Introduction to Graphical model, exercices

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1. **Party Animal**.
   The party animal problem takes into account the following variables:
   - P: been to party
   - H: Got a headache
   - D: Demotivated at work
   - U: Underperform at work
   - A: Boss angry

   The distribution over these variables factorizes as:

   $$p(P, H, D, U, A) = p(P)p(D)p(U|P, D)p(H|P)p(A|U)$$

   The specifications of the individual CPD tables are:

   $$\begin{array}{ll} p(U = tr|P = tr, D = tr) = 0.999 & p(U = tr|P = fa, D = tr) = 0.9 \\ p(U = tr|P = tr, D = fa) = 0.9 & p(U = tr|P = fa, D = fa) = 0.001 \\ p(H = tr|P = tr) = 0.9 & p(H = tr|P = fa) = 0.2 \\ p(A = tr|U = tr) = 0.95 & p(A = tr|U = fa) = 0.5 \\ p(P = tr) = 0.2 & p(D = tr) = 0.4 \end{array}$$

   (a) Draw the graphical model corresponding to this problem.

   (b) Given that the boss is angry and that the worker has a headache, what is the probability that the worker has been to the party ?

2. **Asbestos**.
   There is a synergistic relationship between Asbestos (A) exposure, Smoking (S) and Cancer (C). A model describing this relationship is given by

   $$p(A, S, C) = p(C|A, S)p(A)p(S)$$

   (a) Is $A \perp\!\!\!\perp S$? Is $A \perp\!\!\!\perp S|C$?

   (b) How could you adjust the model to account for the fact that people who work in the building industry have a higher likelihood to also be smokers and also a higher likelihood to be exposed to Asbestos ?

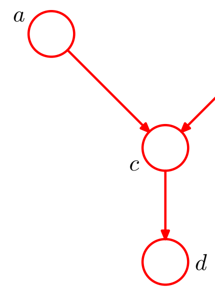

Figure 1: Graphical model. Exploring the conditional independence properties.

3. **Conditional independence**.
In the course, we have studied the explaining away effect of the 'head-to-head' graphical model. Here we show that it still holds when the observed variable is a descendent of the head-to-head configuration. Consider the directed graphical model shown in Fig. 1, in which none of the variables is observed. Show that $a \perp\!\!\!\perp b$ (hint see how the demonstration was shown in the course for the canonical model 3).

(Bonus. Not graded). Suppose now that we observe d. Show that in general $a \not\perp\!\!\!\perp b|d$.

4. **Chest Clinic Network.**[1]
This network concerns the diagnosis of lung disease (tuberculosis, lung cancer, or both, or neither). In this model, a visit to Asia is assumed to increase the probability of tuberculosis. The involved variables are:

- x: positive X-ray
- d: Dyspnea (shortness of breath)
- e: either tuberculosis or lung cancer
- t: Tuberculosis
- l: Lung cancer
- b: Bronchitis
- a: visit to Asia
- s: Smoker

According to this model, the distribution factorizes as:

$$p(x, d, e, t, l, b, a, s) = p(a)p(s)p(t|a)p(l|s)p(b|s)p(d|b, e)p(e|t, l)p(x|e)$$

(a) Draw the graphical model associated with this problem.

(b) State whether the following conditional independence relationships are true of false, and explain why.

- tuberculosis $\perp\!\!\!\perp$ smoking | shortness of breath

[1] S. Lauritzen and D. Spiegelhalter. Local computation with probabilities on graphical structures and their application to expert systems. *Jl of Royal Statistical Society B,* 1988.

- lung cancer $\perp\!\!\!\perp$ bronchitis | smoking
- visit to Asia $\perp\!\!\!\perp$ smoking | lung cancer
- visit to Asia $\perp\!\!\!\perp$ smoking | lung cancer, shortness of breath

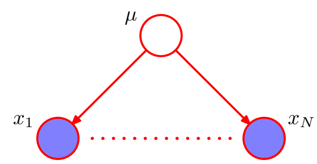

Figure 2: Inference of a Gaussian mean.

5. **Estimation of a Gaussian mean.** Consider the graphical model of Fig. 2, characterized by the following generative process:

$$\boldsymbol{\mu} \sim \mathcal{N}(\boldsymbol{\mu}|\boldsymbol{\mu}_0, \Sigma_0) \tag{1}$$
$$\mathbf{x}_i \sim \mathcal{N}(\mathbf{x}|\boldsymbol{\mu}, \Sigma), \forall i \tag{2}$$

and let us denote $\mathcal{X} = \{\mathbf{x}_i, i = 1 \dots N\}$, and $\Theta = (\boldsymbol{\mu}_0, \Sigma_0, \Sigma)$.

(a) (Bonus) Draw the graphical model using the plate notation (see course appendix and bishop's book)

(b) Using d-separation, state whether $\mathbf{x}_i \perp\!\!\!\perp \mathbf{x}_j \mid \boldsymbol{\mu}$ holds true or not. Similarly, do we have $\mathbf{x}_i \perp\!\!\!\perp \mathbf{x}_j$ in general?

(c) Without computation and exploiting the course, state what is the type of the distribution of $p(\boldsymbol{\mu}|\mathcal{X})$?

(d) Bonus - not graded. Suppose now that the $\mathbf{x}$ are one dimensionnal, so that $\boldsymbol{\mu}_0 = \mu_0, \Sigma_0 = \sigma_0, \Sigma = \sigma$. Compute analytically the parameters defining the distribution $p(\mu|\mathcal{X})$.