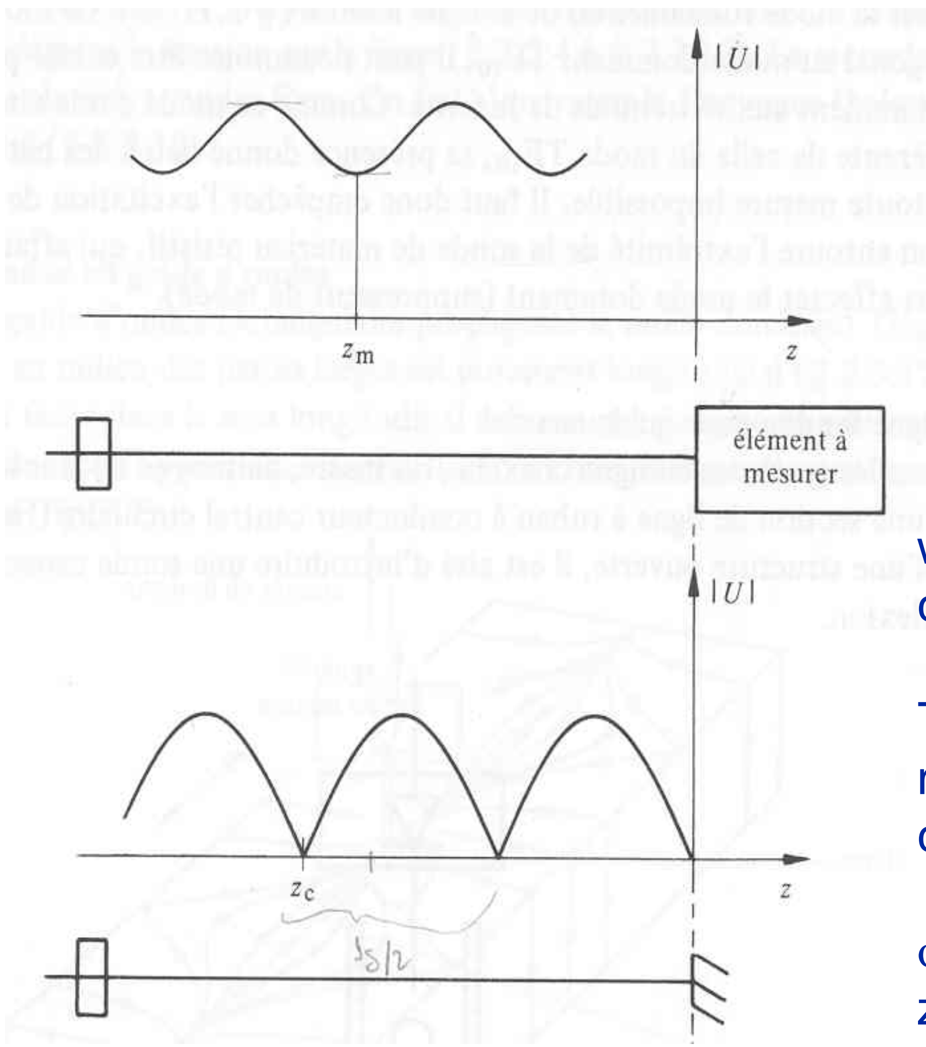
coaxial slotted line

Measurement procedure

- Find a maximum of the voltage by moving the probe in the line
- Adjust the needle on the display to the max. by adjusting the gain
- Move to a minimum
- Read the VSWR on the display

- Take care to stay in the quadratic zone of the detector. If it is not possible, use a substitution procedure with a precision tunable attenuator !

Phase measurement

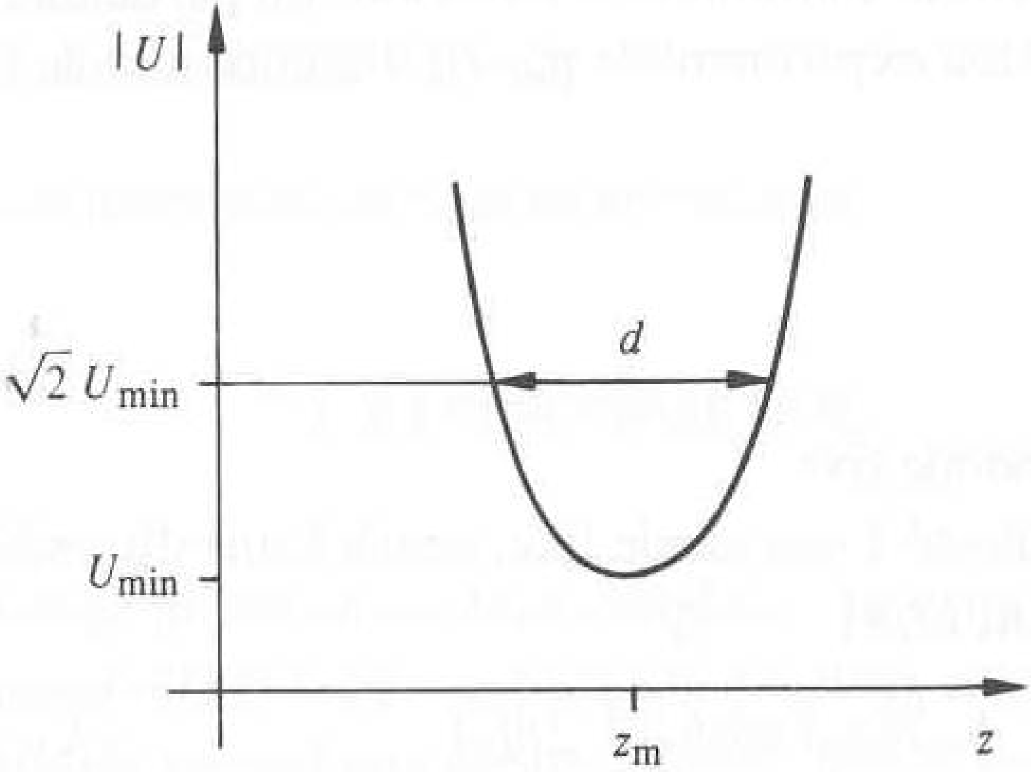


we know that the minimum at z_c corresponds to $\varphi=\pi$. Thus $\beta z_c=n\pi$.

The phase φ of the unknown reflection
coefficient is then given by

$\varphi=2\beta(z_c-z_m)+(2n-1)\pi=4\pi(z_c-z_m)/\lambda_g+\pi\pm2n\pi$

Special case od large VSWR



vswr ≈ $\lambda_g/\pi d$

Modern reflectometry

- Using one coupler
- Using two couplers

The microwave coupler : a short reminder

Outline

- Theoretical approach
- Ideal coupler
- Real coupler
- Applications :
  - Hybrid ring
  - Hybrid coupler

4-port scattering matrix

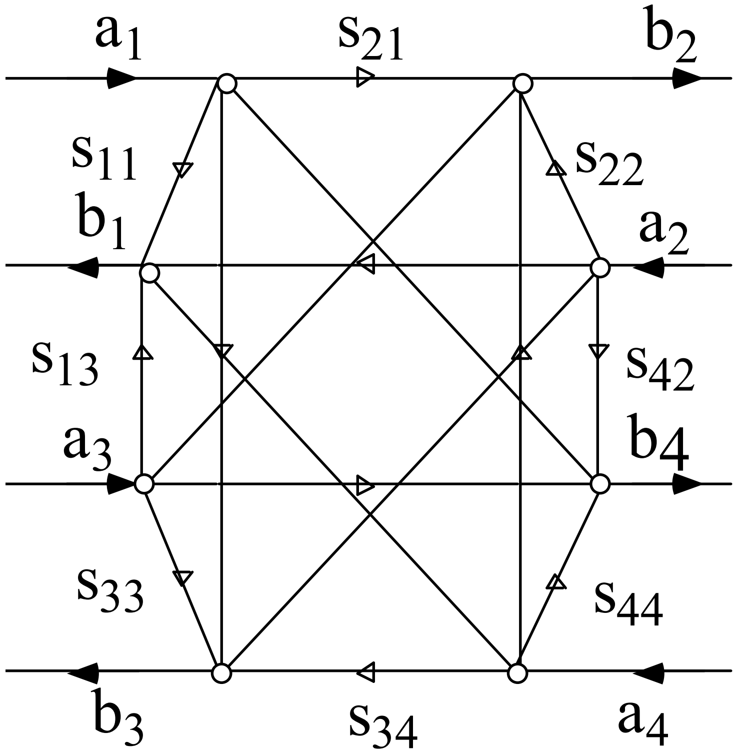
$$\begin{bmatrix} s_{11} & s_{12} & s_{13} & s_{14} \\ s_{21} & s_{22} & s_{23} & s_{24} \\ s_{31} & s_{32} & s_{33} & s_{34} \\ s_{41} & s_{42} & s_{43} & s_{44} \end{bmatrix}$$

General

$$\begin{bmatrix} 0 & s_{12} & s_{13} & s_{14} \\ s_{12} & 0 & s_{23} & s_{24} \\ s_{13} & s_{23} & 0 & s_{34} \\ s_{14} & s_{24} & s_{34} & 0 \end{bmatrix}$$

adapted and reciprocal

flow chart



adapted and reciprocal and lossless 4-port

$$\begin{bmatrix} 0 & s_{12}^* & s_{13}^* & s_{14}^* \\ s_{12}^* & 0 & s_{23}^* & s_{24}^* \\ s_{13}^* & s_{23}^* & 0 & s_{34}^* \\ s_{14}^* & s_{24}^* & s_{34}^* & 0 \end{bmatrix} \begin{bmatrix} 0 & s_{12} & s_{13} & s_{14} \\ s_{12} & 0 & s_{23} & s_{24} \\ s_{13} & s_{23} & 0 & s_{34} \\ s_{14} & s_{24} & s_{34} & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

What does it mean ?

$$s_{13}^{*}s_{14}+s_{23}^{*}s_{24}=0 \qquad (line\,3\,\mathrm{x}\,column\,4) \qquad \mathrm{x}\,s_{14}^{*}$$

$$s_{13}^{*}s_{23}+s_{14}^{*}s_{24}=0 \qquad (line\,1\,\mathrm{x}\,column\,2) \qquad \mathrm{x}\,s_{23}^{*}$$

$-$

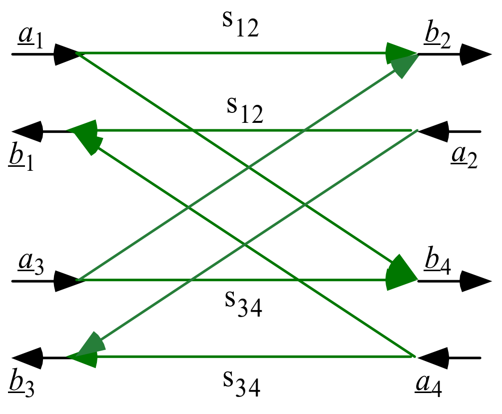
$$s_{13}^{*}\left(\left|s_{14}\right|^{2}-\left|s_{23}\right|^{2}\right)=0$$

$$s_{13}^{*}\left(\left|s_{14}\right|^{2}-\left|s_{23}\right|^{2}\right)=0$$

1st solution : $s_{13}=0\ ,\ s_{14}\neq 0\ \ ,\ s_{23}\neq 0$

thus : $s_{13}^{*}s_{14}+s_{23}^{*}s_{24}=0\ \Rightarrow\ s_{24}=0$

$$\begin{bmatrix} 0 & s_{12} & 0 & s_{14} \\ s_{12} & 0 & s_{23} & 0 \\ 0 & s_{23} & 0 & s_{34} \\ s_{14} & 0 & s_{34} & 0 \end{bmatrix}$$



$$s_{13}^{*}\left(\left|s_{14}\right|^{2}-\left|s_{23}\right|^{2}\right)=0$$

2 nd solution : $\left|s_{14}\right|=\left|s_{23}\right|$

selection of reference planes : $s_{14}=s_{23}=j\beta$

$$\left|s_{12}\right|^{2}+\left|s_{13}\right|^{2}+\left|s_{14}\right|^{2}=1 \qquad (line\,1\,\mathrm{x}\,column\,1)$$

$$\left|s_{13}\right|^{2}+\left|s_{23}\right|^{2}+\left|s_{34}\right|^{2}=1 \qquad (line\,3\,\mathrm{x}\,column\,3)$$

thus : $s_{12}=s_{34}=\alpha$ with adequate ref. planes

2^{nd} solution

$$s_{12}^{*}s_{24}+s_{13}^{*}s_{34}=0=\alpha\left(s_{24}+s_{13}^{*}\right) \qquad (line\,1\,\mathrm{x}\,column\,4)$$

$$s_{13}^{*}s_{14}+s_{23}^{*}s_{24}=0=\beta\left(s_{13}^{*}-s_{24}\right) \qquad (line\,3\,\mathrm{x}\,column\,4)$$

$\alpha=0\,,\,\beta=0$

$$\begin{bmatrix} 0 & 0 & s_{13} & 0 \\ 0 & 0 & 0 & s_{24} \\ s_{13} & 0 & 0 & 0 \\ 0 & s_{24} & 0 & 0 \end{bmatrix}$$

$s_{13}=s_{24}=0$

$$\begin{bmatrix} 0 & s_{12} & 0 & s_{14} \\ s_{12} & 0 & s_{23} & 0 \\ 0 & s_{23} & 0 & s_{34} \\ s_{14} & 0 & s_{34} & 0 \end{bmatrix}$$

Ideal coupler

$$\begin{bmatrix} 0 & s_{12}^* & 0 & s_{14}^* \\ s_{12}^* & 0 & s_{23}^* & 0 \\ 0 & s_{23}^* & 0 & s_{34}^* \\ s_{14}^* & 0 & s_{34}^* & 0 \end{bmatrix} \begin{bmatrix} 0 & s_{12} & 0 & s_{14} \\ s_{12} & 0 & s_{23} & 0 \\ 0 & s_{23} & 0 & s_{34} \\ s_{14} & 0 & s_{34} & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$|s_{12}|^2 + |s_{14}|^2 = 1$$

$$s_{12} s_{23}^* + s_{14} s_{34}^* = 0$$

$$|s_{12}|^2 + |s_{23}|^2 = 1$$

$$s_{12} s_{14}^* + s_{23} s_{34}^* = 0$$

$$|s_{23}|^2 + |s_{34}|^2 = 1$$

$$|s_{14}|^2 + |s_{34}|^2 = 1$$

Ideal coupler

$$|s_{12}| = |s_{34}| = \alpha$$

$$|s_{14}| = |s_{23}| = \beta$$

$$\alpha^2 + \beta^2 = 1$$

$$s_{12} = \alpha e^{j\varphi}$$

$$s_{34} = \alpha e^{j\eta}$$

$$s_{14} = \beta e^{j\psi}$$

$$s_{23} = \beta e^{j\theta}$$

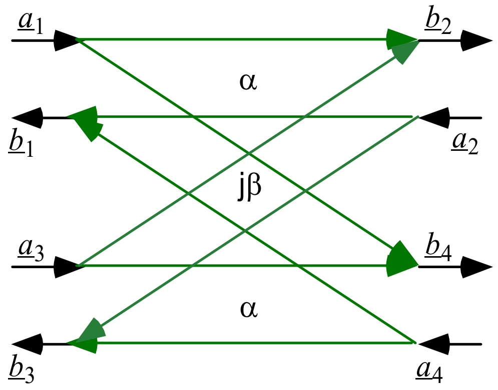
with $(\varphi + \eta) = \pi + (\psi + \theta) + 2n\pi$

and $\alpha > \beta$ by convention

symmetric coupler

$$\psi = \theta = \pi/2$$

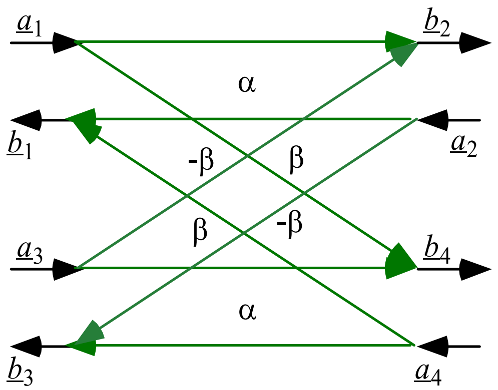
$$\begin{bmatrix} 0 & \alpha & 0 & j\beta \\ \alpha & 0 & j\beta & 0 \\ 0 & j\beta & 0 & \alpha \\ j\beta & 0 & \alpha & 0 \end{bmatrix}$$



asymmetric coupler

$$\psi = 0 \ , \ \theta = \pi$$

$$\begin{bmatrix} 0 & \alpha & 0 & \beta \\ \alpha & 0 & -\beta & 0 \\ 0 & -\beta & 0 & \alpha \\ \beta & 0 & \alpha & 0 \end{bmatrix}$$

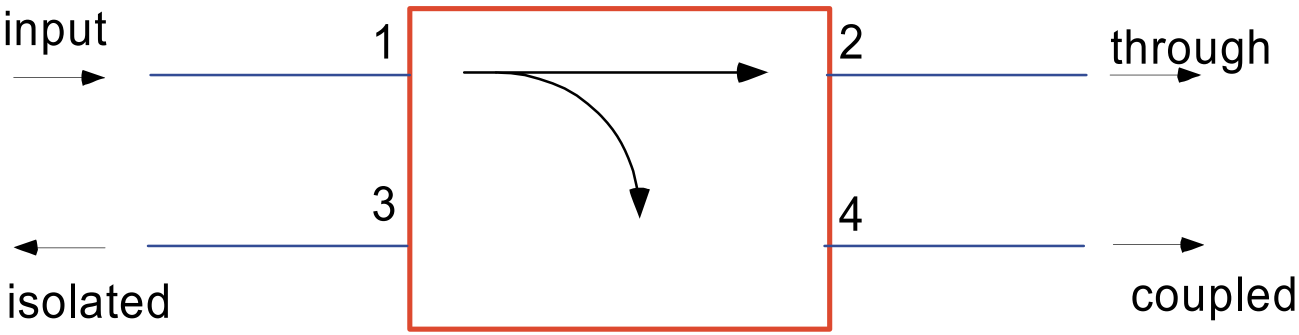


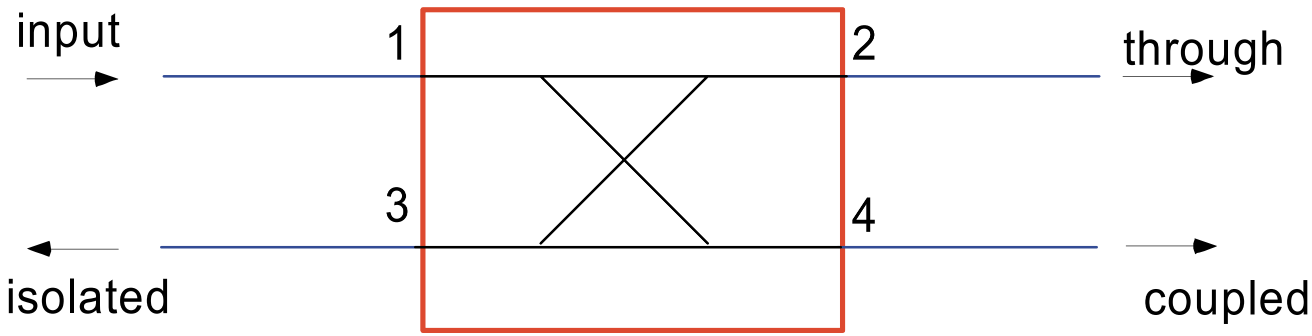
Summary : ideal coupler

Coupling : LC = -20log10 β

Attenuation : LA = -20log10 α

$\alpha \geq \beta$



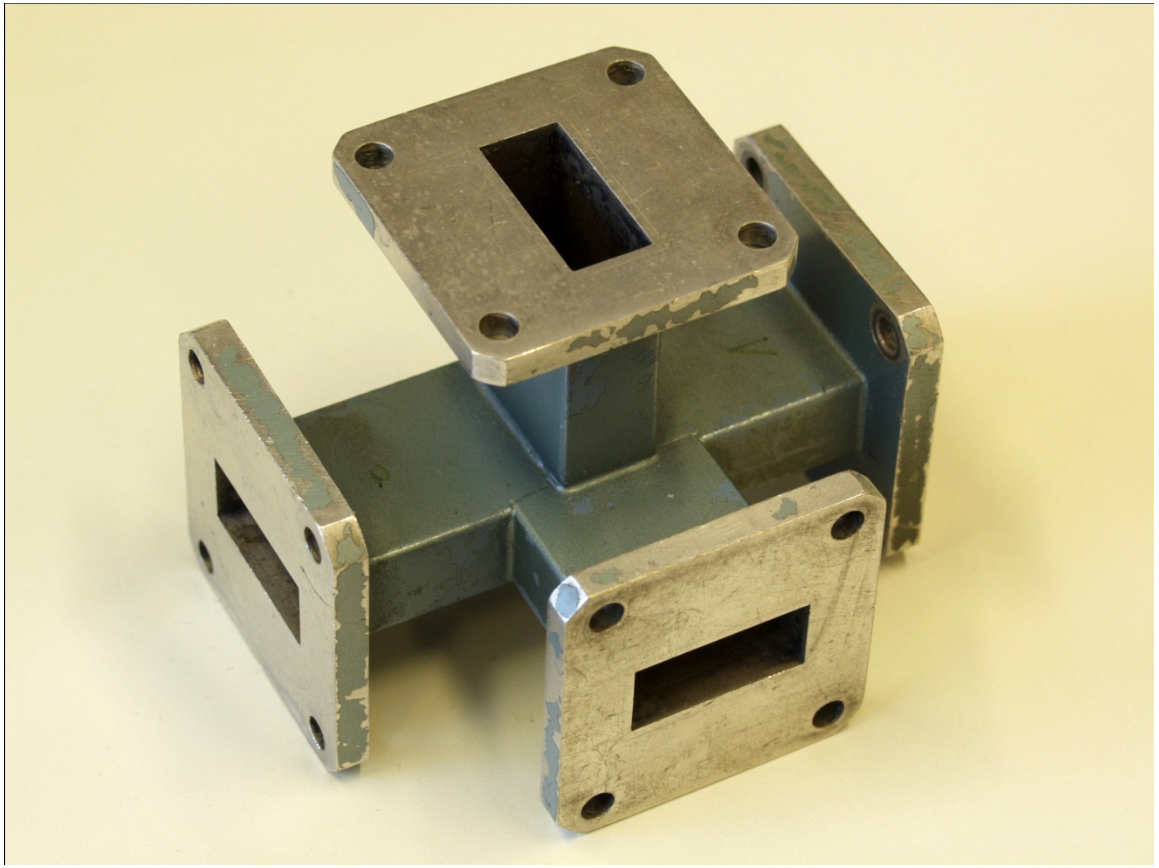


real coupler

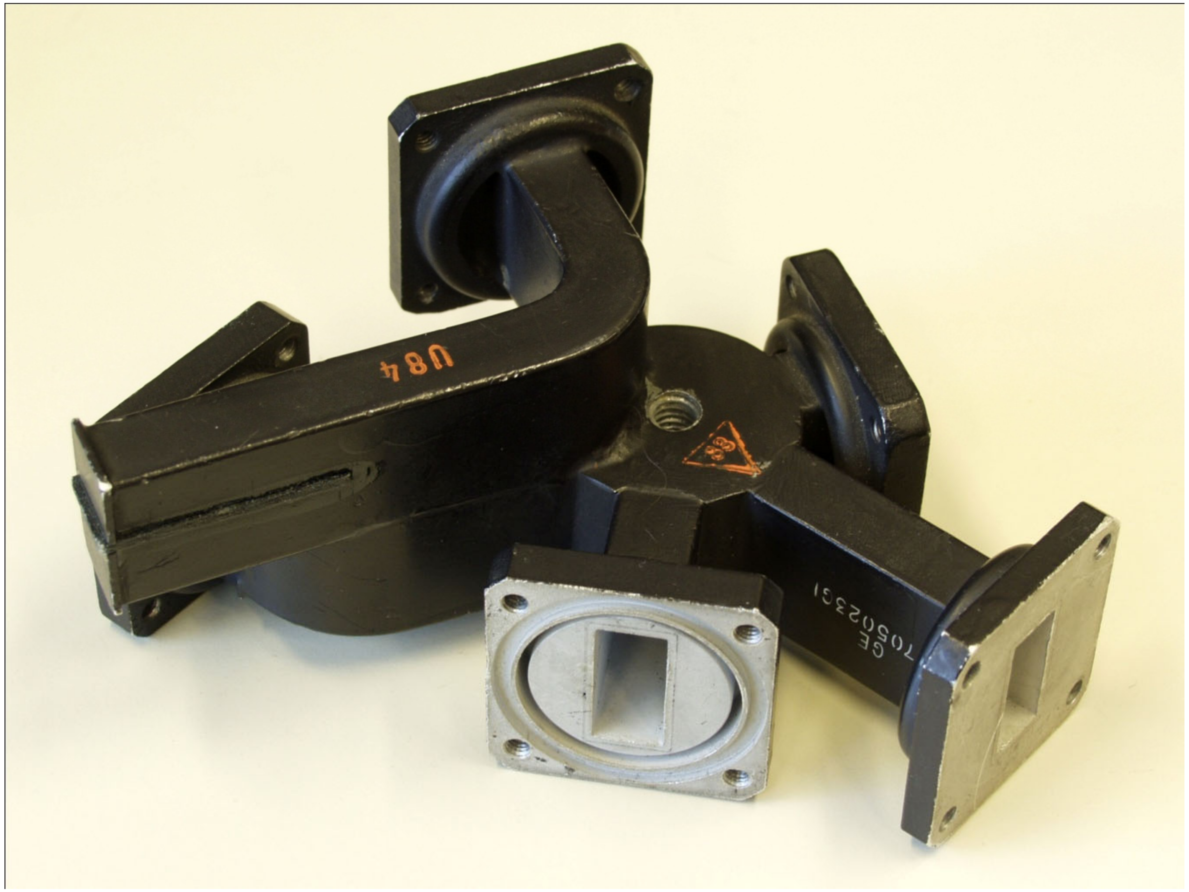
- $\alpha \geq \beta$
- Attenuation : $LA = -20\log_{10} \alpha$
- Coupling : $LC = -20\log_{10} \beta$
- Isolation : $LI_{31} = -20\log_{10} |s_{31}|$
- Isolation : $LI_{24} = -20\log_{10}|s_{24}|$
- Directivity : $LD_{13} = LI_{13}\text{-}LC = -20\log_{10} |s_{31}|/\beta$
- Directivity : $LD_{24} = LI_{24}\text{-}LC = -20\log_{10} |s_{24}|/\beta$

example : 10 dB waveguide coupler

example : magic T



example : magic T combined with 10dB coupler



Example : E Band 10 dB coupler

Example

LC [dB]	β	α	LA[dB]
40	0.01	0.99995	0.0004
30	0.0316	0.9995	0.004
20	0.1	0.995	0.04
10	0.316	0.948	0.45
3	0.707	0.707	3

Directivity : measure of quality

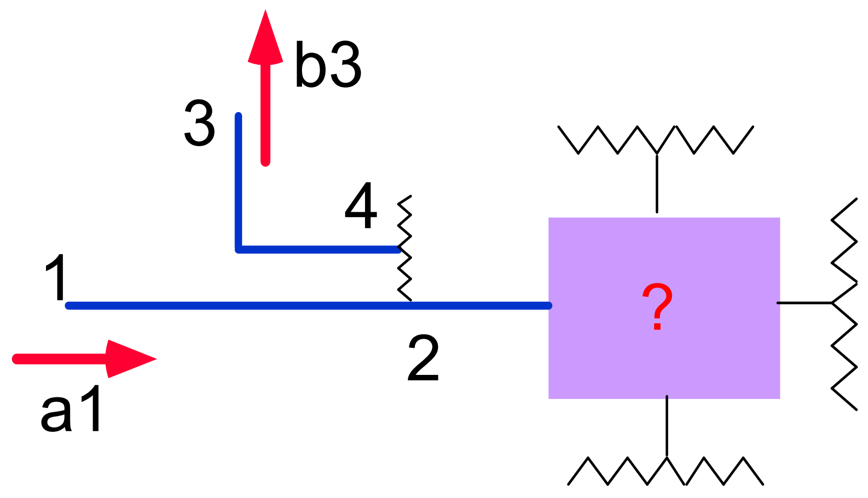
- Microstrip hybrid coupler : 30 dB
- Multihole waveguide couplers : 40 dB
- Waveguide couplers used in metrology : 50-60 dB

Couplers used in measurement

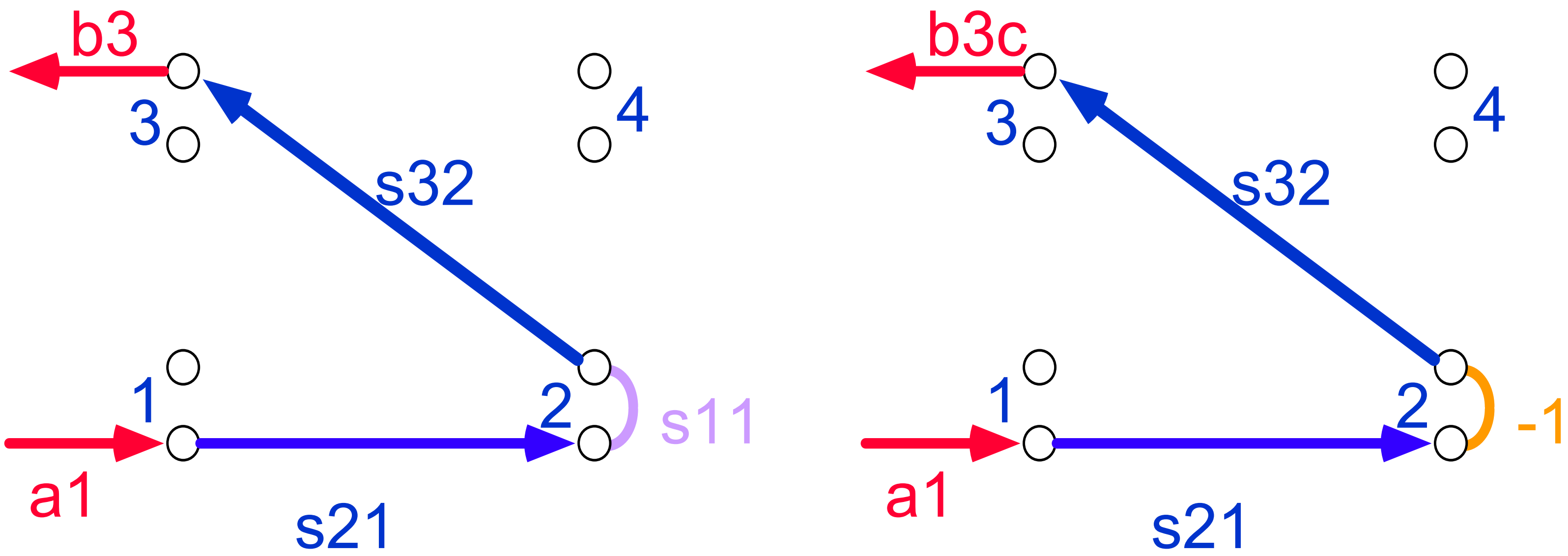
Reflectometry

- with 1 coupler
  - amplitude measurements
- with 2 couplers
  - precision measurements
  - phase and amplitude measurements

1 coupler reflectometry



1 coupler reflectometry



$$b_3 = s_{21} s_{ii} s_{32} a_1$$

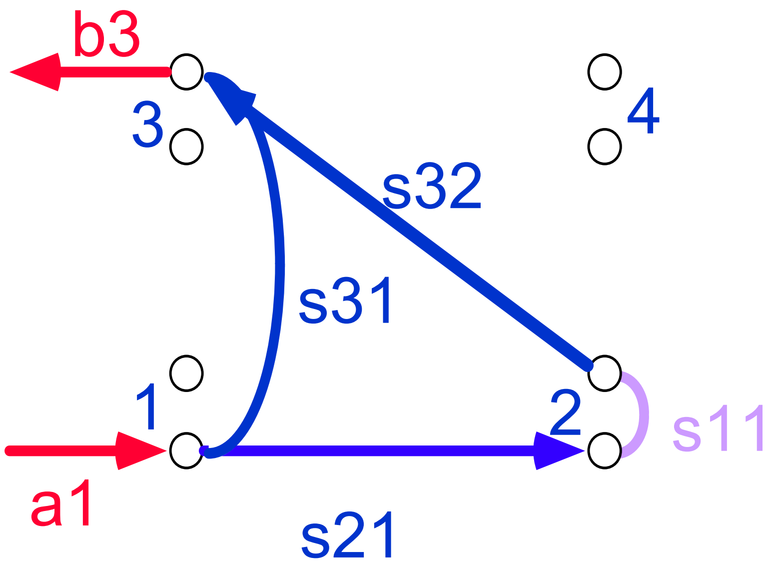
$$\boxed{\frac{b_3}{b_{3c}} = -s_{ii}}$$

$$b_{3c} = -s_{21} s_{32} a_1$$

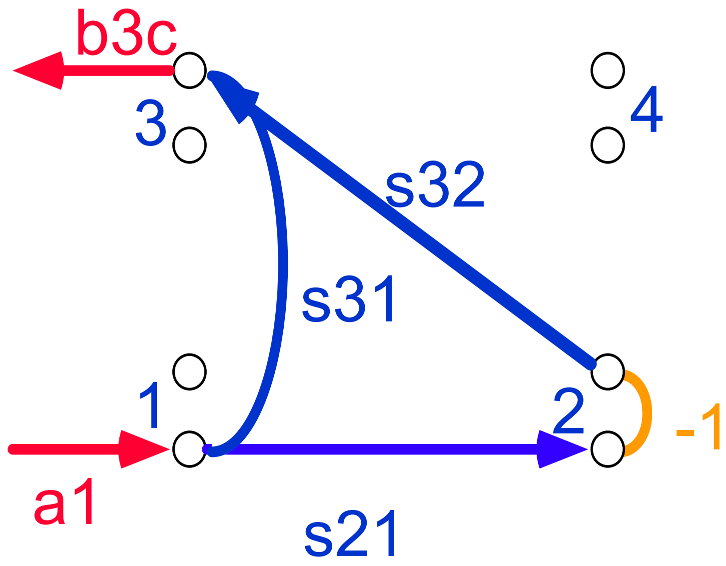
real coupler

- s_{ii}, s_{13}, s_{24} <<1, but non equal to zero
- we need :
  - coupling $LC=-20\log\beta$
  - isolation $LI_{13}=-20\log|s_{13}|$
  - $LI_{14}=-20\log|s_{24}|$
- The quality of the coupler is given by :
  - directivity $LD_{13}=LI_{13}-LC=-20\log|s_{13}|/\beta$
  - $LD_{24}=LI_{24}-LC=-20\log|s_{24}|/\beta$

1 coupler reflectometry



$$b_3 = a_1\left(s_{21}s_{ii}s_{32} + s_{31}\right)$$



$$b_{3c} = a_1\left(-s_{21}s_{32} + s_{31}\right)$$

$$\boxed{\frac{b_3}{b_{3c}} = \frac{s_{21}s_{ii}s_{32} + s_{31}}{-s_{21}s_{32} + s_{31}}}$$

1 coupler reflectometry

- s_{31} is non negligible at the numerator if s_{ii} <<1
- s_{31} is negligible at the denominator if the couple is of good quality : $\frac{b_3}{b_{3c}} = -s_{ii} - \frac{s_{31}}{s_{21}s_{32}}$
- The phases of the signals are not known

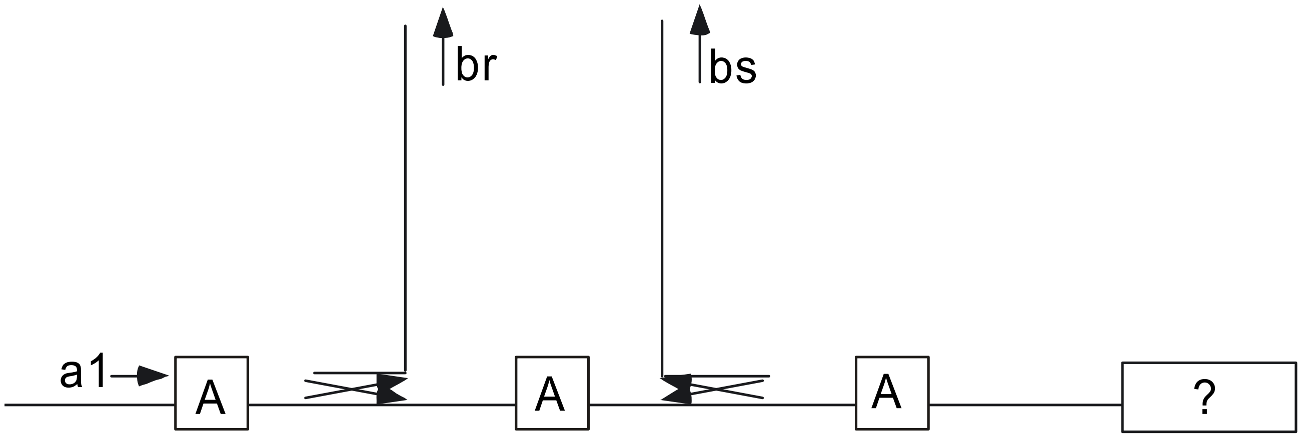
$$\left|\frac{b_3}{b_{3c}}\right| - \left|\frac{s_{31}}{s_{21}s_{32}}\right| \le s_{ii} \le \left|\frac{b_3}{b_{3c}}\right| + \left|\frac{s_{31}}{s_{21}s_{32}}\right|$$

1 coupler reflectometry

- For a weak coupling (LC large) S21 ≈ 1
- The error term is directly the directivity of the coupler : $$\left|\frac{b_3}{b_{3c}}\right| - \left|\frac{s_{31}}{s_{32}}\right| \leq s_{ii} \leq \left|\frac{b_3}{b_{3c}}\right| + \left|\frac{s_{31}}{s_{32}}\right|$$
-

LD [dB]	$\lvert s_{31}/s_{32} \rvert$
10	0.3162
20	0.1
30	0.03162
40	0.01
50	0.003162
60	0.001

2 coupler reflectometry



$$\frac{b_s}{b_r} = \frac{As_{ii} + B}{Cs_{ii} + D}$$

$$A = s_{21}s_{32} - s_{31}s_{22}$$

$$B = s_{31}$$

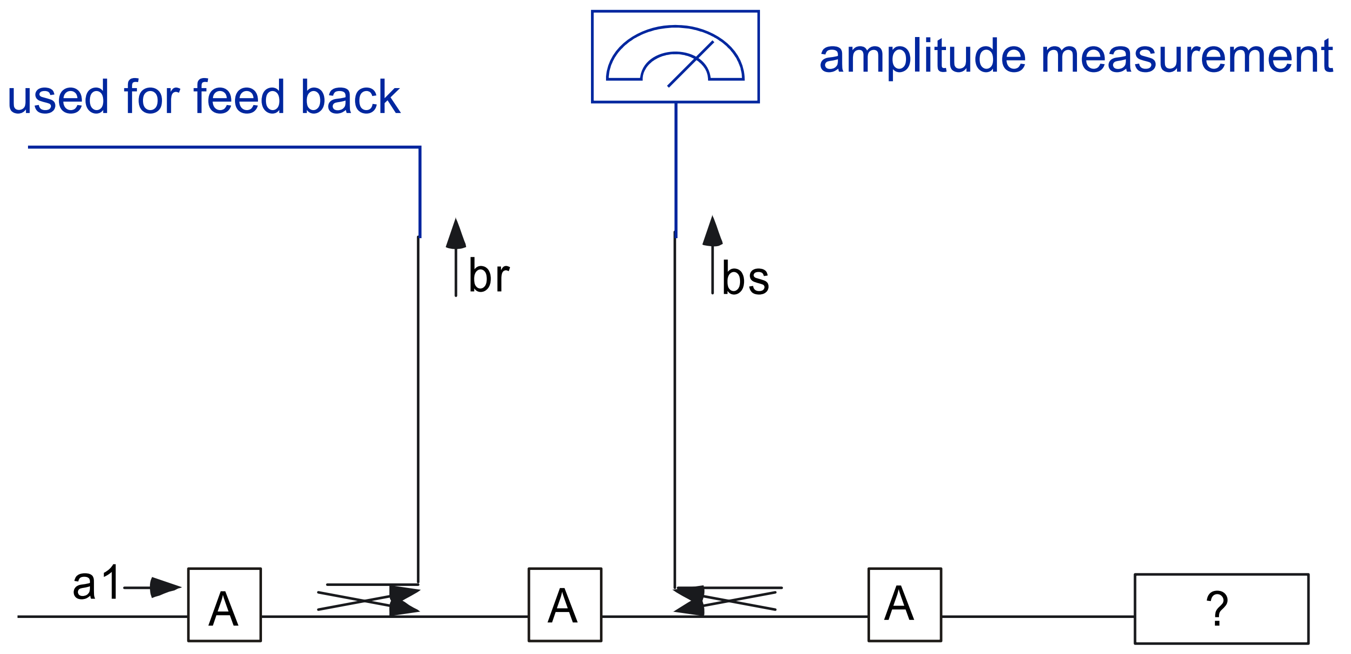
$$C = s_{21}s_{42} - s_{41}s_{22}$$

$$D = s_{41}$$

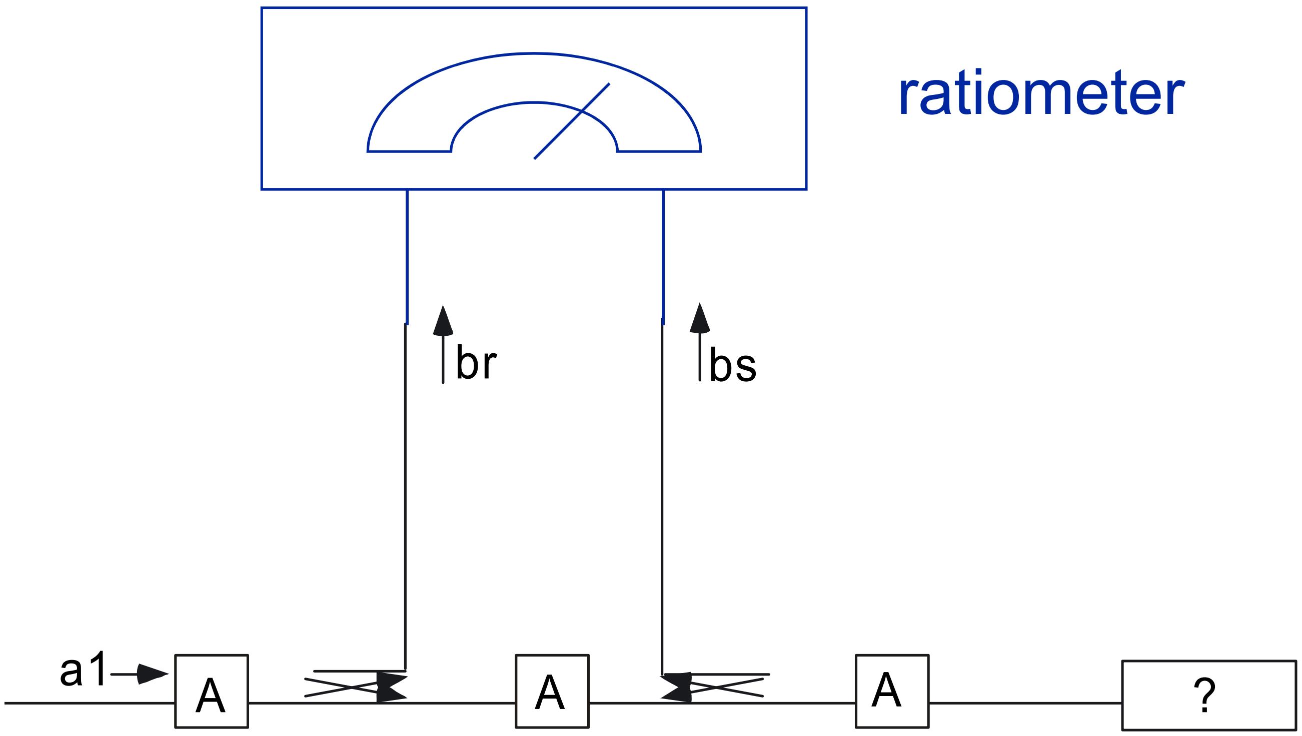
Use adapters to put B=C=0

$$\frac{b_s}{b_r} = \frac{s_{21}s_{ii}s_{32}}{s_{41}}$$

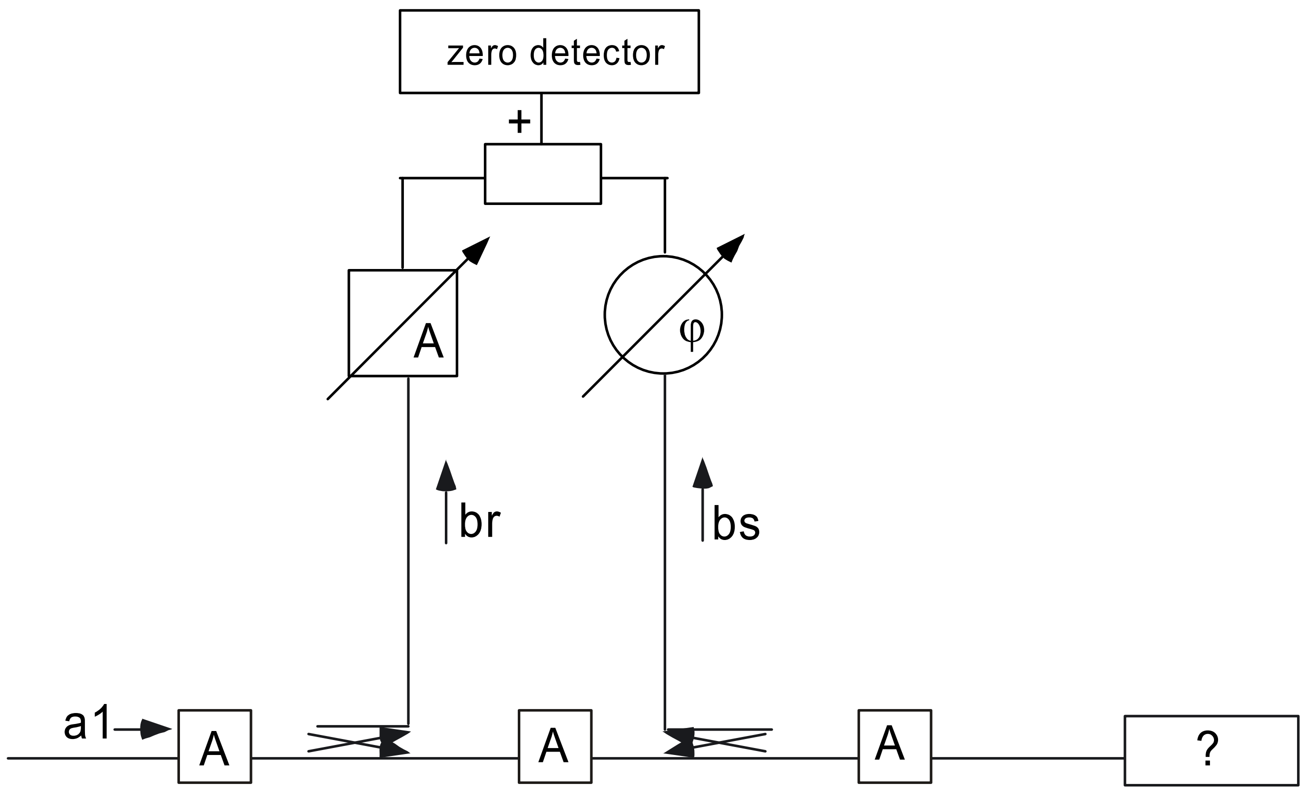
2 coupler reflectometry



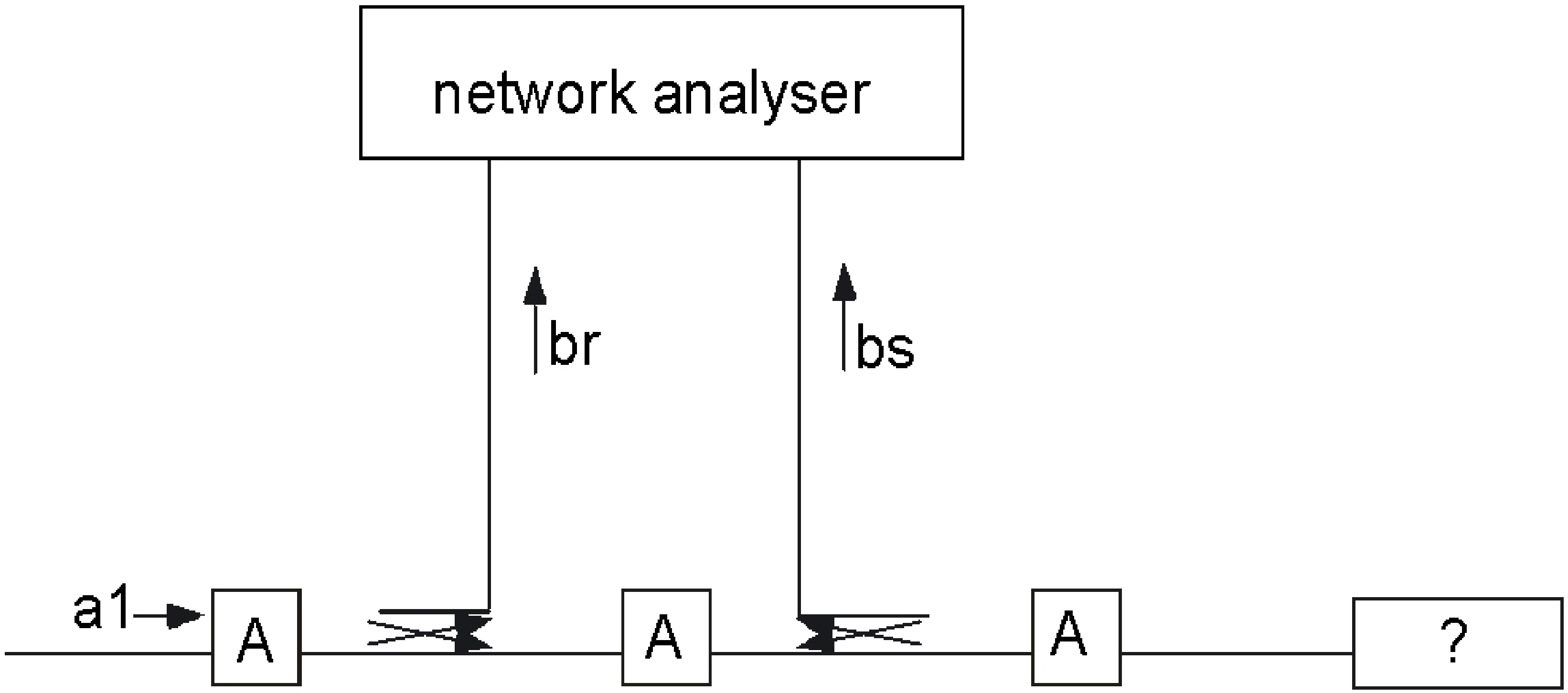
2 coupler reflectometry



2 coupler reflectometry



2 coupler reflectometry



Transmission measurements

(Attenuation measurements)

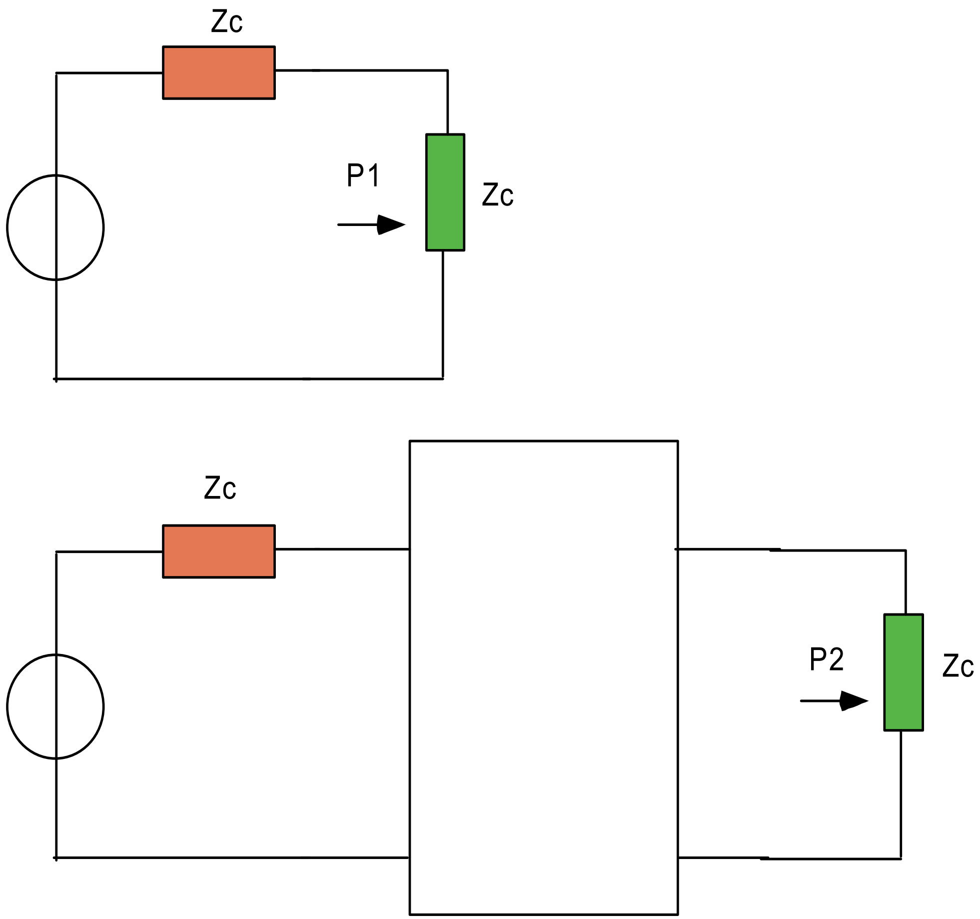
Transmission measurements

- Attenuation (very common)
- Phase (more seldom)

Causes of attenuation

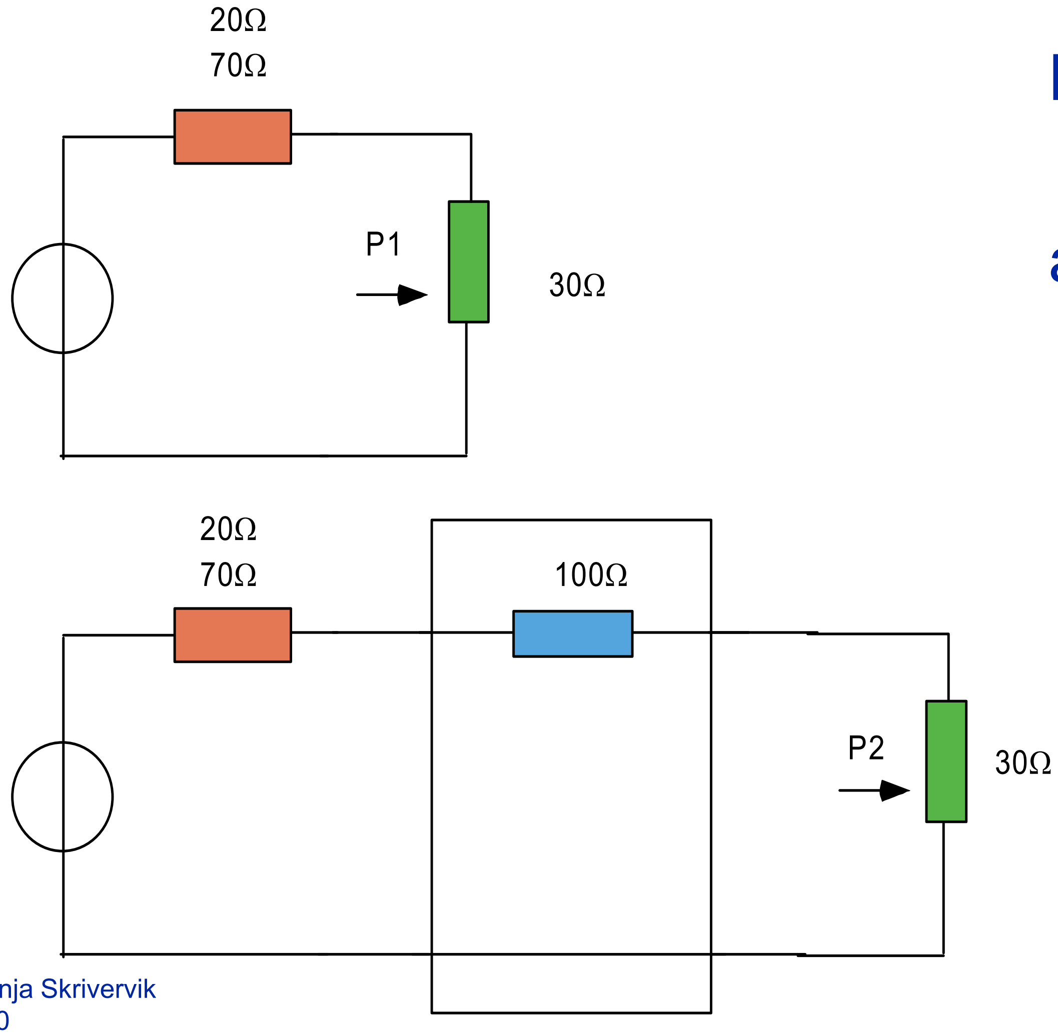
- Absorbtion in the component
- Mismatch of the component

Attenuation



LA=-10 Log(P2/P1)   dB

Attenuation : example



LA=-10 Log(P2/P1)   dB

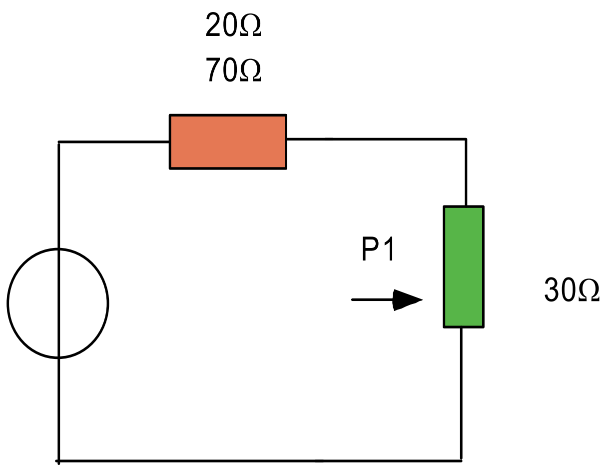
a) Rg = 20 Ω

$$P_1 = \left(\frac{100}{20+30}\right)^2 30 = 120W$$

$$P_2 = \left(\frac{100}{20+30+100}\right)^2 30 = \frac{40}{3}W$$

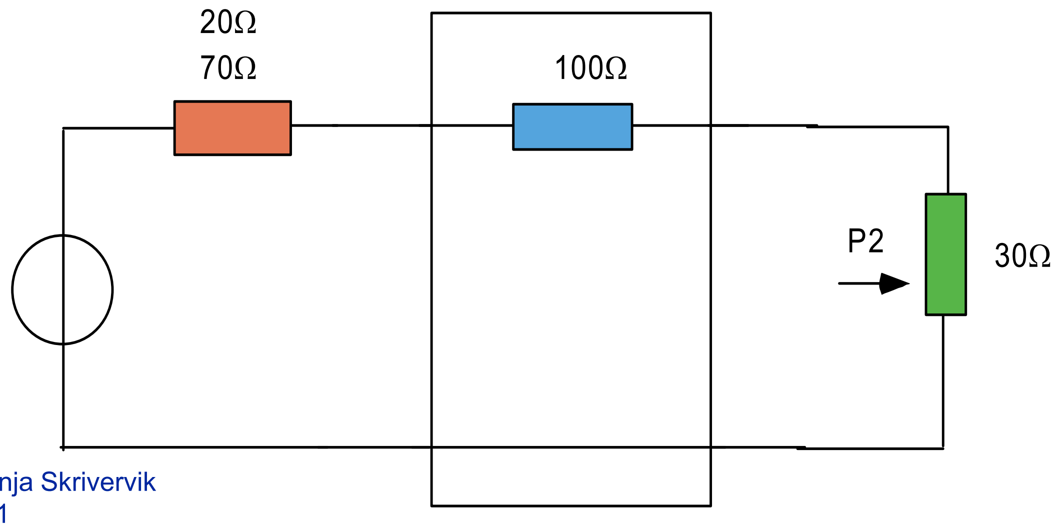
$$LA = 10Log9 = 9.5dB$$

Attenuation : example



LA=-10 Log(P2/P1)   dB

b) Rg = 70 Ω

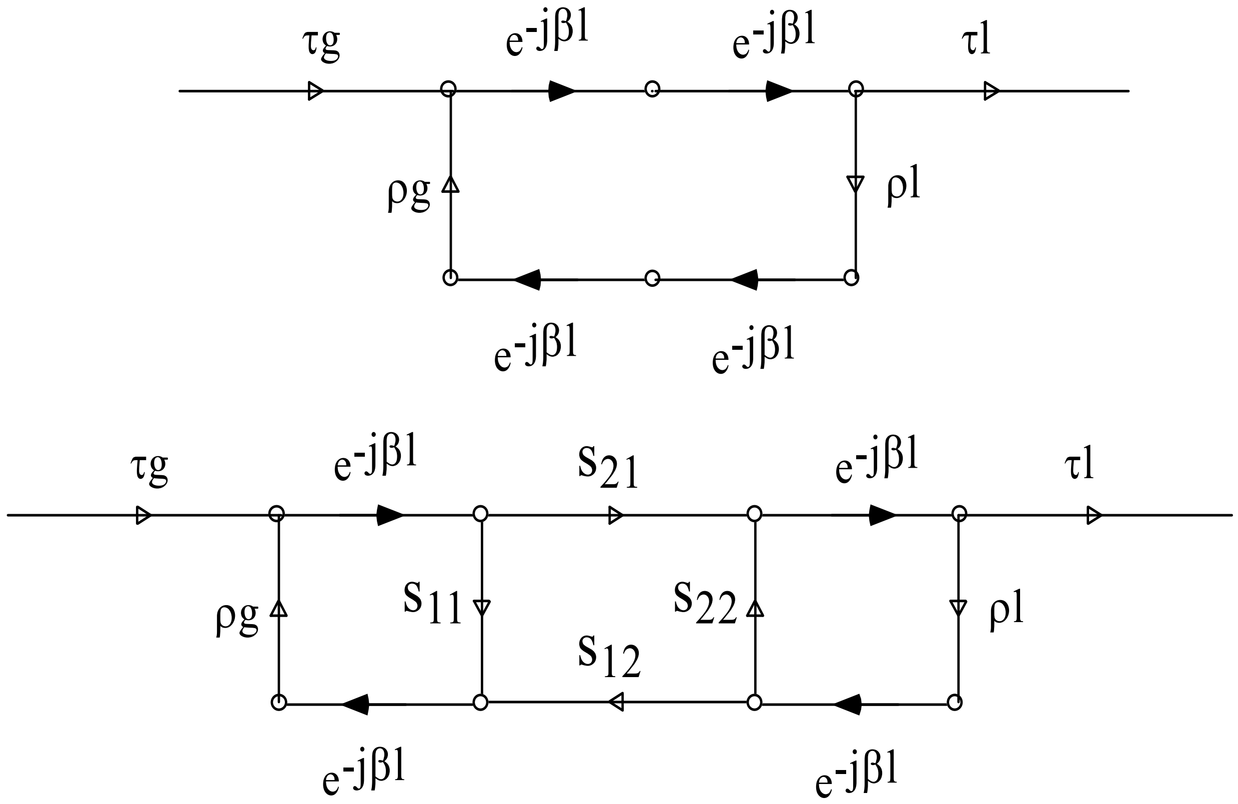


$$P_1 = \left(\frac{100}{70+30}\right)^2 30 = 30W$$

$$P_2 = \left(\frac{100}{70+30+100}\right)^2 30 = \frac{30}{4}W$$

$$LA = 10 Log 4 = 6dB$$

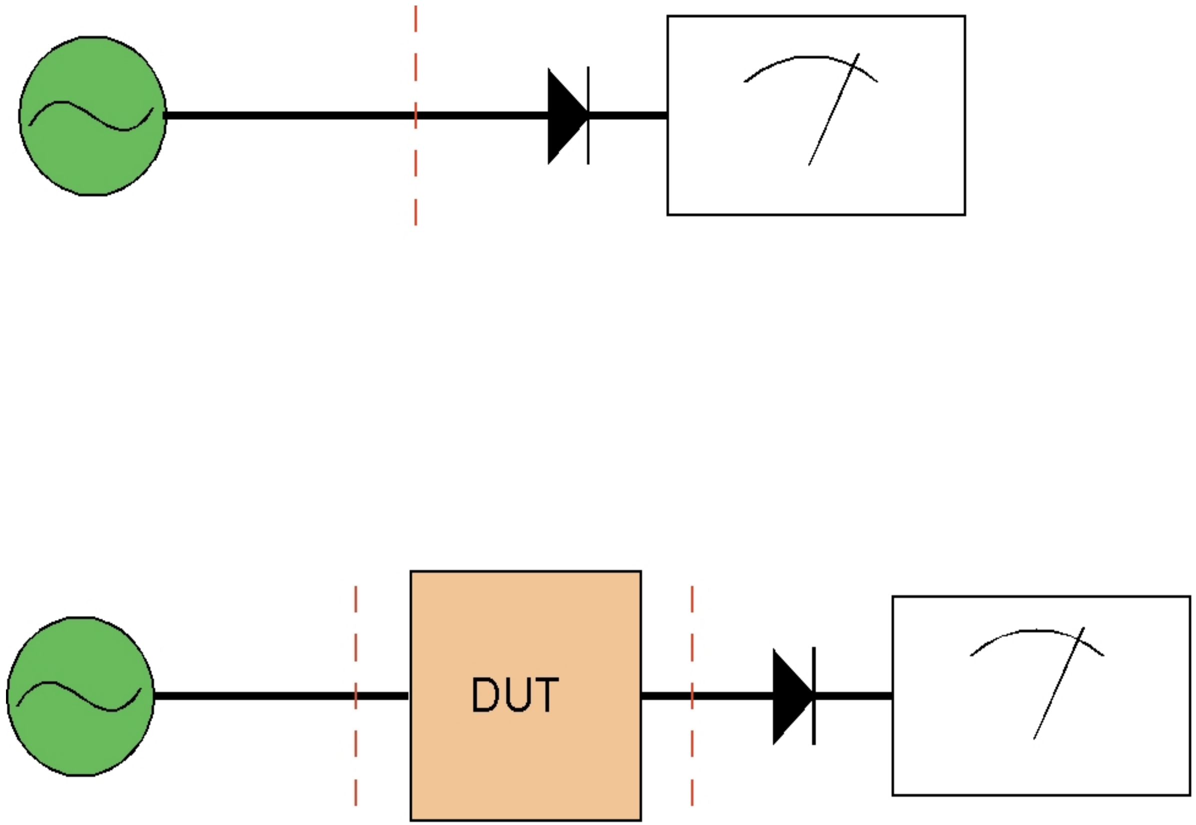
Effet od the mismatch of the source and the load



Methods

- Direct measurement
- Substitution measurement
- Measurement through reflection measurement
- Method using couplers

Direct measurement

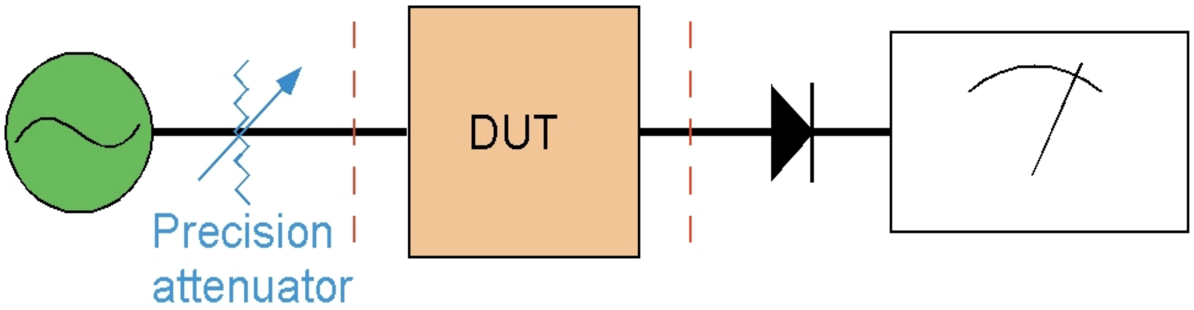


Direct measurement

- The detector should be adapted (use attnenuator or isolator)
- The detector should be quadratic (limited dynamic range)
- The connexions should be reproductible (sometimes difficult)
- The generator should deliver a signal which is stable over time

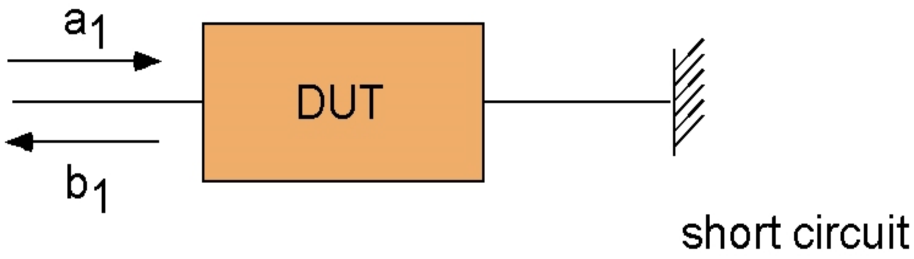
Cheap , but not very precise and quite limited !!!

Substitution measurements

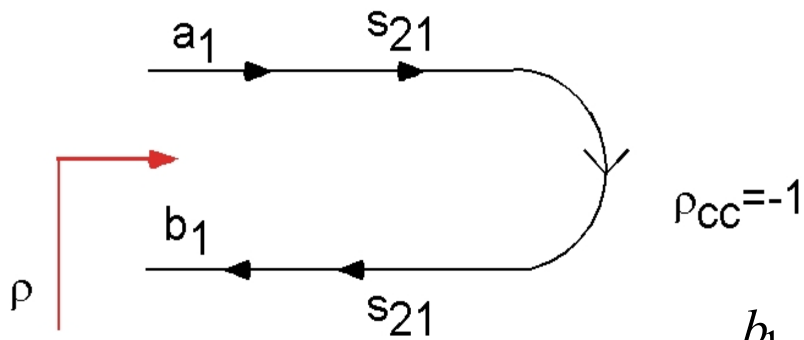


- The precision attenuator (rotative blade) allows a measurement precision od 1-2%, over a 60 dB dynamic range
- Only for wavguides
- No precision attenuators for coaxial cables

Measurement through reflection measurement



1) the device is matched but reciprocal

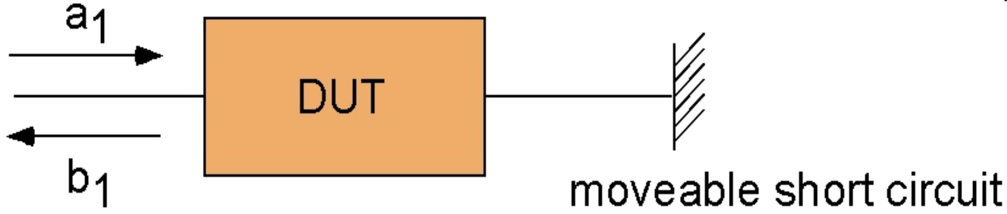


$$\rho = \frac{b_1}{a_1} = s_{21}^2 \rho_{cc} \simeq s_{21}^2$$

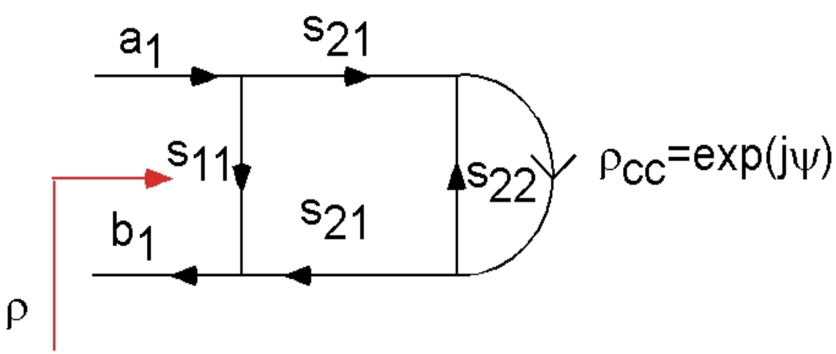
$$LA = -10\log|\rho| + 10\log|\rho_{cc}| \simeq -10\log|\rho|$$

$$\varphi = \frac{1}{2}\arg(\rho) - \frac{1}{2}\arg(\rho_{cc}) \pm n\pi \simeq \frac{1}{2}\arg(\rho) + \frac{\pi}{2} \pm n\pi$$

Measurement through reflection measurement



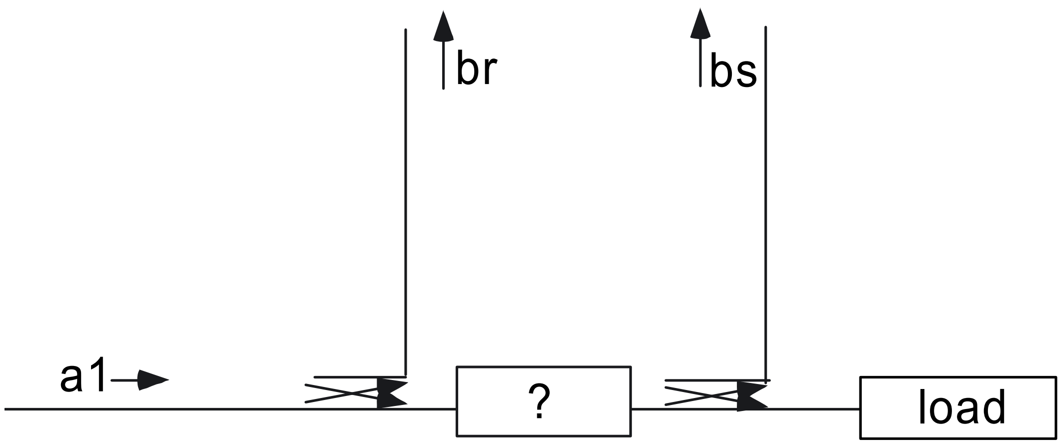
2) the device is mis-matched but reciprocal



$$\rho = \frac{b_1}{a_1} = s_{11} + \frac{s_{21}^2\, e^{-j\psi}}{1 - s_{21}\, e^{-j\psi}}$$

f(ψ) is a circle on the Smith chart

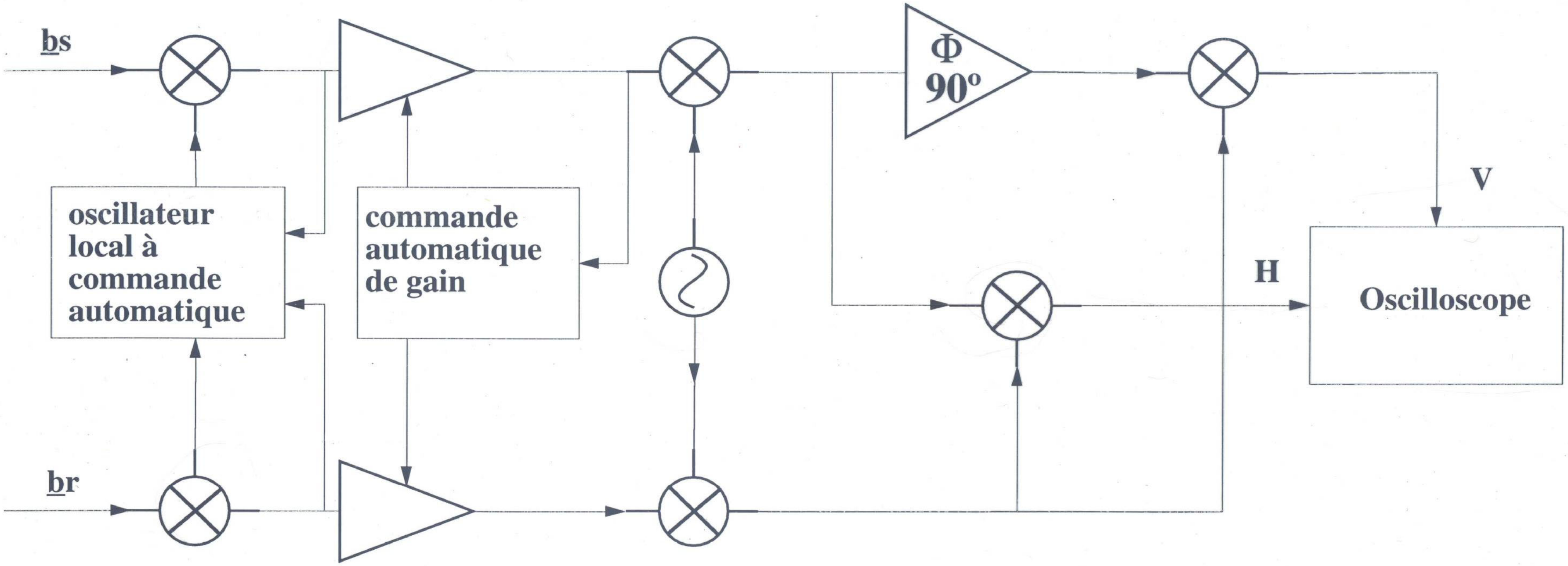
Attenuation measurement



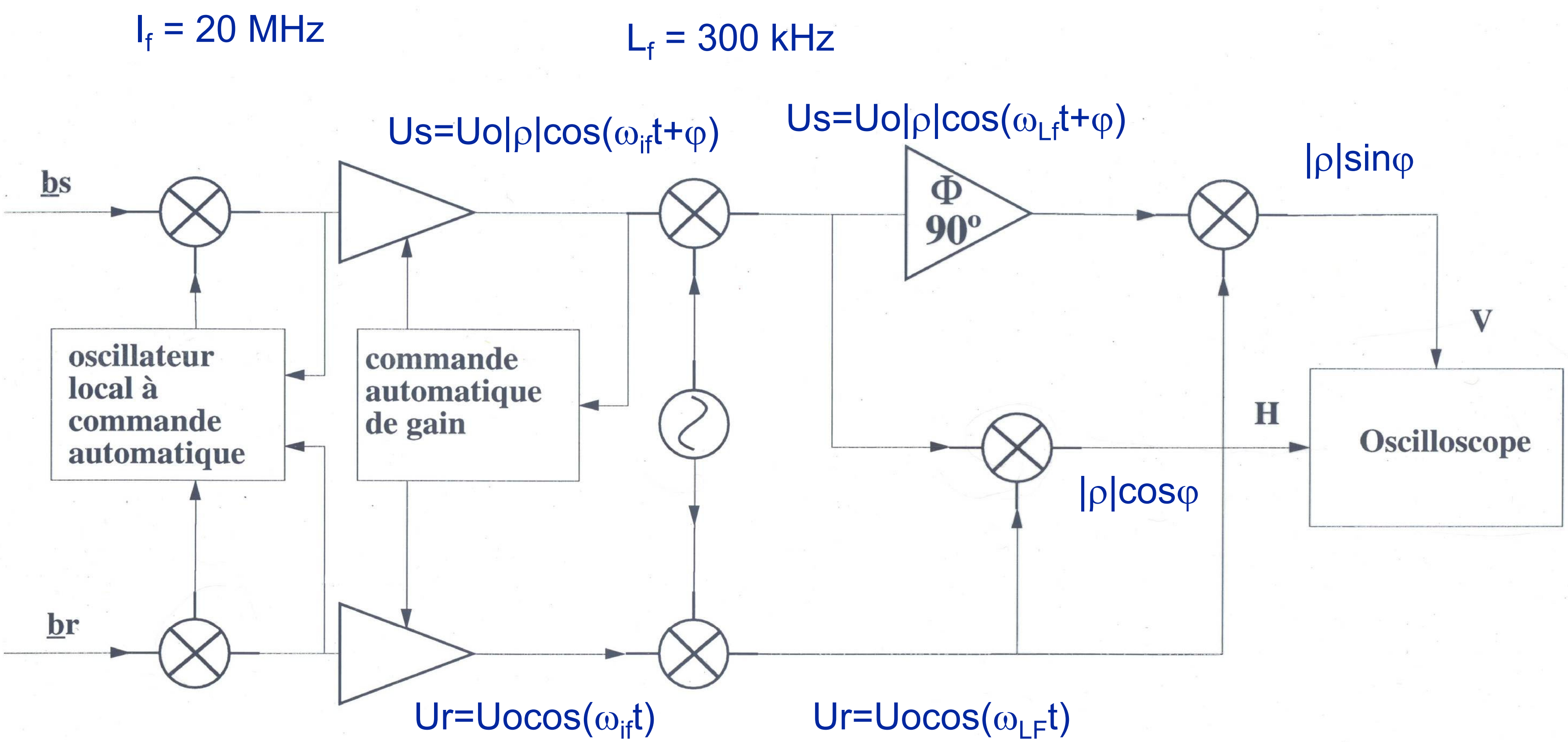
Same principle than reflectometry

Network analyser measurements

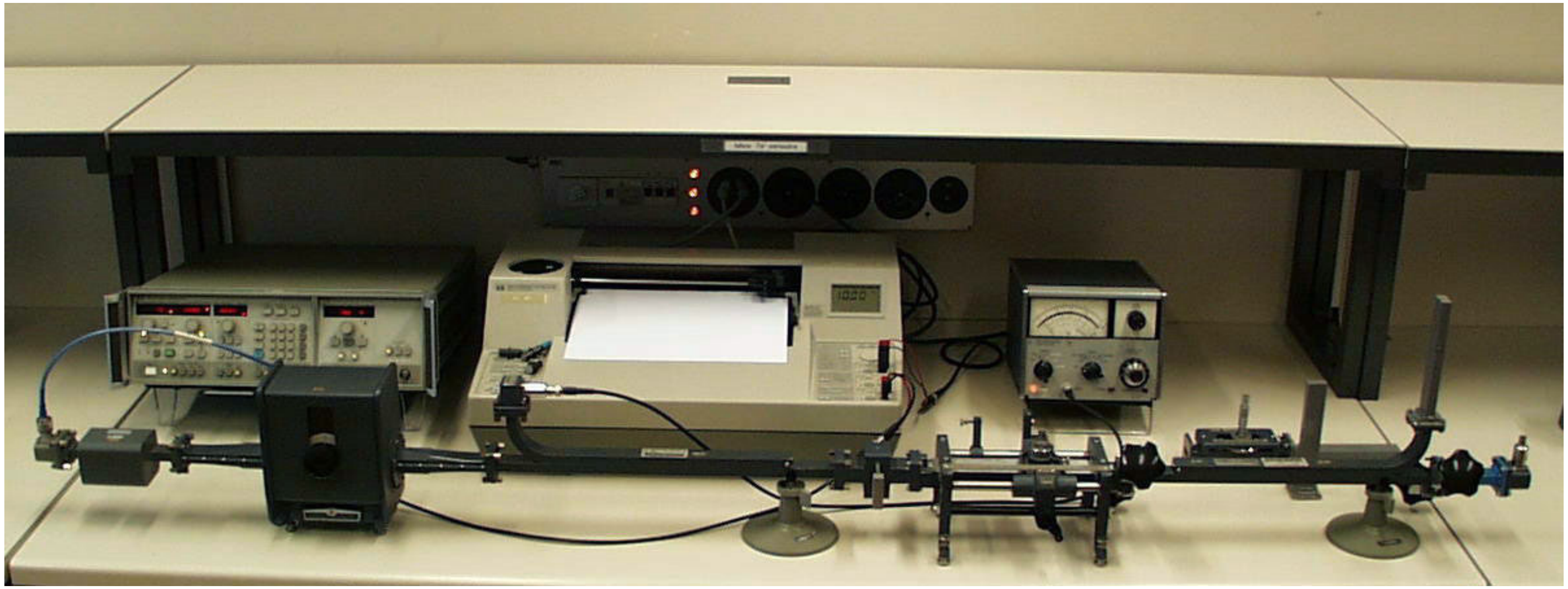
Network analyser (principle)



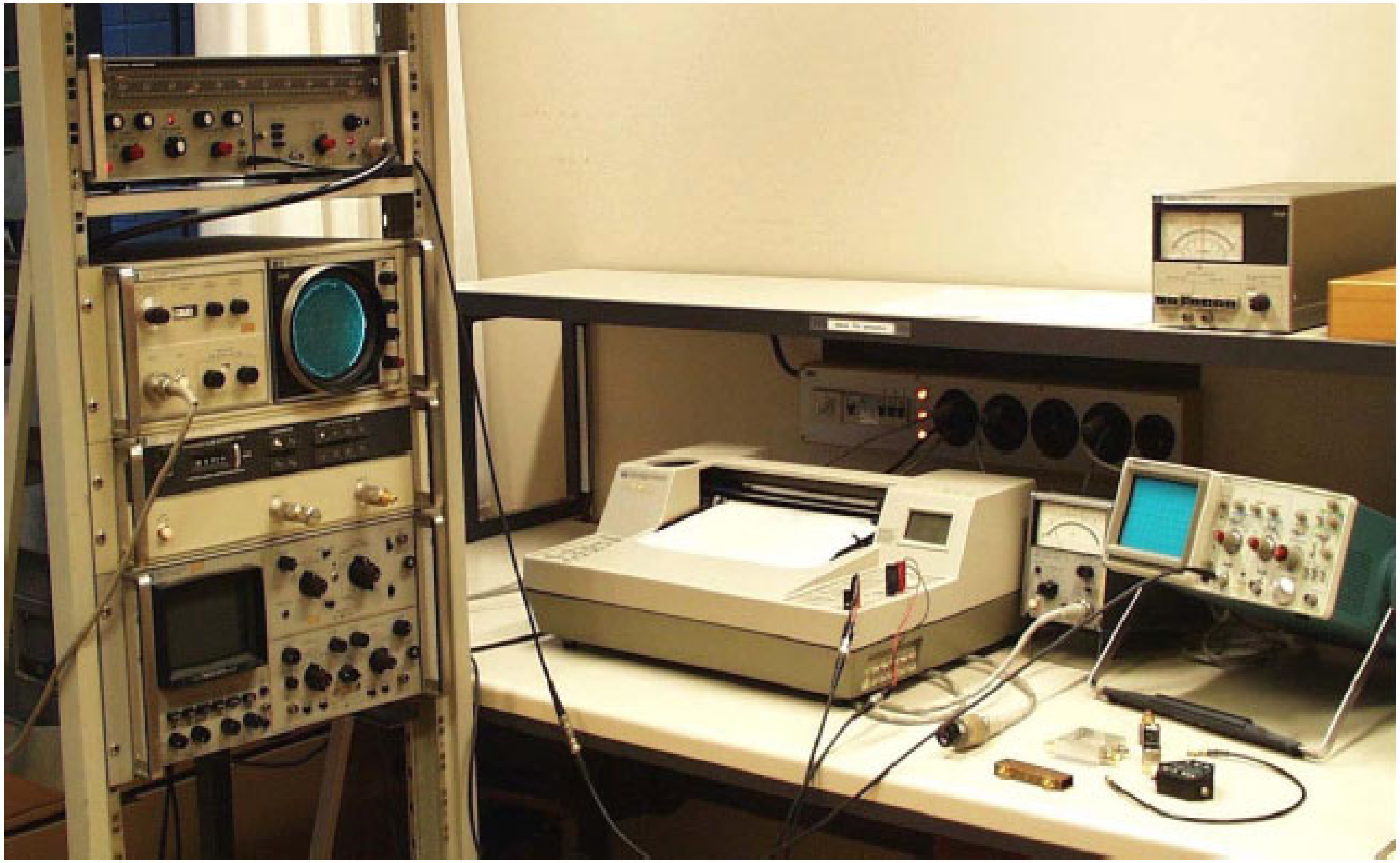
Network analyser (principle)



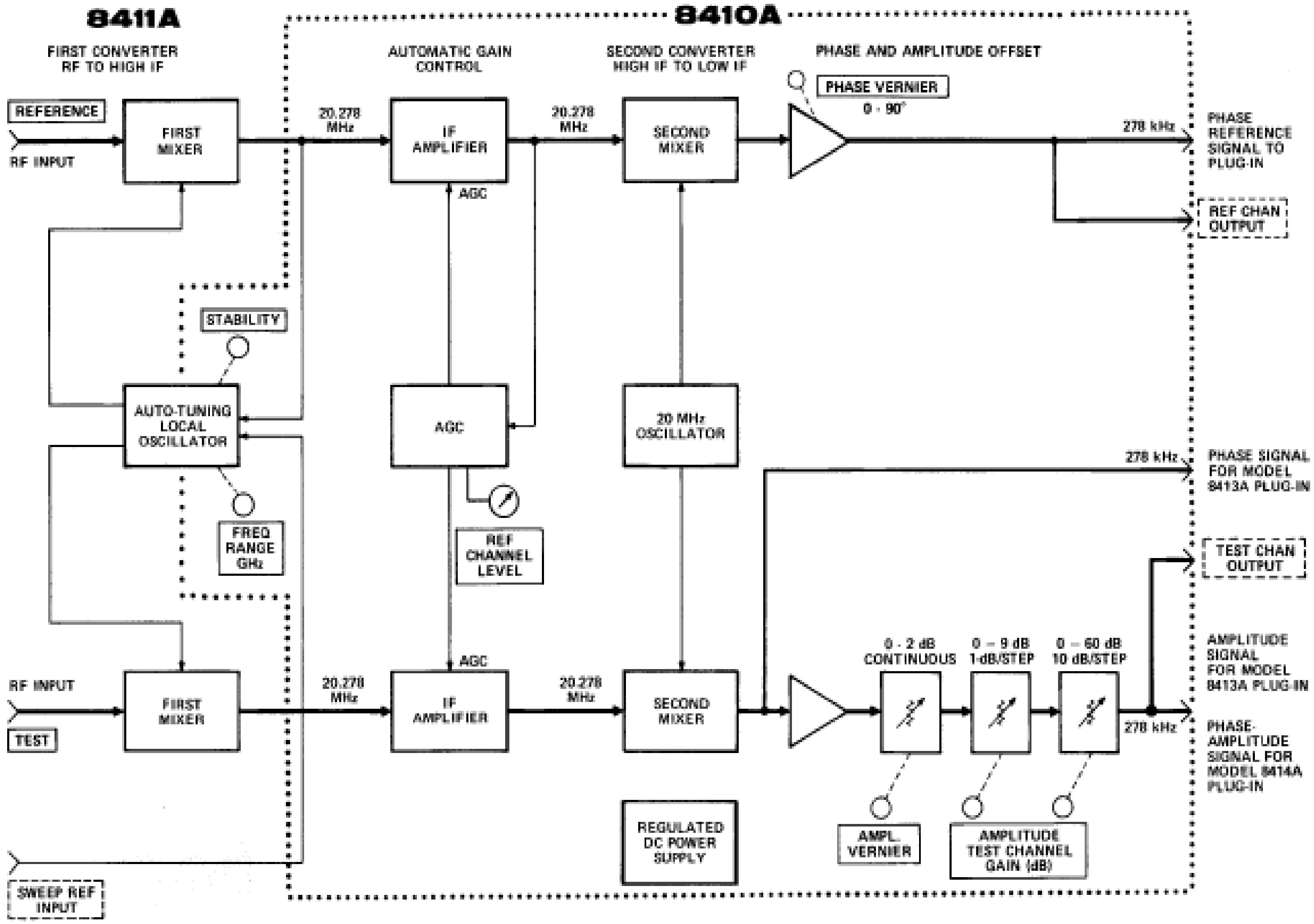
Microwave measurements



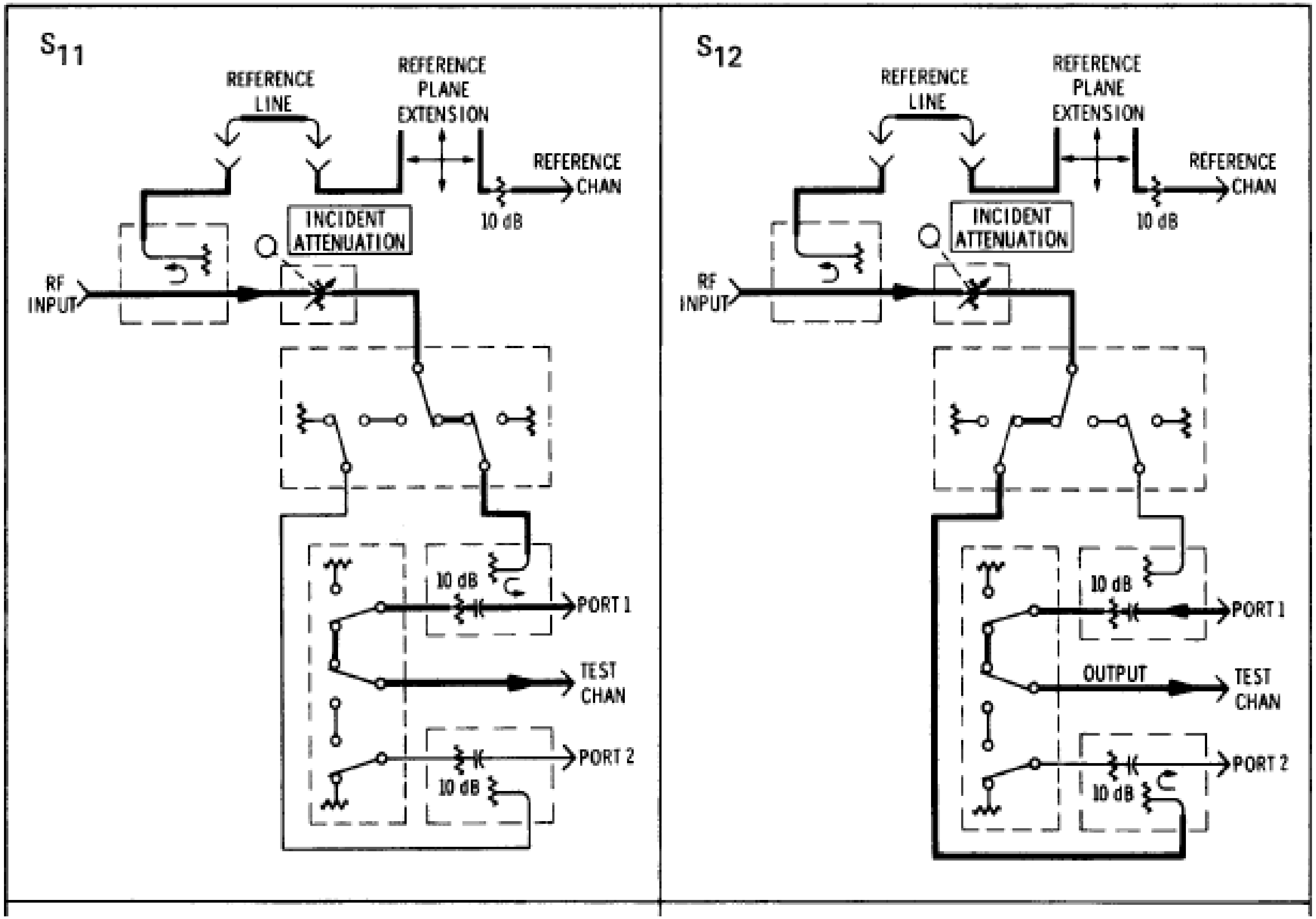
Analog NWA



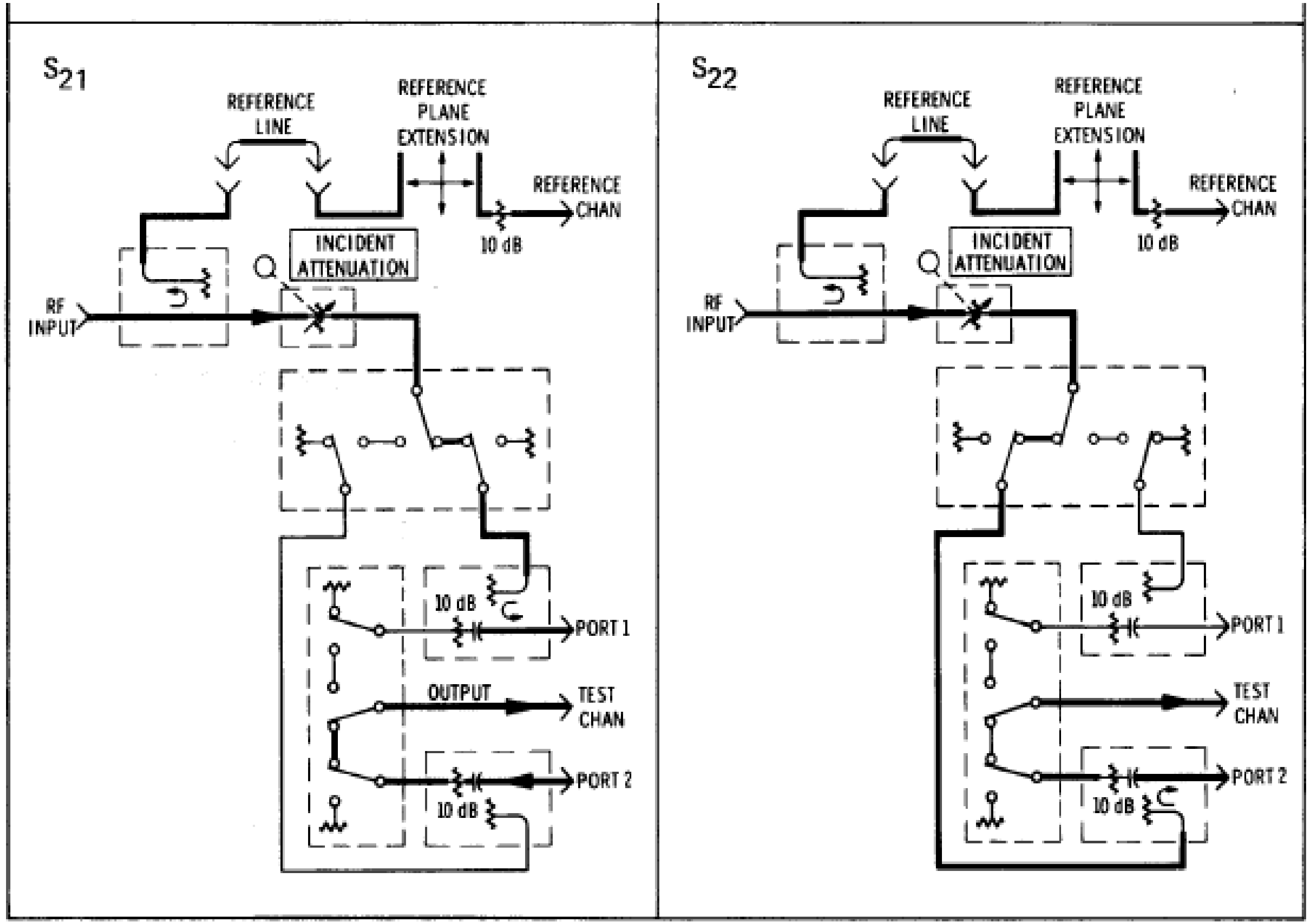
Analog NWA


8411A
8410A
FIRST CONVERTER RF TO HIGH IF
AUTOMATIC GAIN CONTROL
SECOND CONVERTER HIGH IF TO LOW IF
PHASE AND AMPLITUDE OFFSET
REFERENCE
RF INPUT
FIRST MIXER
20.278 MHz
IF AMPLIFIER
SECOND MIXER
PHASE VERNIER
0 - 90°
278 kHz
PHASE REFERENCE SIGNAL TO PLUG-IN
REF CHAN OUTPUT
AGC
STABILITY
AUTO-TUNING LOCAL OSCILLATOR
20 MHz OSCILLATOR
PHASE SIGNAL FOR MODEL 8413A PLUG-IN
FREQ RANGE GHz
REF CHANNEL LEVEL
TEST CHAN OUTPUT
0 - 2 dB CONTINUOUS
0 – 9 dB 1-dB/STEP
0 – 60 dB 10 dB/STEP
AMPLITUDE SIGNAL FOR MODEL 8413A PLUG-IN
TEST
PHASE-AMPLITUDE SIGNAL FOR MODEL 8414A PLUG-IN
REGULATED DC POWER SUPPLY
AMPL. VERNIER
AMPLITUDE TEST CHANNEL GAIN (dB)
SWEEP REF INPUT

Calibration : the old analog way

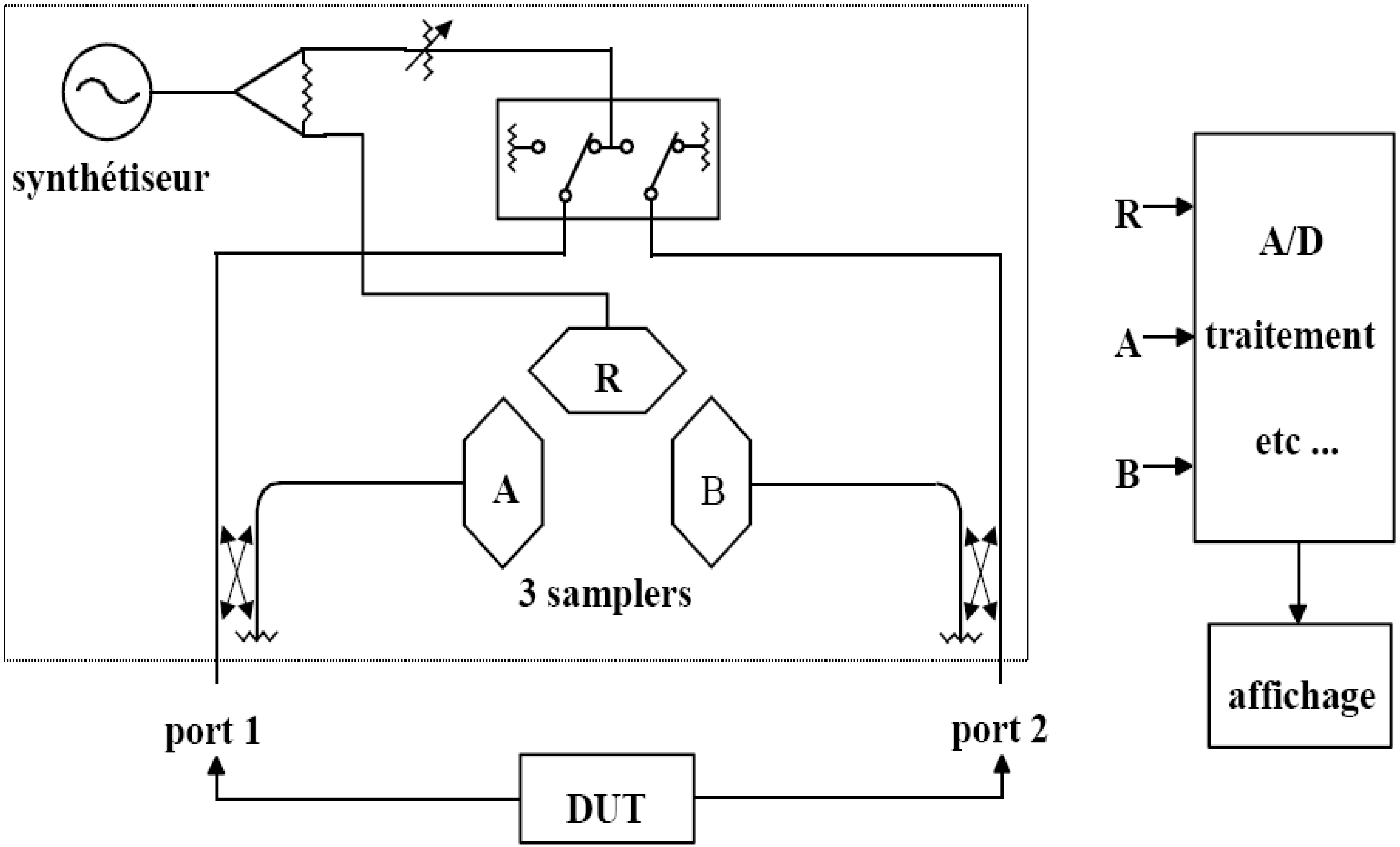


Calibration : the old analog way



Digital Network Analysor

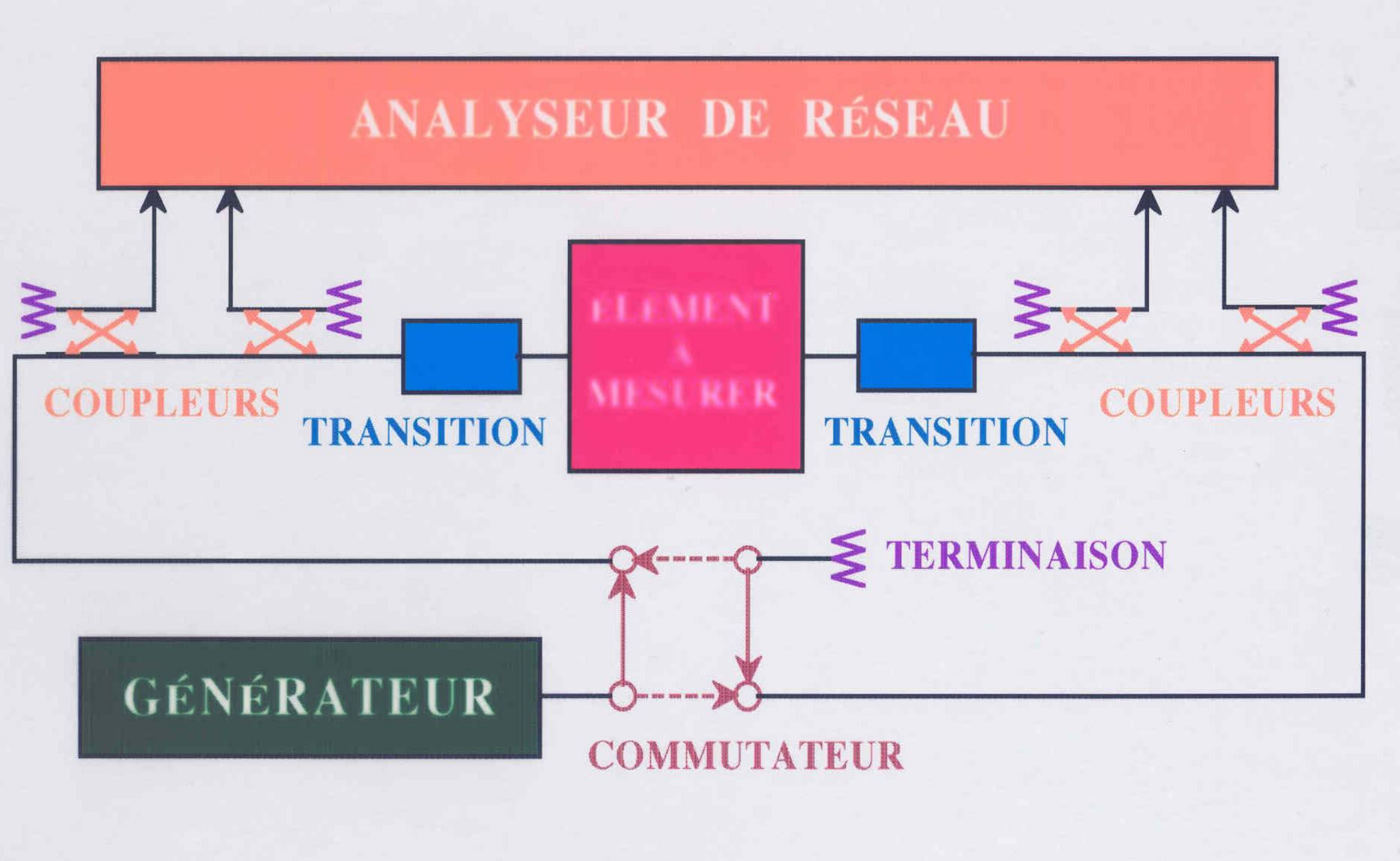
Digital Network Analysor



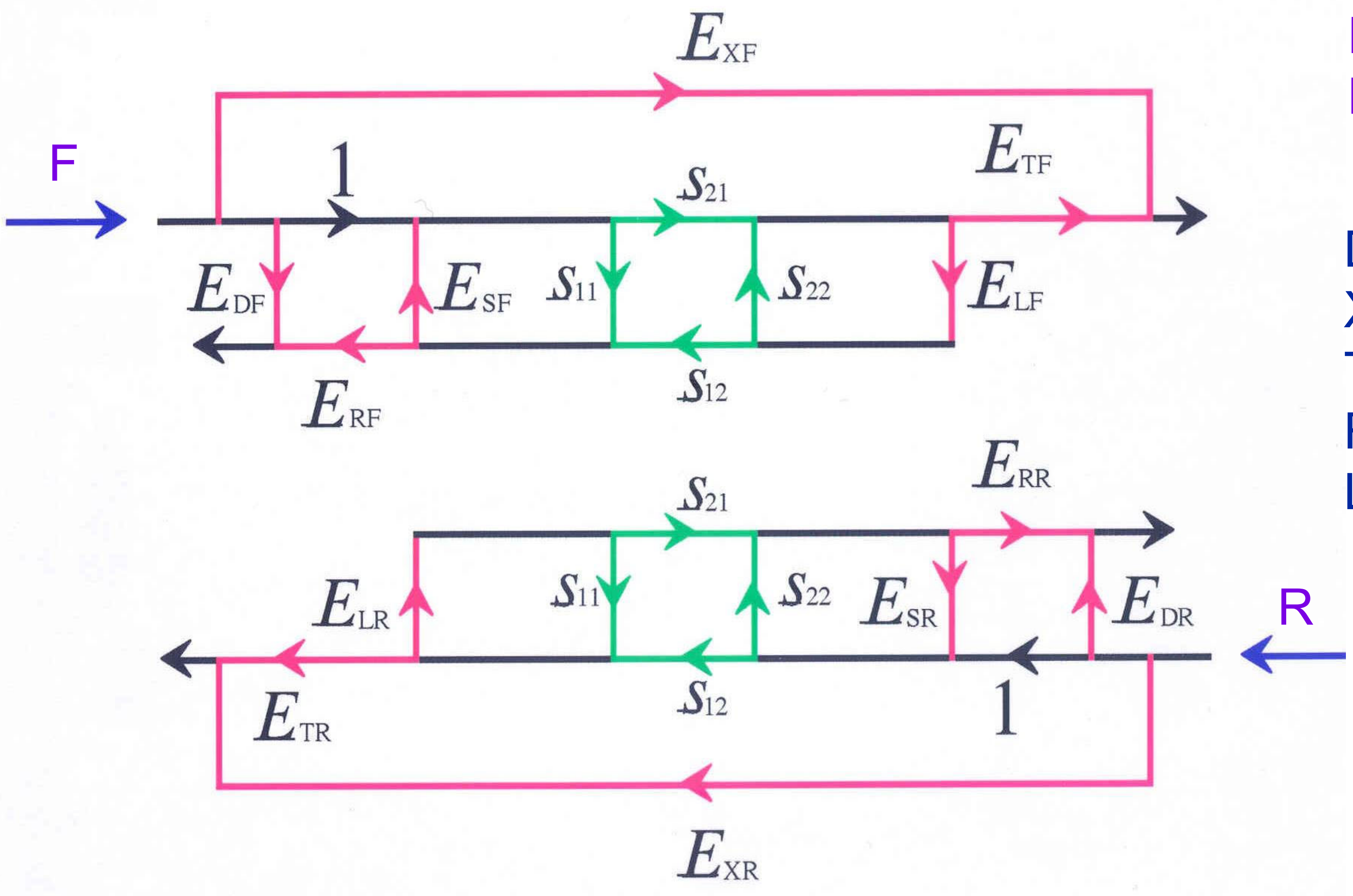
Errors

- 1. Systematic errors:
- Component imperfections
- Time invariant and predictable
- Can be characterized and taken into account mathematically during the measurements
- 2. random errors:
- vary randomly with time
- cannot be eliminated with calibration
- due to noise, connexion repeatability, cables (flexion), etc.
- 3. Errors linked to drift
- Due to variations in the instrument after calibration
- Due mainly to temperature shift
- Can be eliminated by a new calibration

Errors and calibration : the 12 error terms



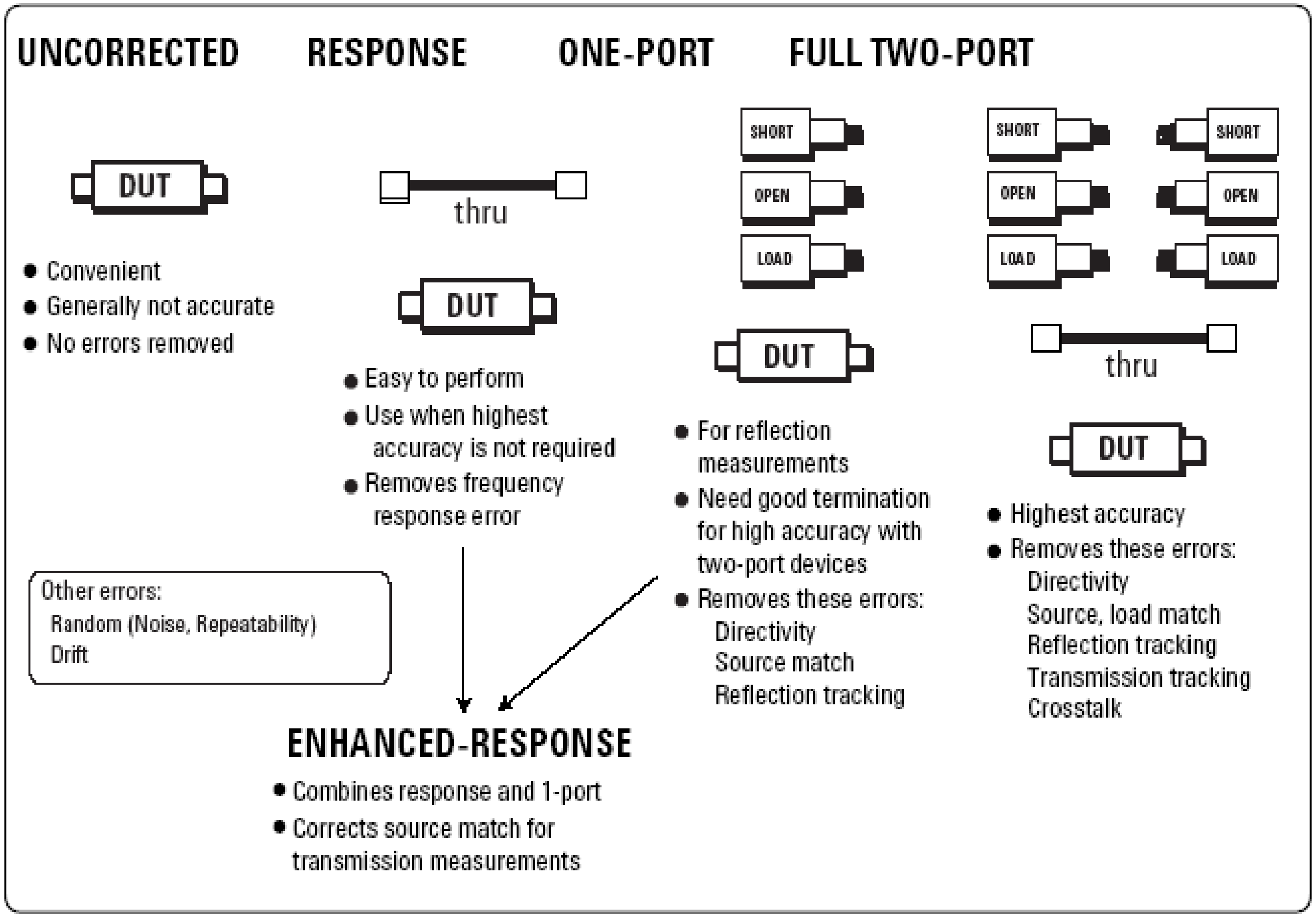
Errors and calibration : the 12 error terms



F : forward
R : reverse

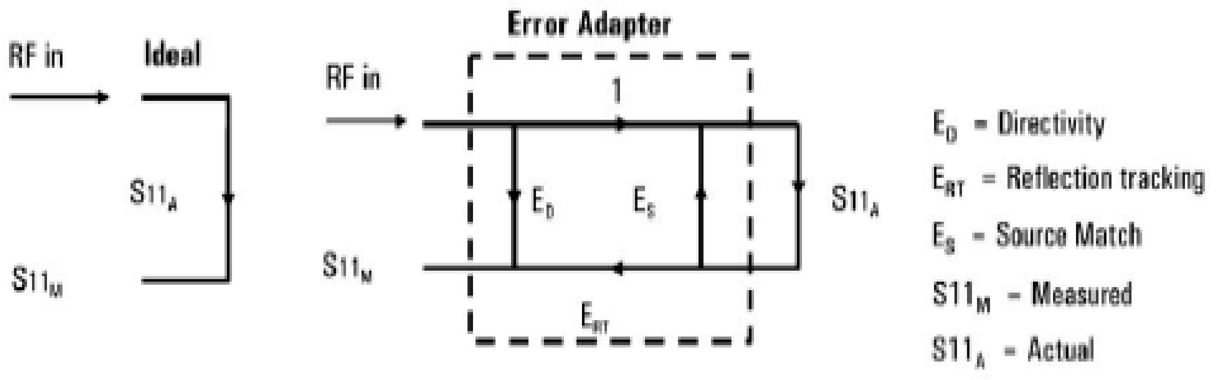
D: directivity
X : Isolation
T : transmission
R : reflexion
L : match of the load

SOLT calibration (Short, Open, Load, Trough)



source : Agilent application note Nr 5965-7709E

SOLT calibration : 1port model



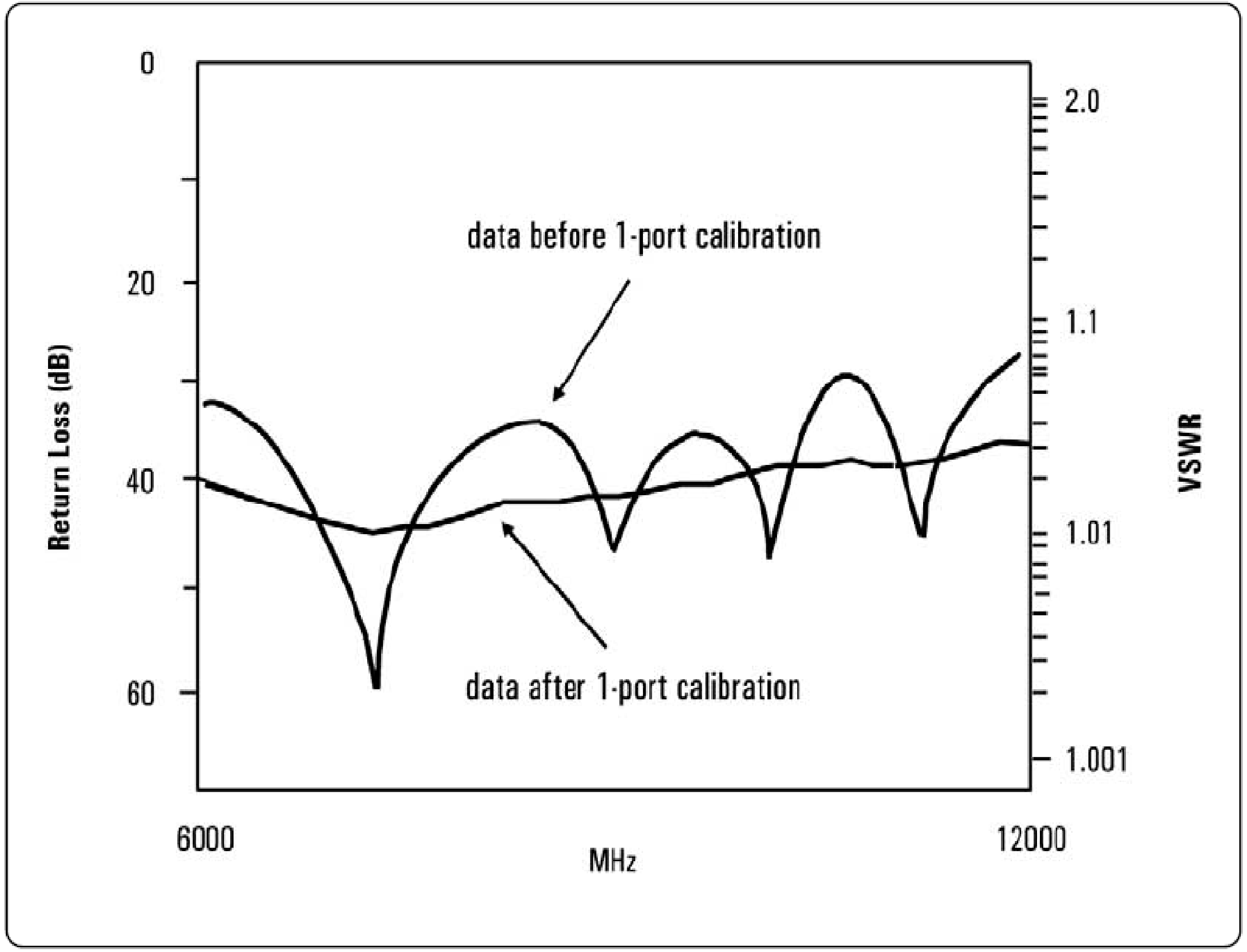
To solve for error terms, we measure 3 standards to generate 3 equations and 3 unknowns

$$S_{11M} = E_D + E_{RT}\left[\frac{S_{11A}}{1 - E_S S_{11A}}\right]$$

- Assumes good termination at port two if testing two-port devices
- If using port 2 of NA *and* DUT reverse isolation is low (e.g., filter passband):
  - assumption of good termination is not valid
  - two-port error correction yields better results

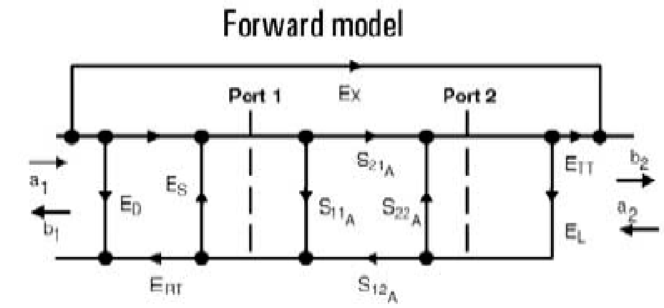
source : Agilent application note Nr 5965-7709E

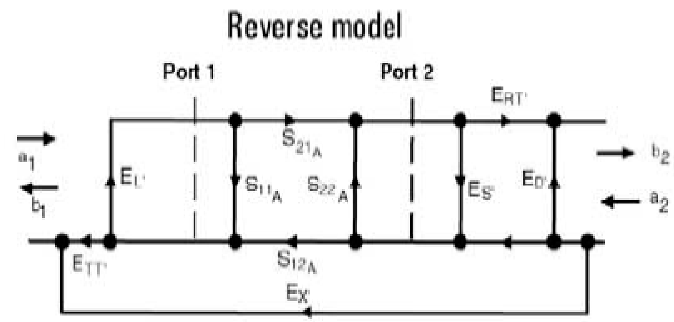
SOLT calibration : 1port model



source : Agilent application note Nr 5965-7709E

SOLT calibration : 2 port model





E_D = fwd directivity
E_S = fwd source match
E_{RT} = fwd reflection tracking
$E_{D'}$ = rev directivity
$E_{S'}$ = rev source match
$E_{RT'}$ = rev reflection tracking

E_L = fwd load match
E_{TT} = fwd transmission tracking
E_X = fwd isolation
$E_{L'}$ = rev load match
$E_{TT'}$ = rev transmission tracking
$E_{X'}$ = rev isolation

- Each actual S-parameter is a function of all four measured S-parameters
- Analyzer must make forward *and* reverse sweep to update any one S-parameter
- Luckily, you don't need to know these equations to ***use*** network analyzers!!!

$$S_{11a} = \frac{\left(\frac{S_{11m}-E_D}{E_{RT}}\right)\left(1+\frac{S_{22m}-E_D'}{E_{RT}'}E_S'\right)-E_L\left(\frac{S_{21m}-E_X}{E_{TT}}\right)\left(\frac{S_{12m}-E_X'}{E_{TT}'}\right)}{\left(1+\frac{S_{11m}-E_D}{E_{RT}}E_S\right)\left(1+\frac{S_{22m}-E_D'}{E_{RT}'}E_S'\right)-E_L'E_L\left(\frac{S_{21m}-E_X}{E_{TT}}\right)\left(\frac{S_{12m}-E_X'}{E_{TT}'}\right)}$$

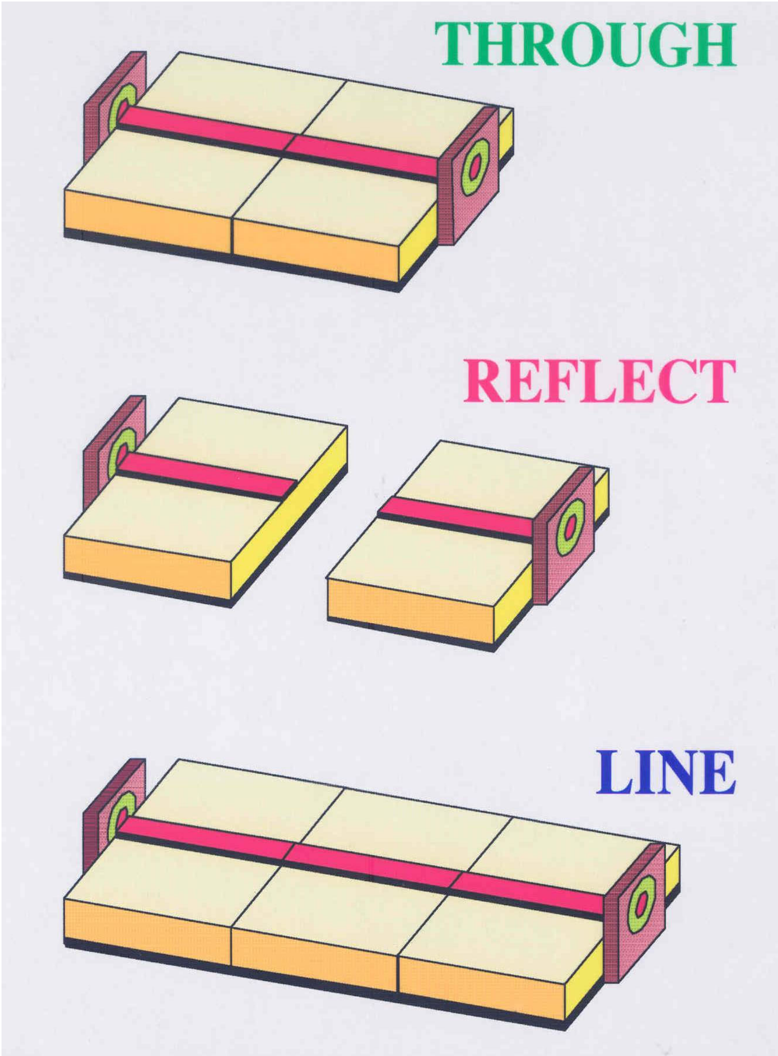
$$S_{21a} = \frac{\left(\frac{S_{21m}-E_X}{E_{TT}}\right)\left(1+\frac{S_{22m}-E_D'}{E_{RT}'}(E_S'-E_L)\right)}{\left(1+\frac{S_{11m}-E_D}{E_{RT}}E_S\right)\left(1+\frac{S_{22m}-E_D'}{E_{RT}'}E_S'\right)-E_L'E_L\left(\frac{S_{21m}-E_X}{E_{TT}}\right)\left(\frac{S_{12m}-E_X'}{E_{TT}'}\right)}$$

$$S_{12a} = \frac{\left(\frac{S_{12m}-E_X'}{E_{TT}'}\right)\left(1+\frac{S_{11m}-E_D}{E_{RT}}(E_S-E_L')\right)}{\left(1+\frac{S_{11m}-E_D}{E_{RT}}E_S\right)\left(1+\frac{S_{22m}-E_D'}{E_{RT}'}E_S'\right)-E_L'E_L\left(\frac{S_{21m}-E_X}{E_{TT}}\right)\left(\frac{S_{12m}-E_X'}{E_{TT}'}\right)}$$

$$S_{22a} = \frac{\left(\frac{S_{22m}-E_D'}{E_{R}'}\right)\left(1+\frac{S_{11m}-E_D}{E_{RT}}E_S\right)-E_L'\left(\frac{S_{21m}-E_X}{E_{TT}}\right)\left(\frac{S_{12m}-E_X'}{E_{TT}'}\right)}{\left(1+\frac{S_{11m}-E_D}{E_{RT}}E_S\right)\left(1+\frac{S_{22m}-E_D'}{E_{RT}'}E_S'\right)-E_L'E_L\left(\frac{S_{21m}-E_X}{E_{TT}}\right)\left(\frac{S_{12m}-E_X'}{E_{TT}'}\right)}$$

source : Agilent application note Nr 5965-7709E

TRL (Trough, Reflect, Line) calibration



- Used for non coaxial or non waveguide measurements
- 16 measurements for 12 unkown parameters
- => redundancy ! We need not to know the standarts very precisely

+ measurements of incident signals
+ measurements of isolations with matched loads